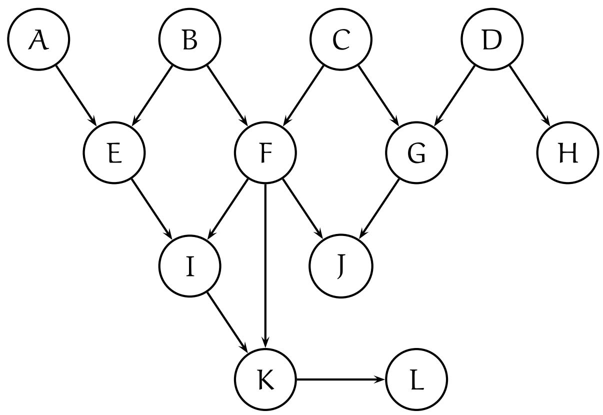
Bayesian Networks and Decision Graphs: Home Assignments #1

28 April 2005

Question 1

Consider the causal network below:



a) What probability distributions (e.g. on the form $P(X|Y)$) should be specified in order to obtain a Bayesian network from the causal network?

b) Which variables are d-separated from A given hard evidence on K?

c) Which variables are d-separated from A given hard evidence on F and I?

Answers

a) $P(A)$, $P(B)$, $P(C)$, $P(D)$, $P(E|A, B)$, $P(F|B, C)$, $P(G|C, D)$, $P(H|D)$, $P(I|E, F)$, $P(J|F, G)$, $P(K|I, F)$, $P(L|K)$.

b) D, H, L.

c) K, L, D, H.

Question 2

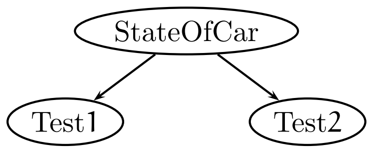
A used car sales man offers all potential costumers to have a test performed on the car they are interested in buying. The test should reveal whether the car has either no defects or one (or more) defects; the prior probability that a car has one or more defects is 0.3. There are two possible tests: Test1 has three possible outcomes, namely no-defects, defects and inconclusive. If the car doesn't have any defects, then the probabilities for these test results are 0.8, 0.05 and

0.15, respectively. On the other hand, if the car has defects, then the probabilities for the test results are 0.05, 0.75 and 0.2. For Test2 there are only two possible outcomes (no-defects and defects). If the car doesn't have any defects, then the probabilities for the test results are 0.8 and 0.2, respectively, and the if the car has defects then the probabilities are 0.25 and 0.75.

a) Construct a Bayesian network (both structure and probabilities) representing the relations between the two tests and the state of the car.

b) Calculate the probabilities $\mathsf{P}(\text{StateOfCar}|\text{Test1})$ and $\mathsf{P}(\text{Test1})$.

Answer for question a



The variable StateOfCar (SOC) is associated with $\mathsf{P}(\text{SOC}) = (0.7, 0.3)$ and for the other two variables we have:

		Test1		
		$\neg$d	d	inc.
SOC	$\neg$d	0.8	0.05	0.15
	d	0.05	0.75	0.2

		Test2	
		$\neg$d	d
SOC	$\neg$d	0.8	0.2
	d	0.25	0.75

Answer for question b

We have:

$$\mathsf{P}(\text{SOC}|\text{Test1}) = \frac{\mathsf{P}(\text{SOC}, \text{Test1})}{\mathsf{P}(\text{Test1})} = \frac{\mathsf{P}(\text{Test1}|\text{SOC})\mathsf{P}(\text{SOC})}{\sum_{\text{SOC}} \mathsf{P}(\text{Test1}|\text{SOC})\mathsf{P}(\text{SOC})}$$

Using the tables we get (for $\mathsf{P}(\text{SOC}, \text{Test1})$):

		Test1		
		$\neg$d	d	inc.
SOC	$\neg$d	0.56	0.035	0.105
	d	0.015	0.225	0.06

$\mathsf{P}(\text{SOC}, \text{Test1})$

Hence, $\mathsf{P}(\text{Test1}) = (0.575, 0.26, 0.165)$ and finally:

		Test1		
		$\neg$d	d	inc.
SOC	$\neg$d	0.974	0.135	0.636
	d	0.026	0.865	0.364

$\mathsf{P}(\text{SOC}|\text{Test1})$