# Consensus

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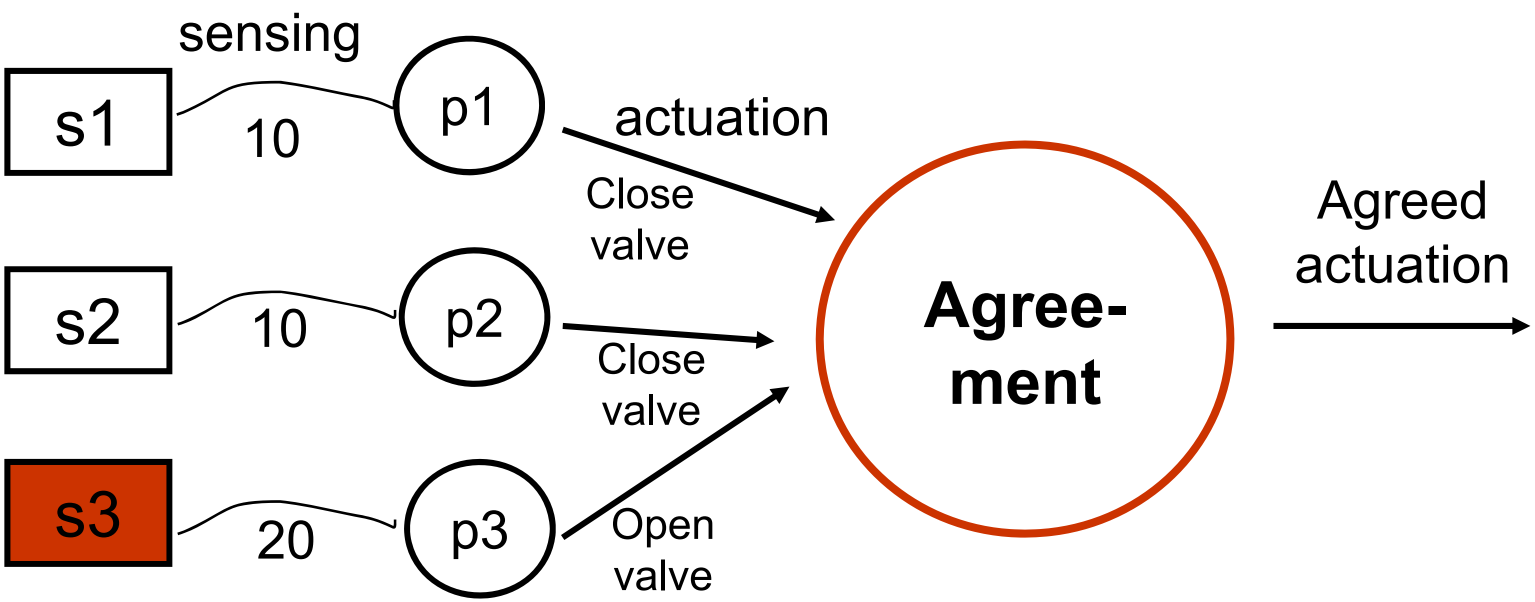
# Consensus problems

- Examples
  - Mutex: which process is granted access
  - Reliable and ordered Multicast
  - Election
  - Abort/proceed in space shuttle launch
  - Consistent credit/debit bank account
- Fault Tolerance
  - Crash, Omission
  - Byzantine (Arbitrary) failures
  - No message signing
    - Message signing limits the harm a faulty process can do
- Problems
  - Consensus
  - Byzantine generals
  - Interactive consistency

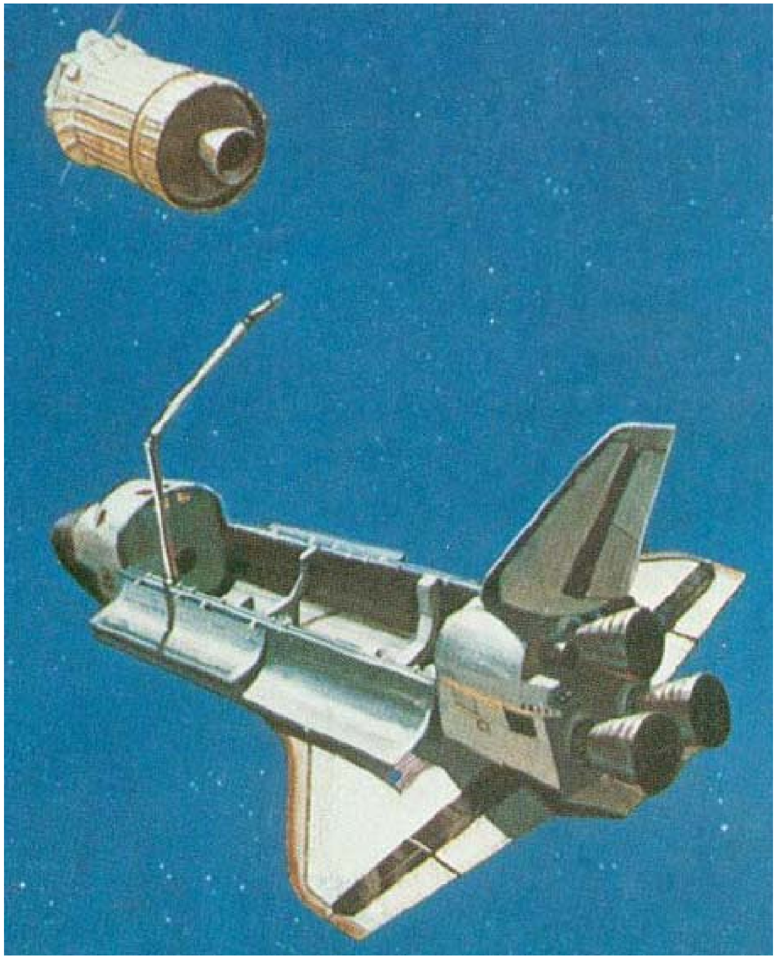
# Redundancy

- •Components (sensors / memory / processors/processes) may fail
- •Critical systems: space / nuclear / train control
- •Increase availabiliy ⇒ Dublicate components/functionality

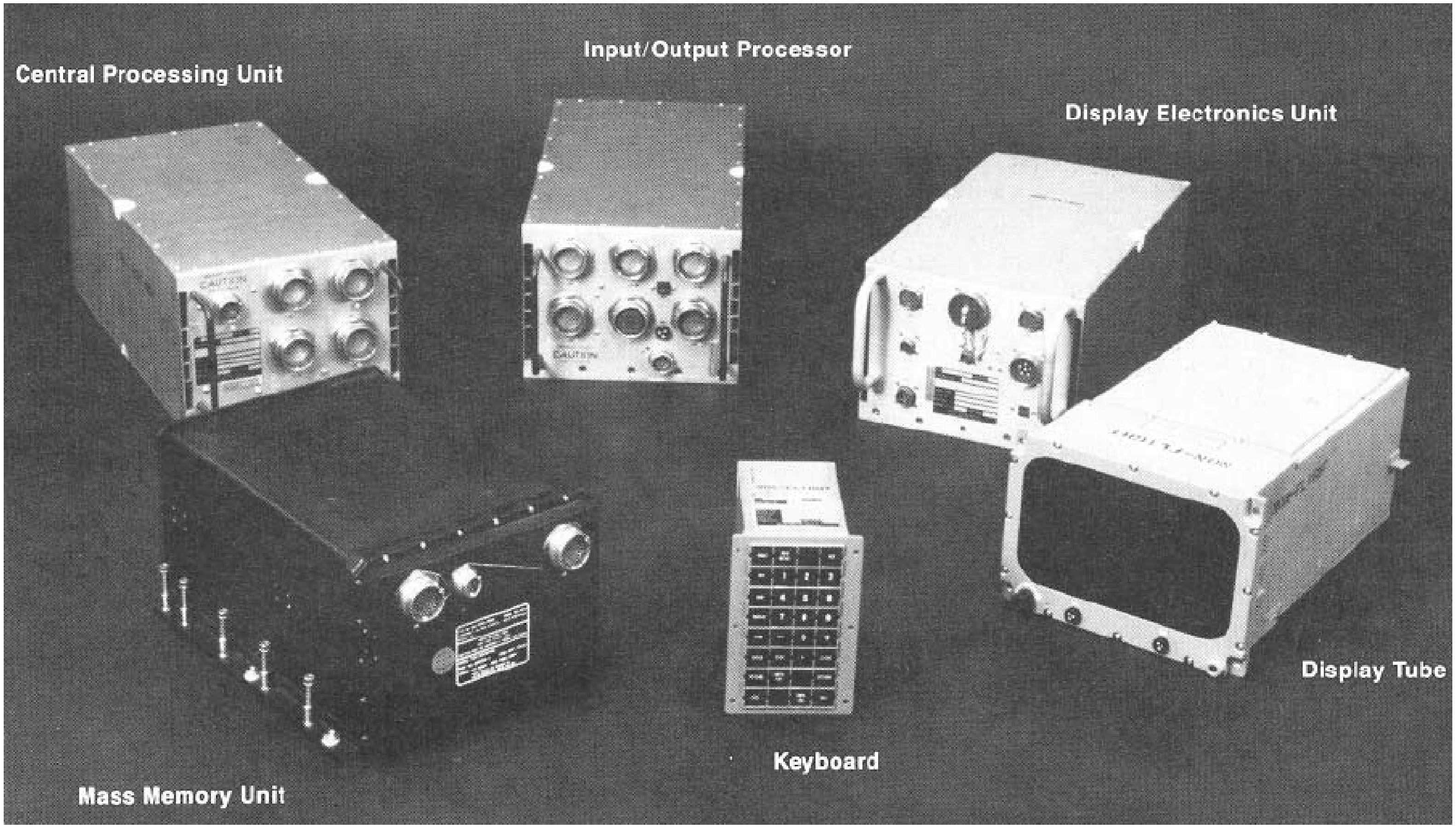


# Example

- The PASS (Primary Avionics Software System) developed by IBM in 1981, was used in a space shuttle
  - Could have been done on one computer
  - But ***4*** separate processors were used for fault-tolerance
    - Voting on the outcome

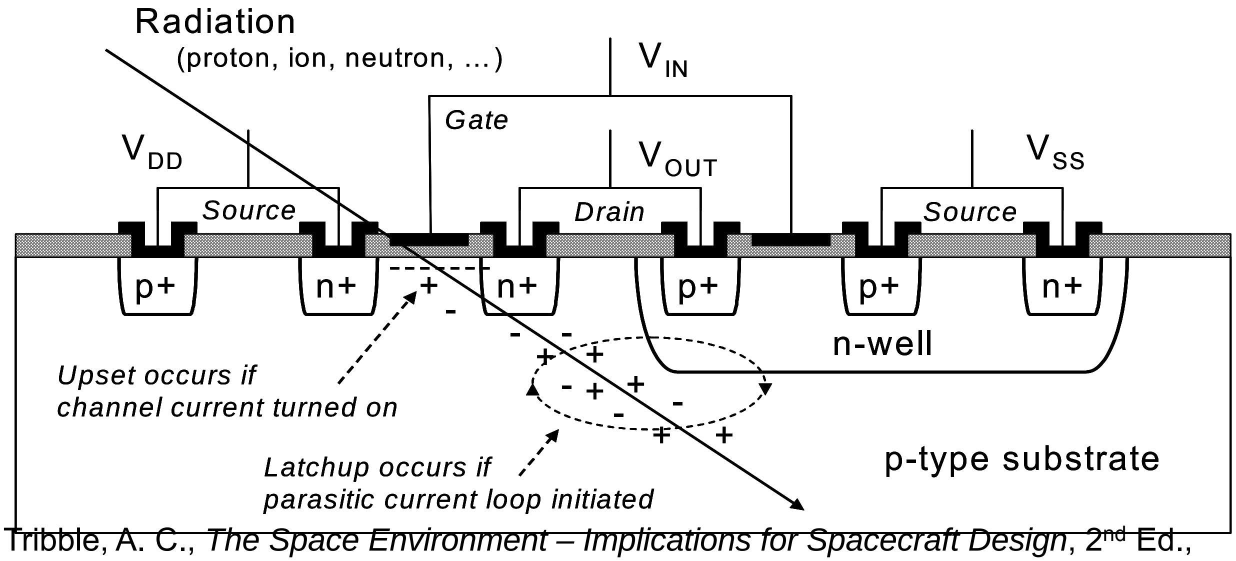


# Space Shuttle DS hardware



# Radiation

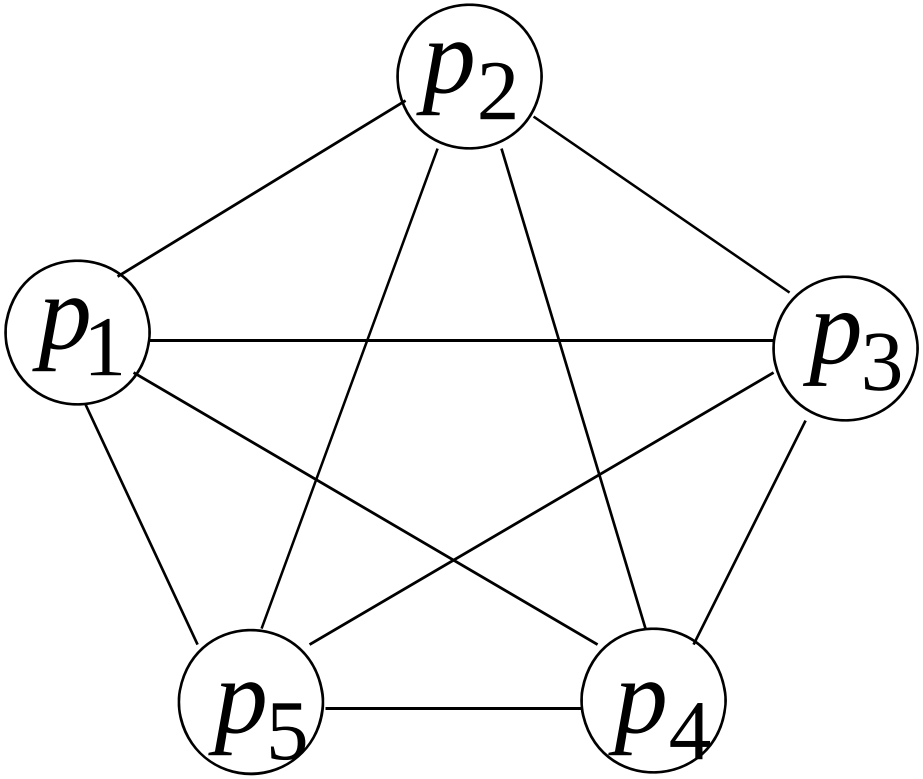
- The Natural (and Hostile) Radiation Environment Poses a Significant Threat to Many Electronic Devices
  - Single Event Upset (SEU), Single Event Latchup (SEL), …



Tribble, A. C., *The Space Environment – Implications for Spacecraft Design*, 2nd Ed., (Princeton, NJ: Princeton University Press, 2003).

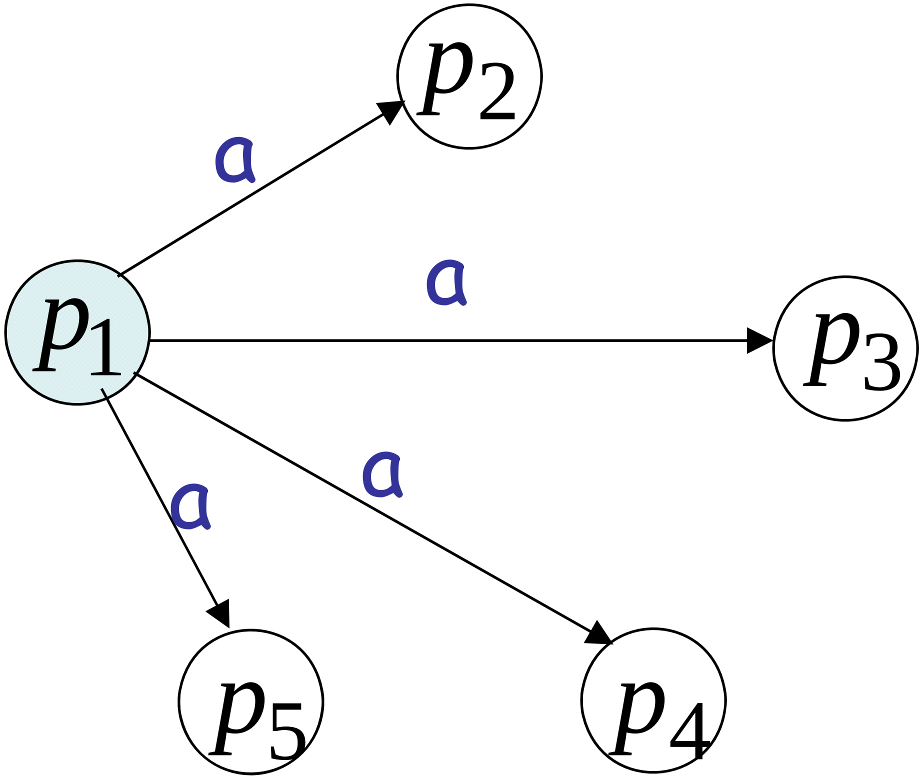
# Consensus in a synchronous systems w. crash failures

# Communication Model



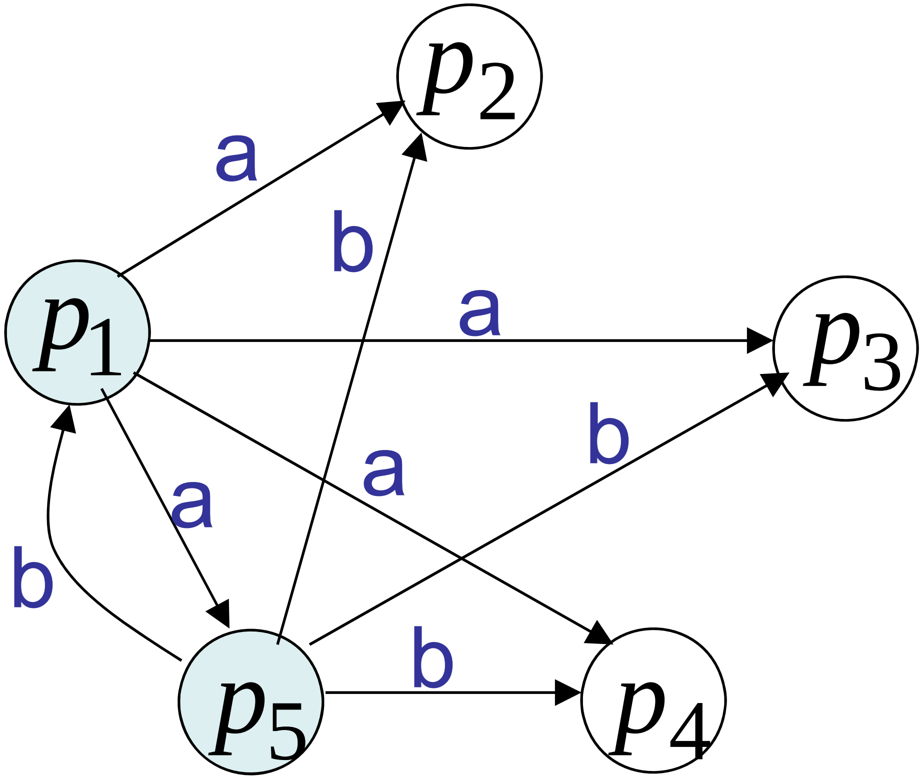
- Reliable point-to-point communication
- Pairwise channels (complete graph)
- Synchronous system

# B-Multicast



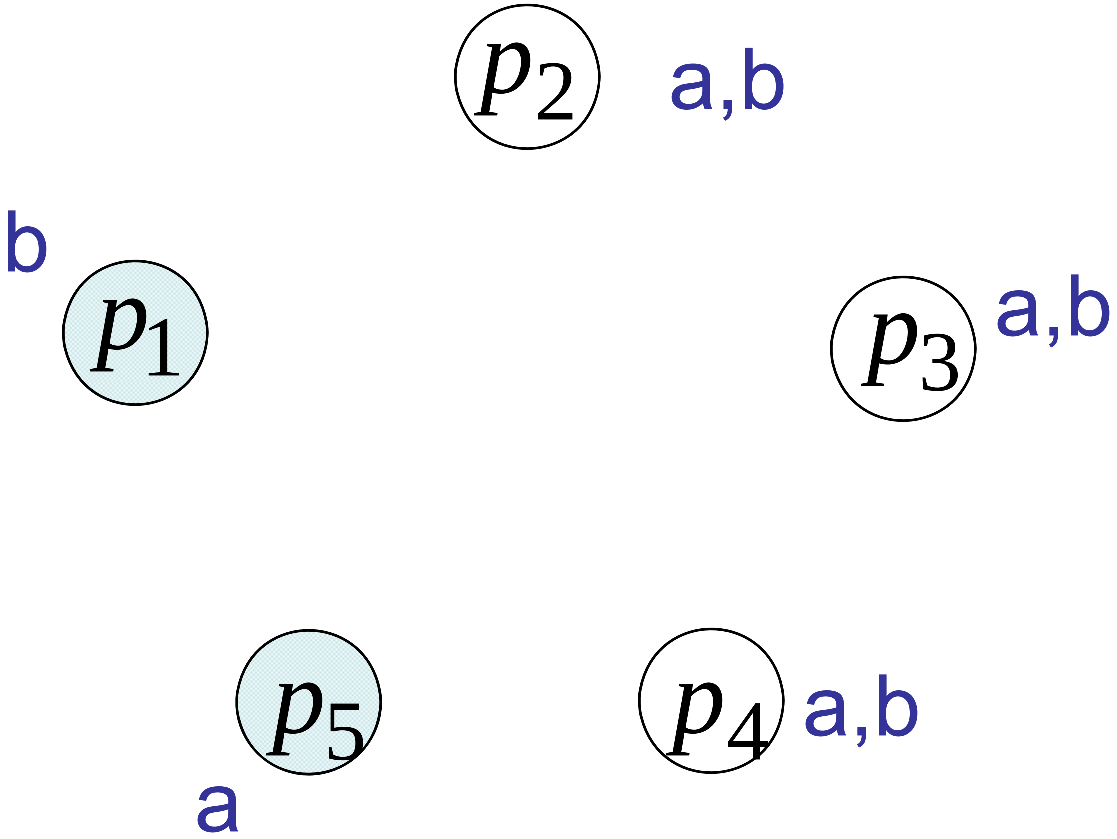
Send a message to all processors in one round

# Concurrent Multicast

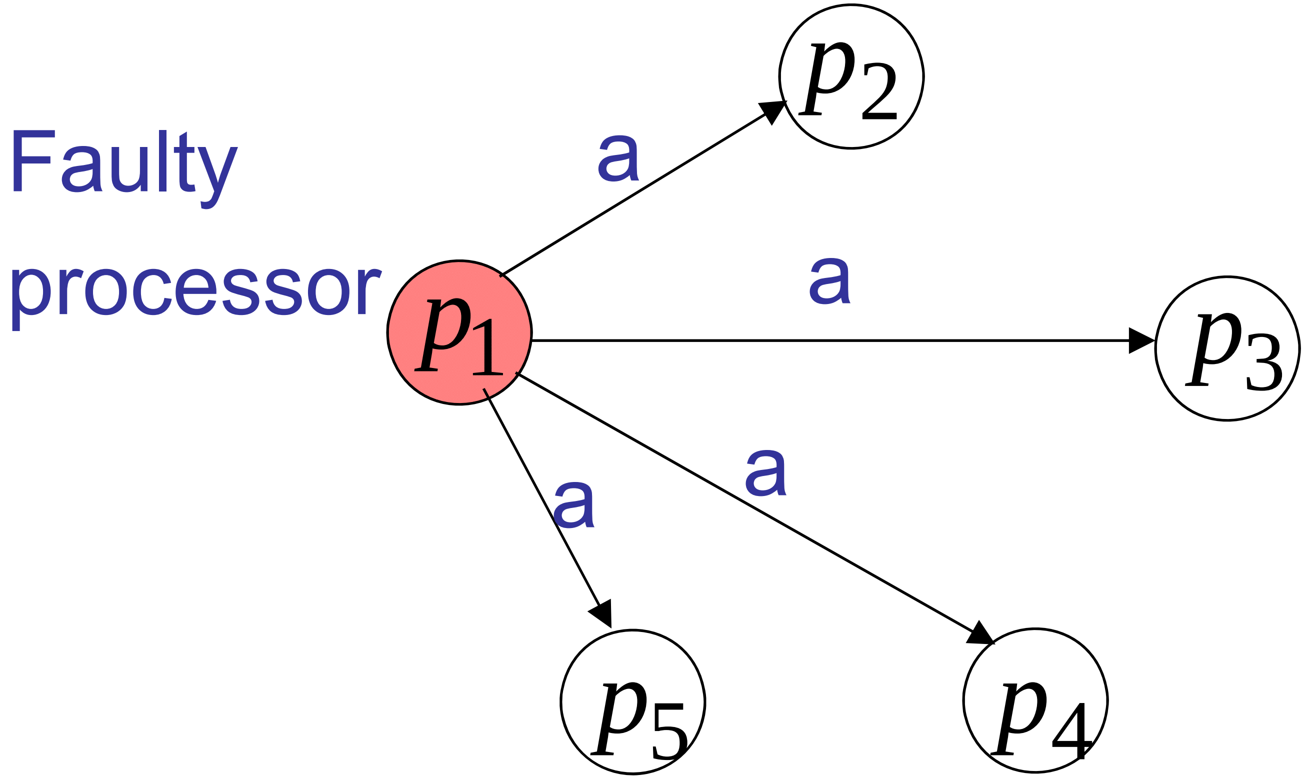


- More processes can multicast at the same round

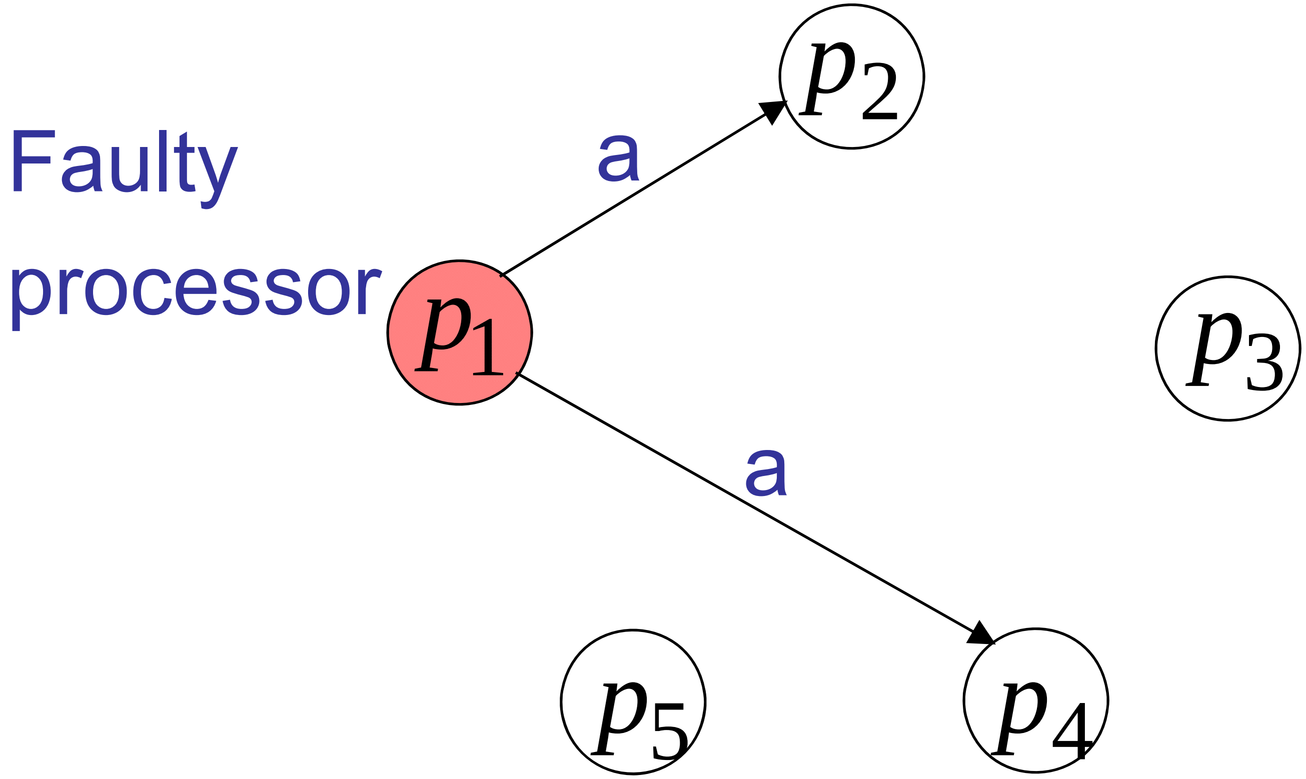
# Concurrent Multicast



# Crash Failures



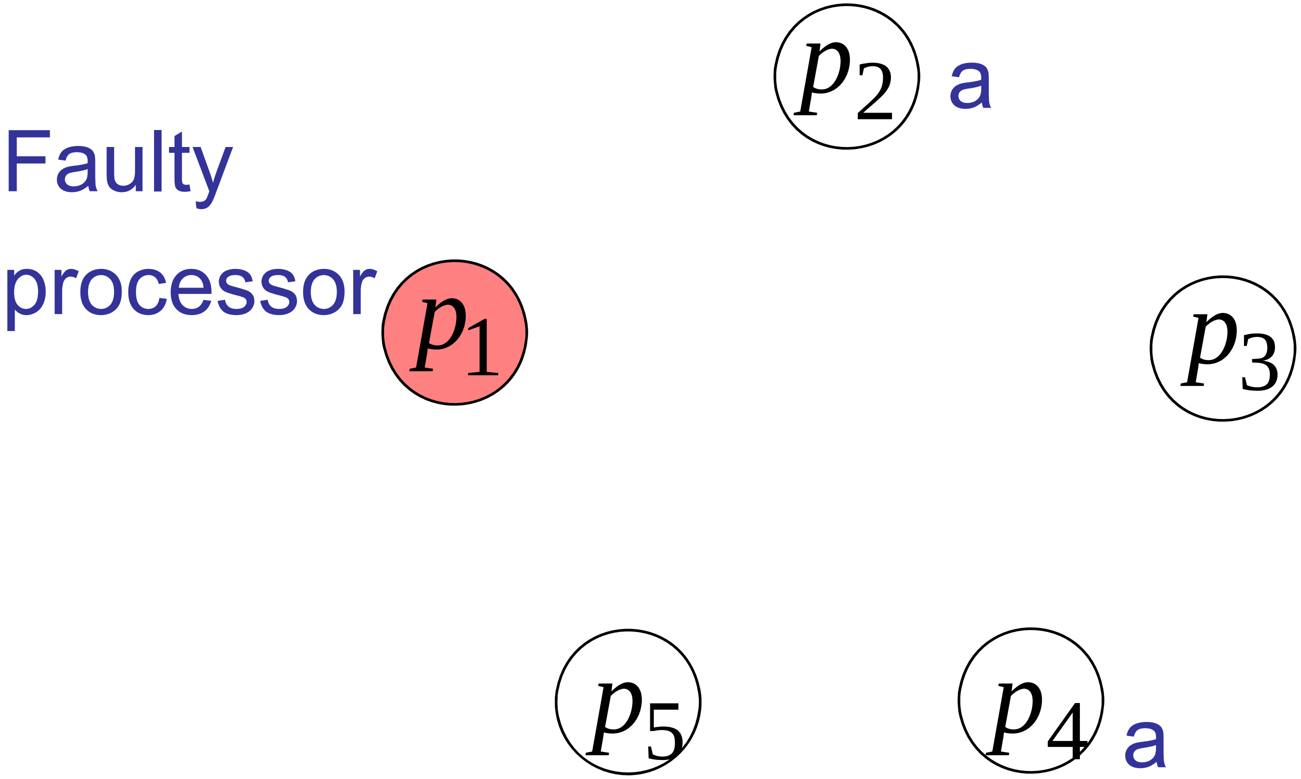
# Un-reliable multicast



B-multicast is unreliable

- Some of the messages are never delivered, if sender crashes

# Un-reliable multicast



Faulty processor

$p_1$

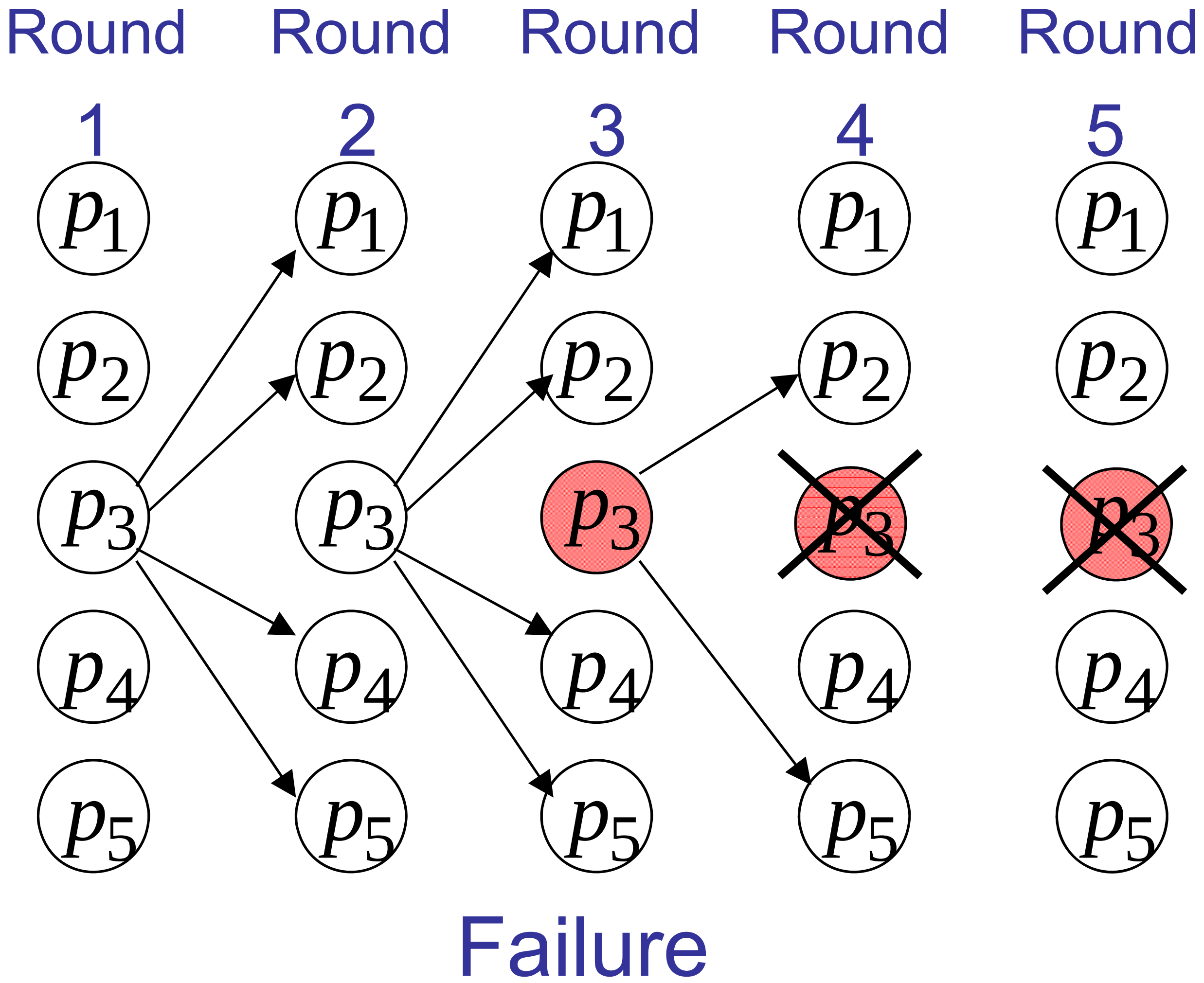
$p_2$ a

$p_3$

$p_4$ a

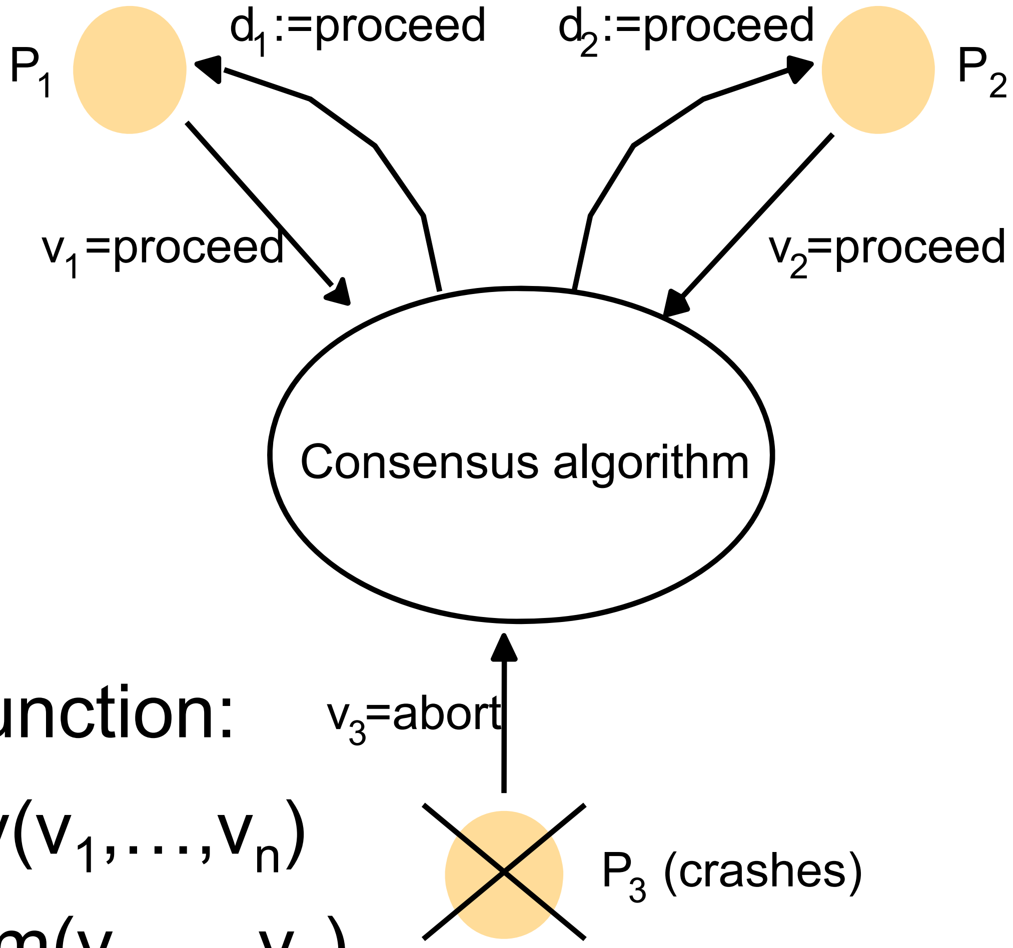
$p_5$

# Crash-failures



After failure the process disappears from the network

# Consensus for three processes



Selection function:

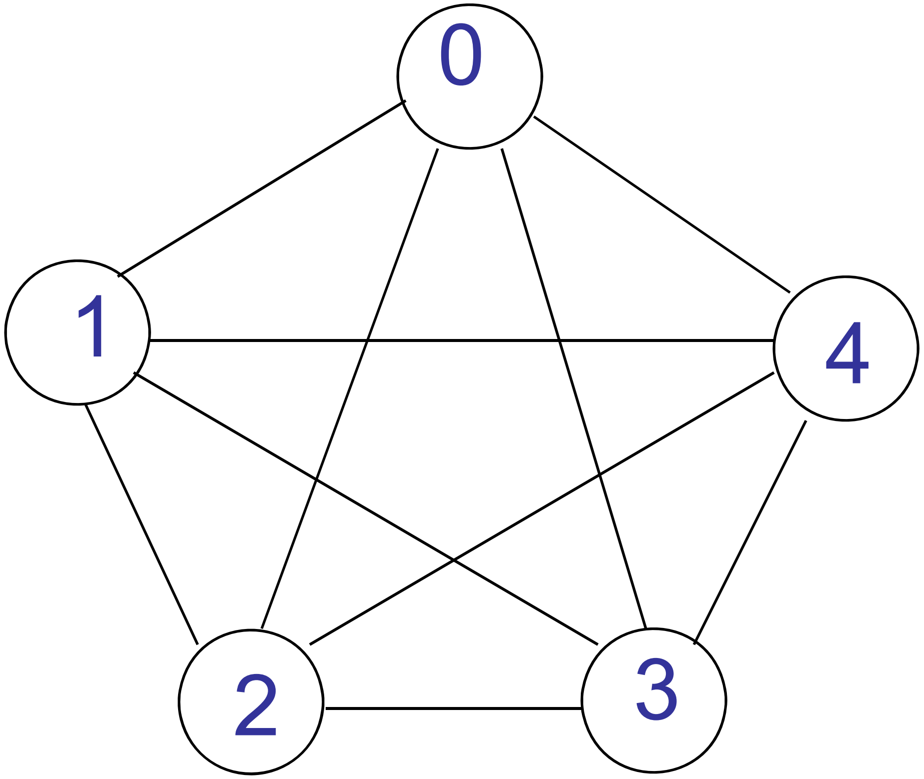
- $d_i$=majority($v_1,\ldots,v_n$)
- $d_i$=minimum($v_1,\ldots,v_n$)
- …

# Consensus

- ***Termination:*** Eventually each correct process $p_i$ sets its decision variable $d_{\mathrm{i}}$.
- ***Agreement:*** The decision value of all correct processes is the same: if $p_i$ and $p_j$ are correct and have entered their *decided* state, then $d_i=d_j$ (for all *i,j2 1..N*).
- ***Integrity:*** If the correct processes all proposed the same value, then any correct process in the *decided* state has chosen that value.

# Consensus

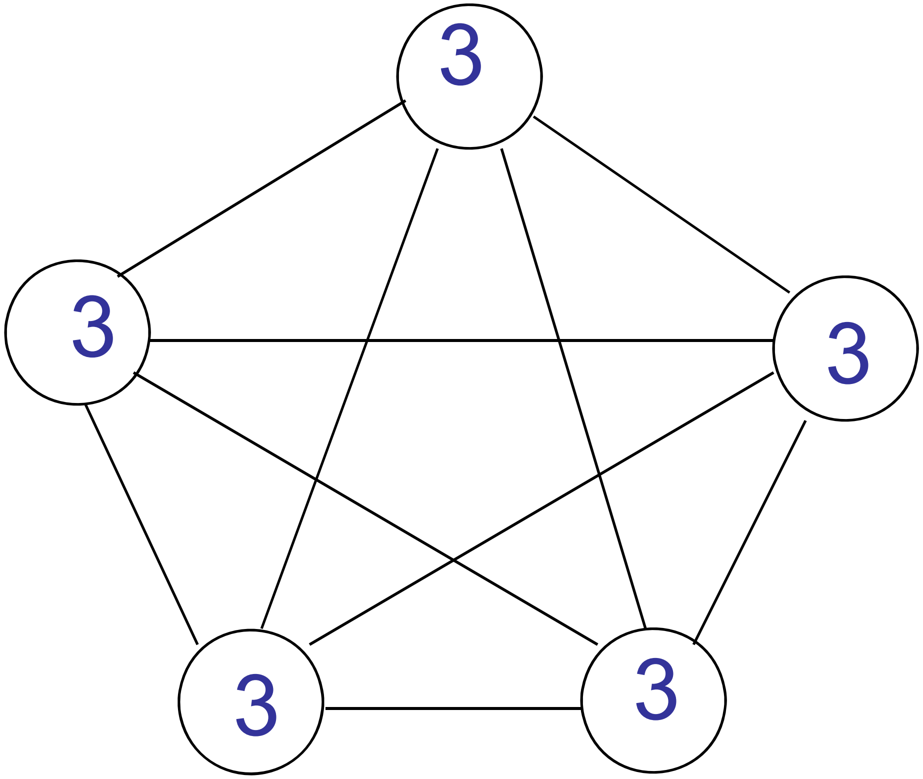
Start



Everybody has an initial proposed value $v_i$

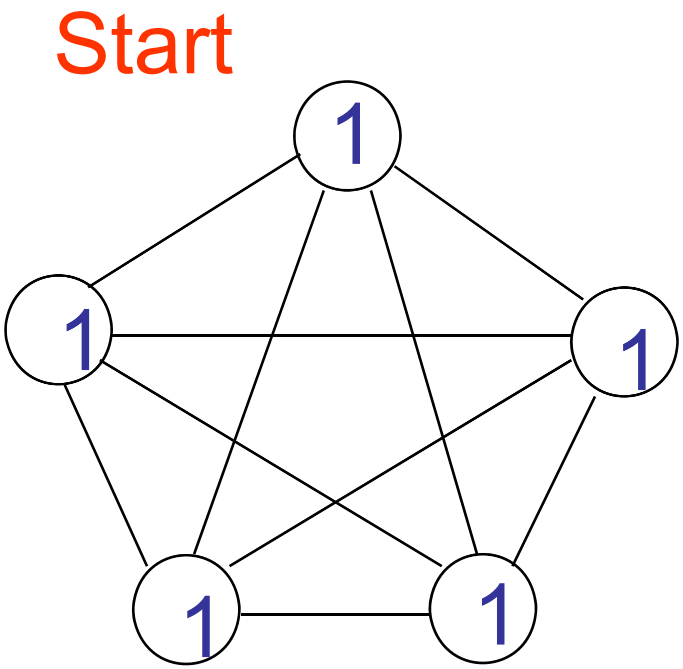
# Consensus

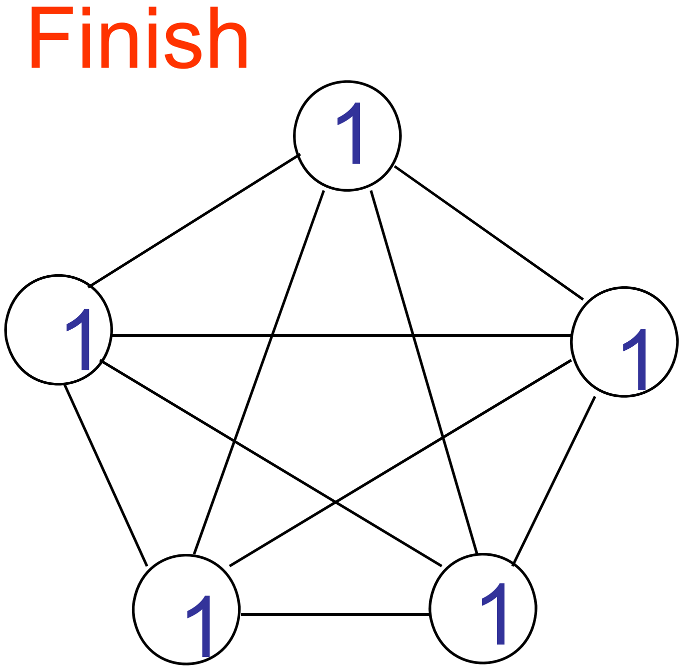
Finish



***Agreement:*** Everybody decides on the same value: $d_i$=$d_j$ (for all *i,j2 1..N)*

# Consensus





***Integrity:*** If the correct processes all proposed the same value, then any correct process in the *decided* state has chosen that value

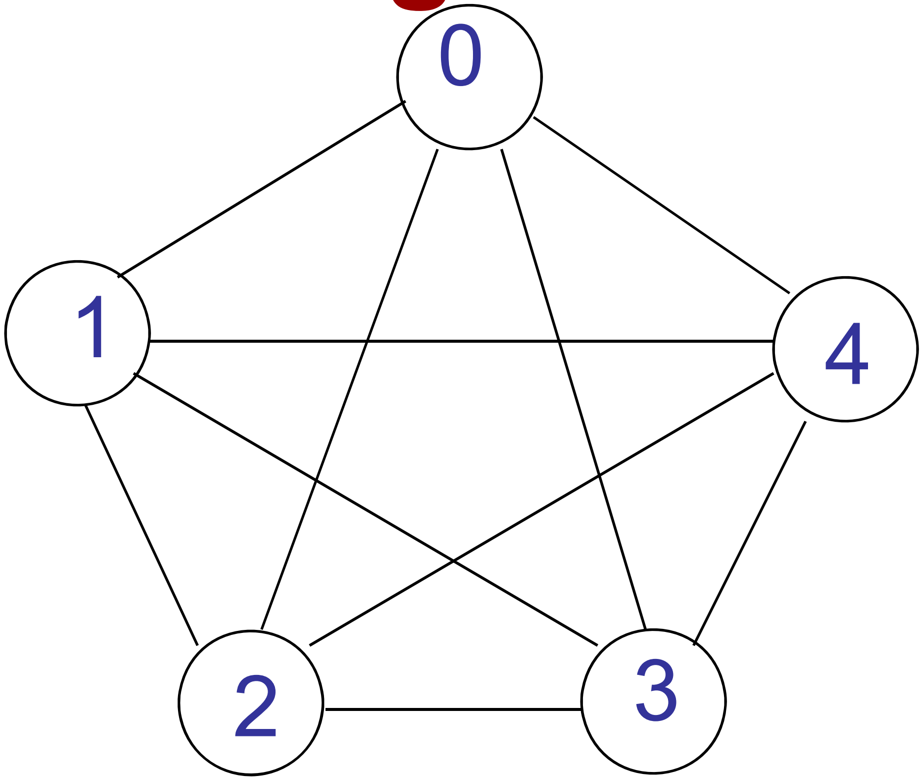
# An Algorithm?

Each proces $p_i$:

1. B-multicast its value to all processes
2. Decide on the minimum

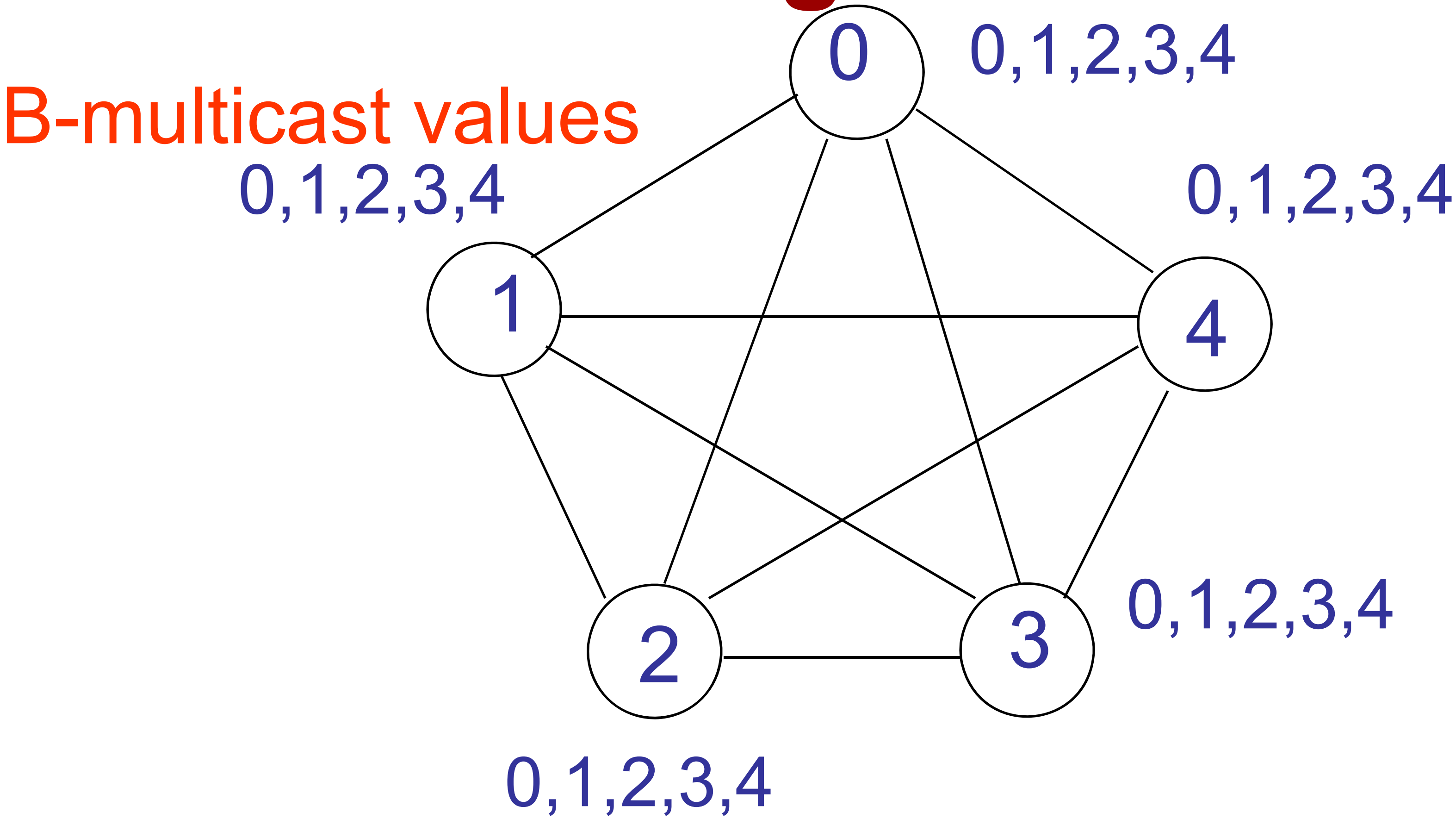
(only one round is needed)

# An Algorithm?



Start

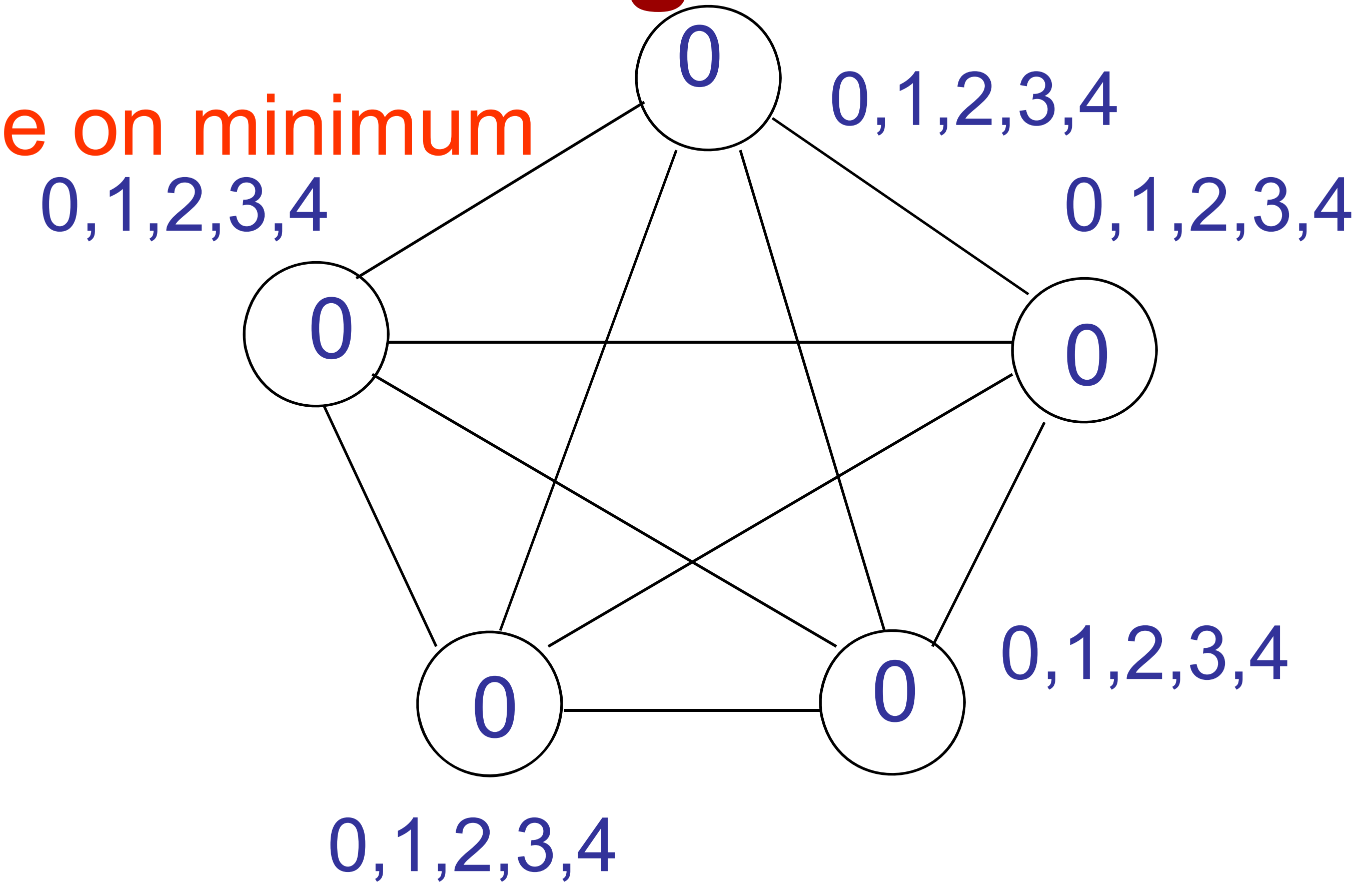
# An Algorithm?



B-multicast values

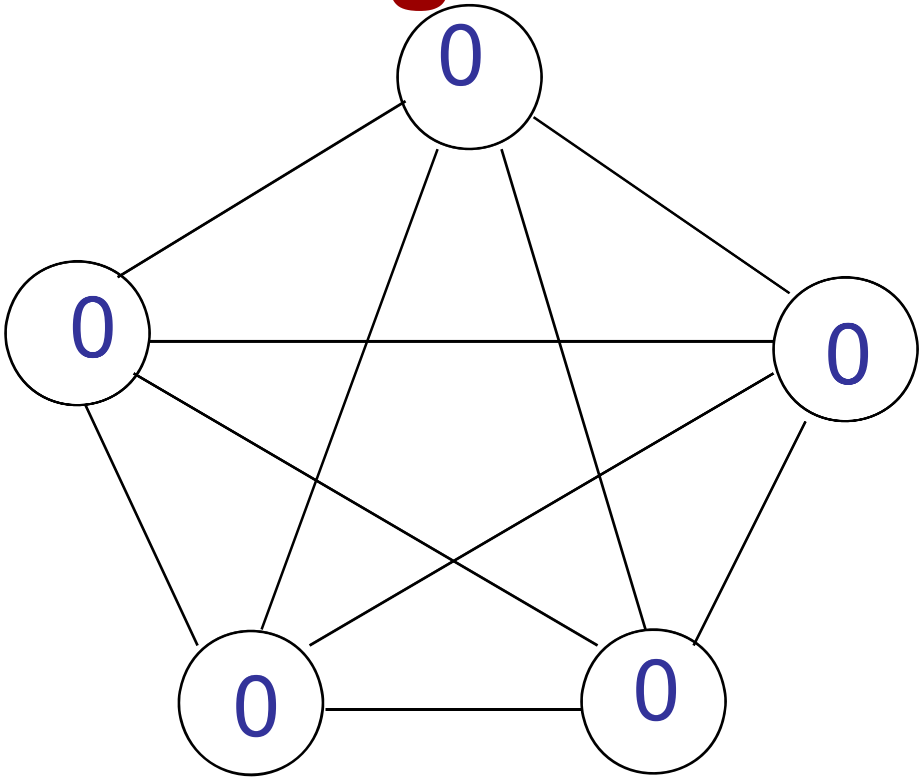
# An Algorithm?

Decide on minimum

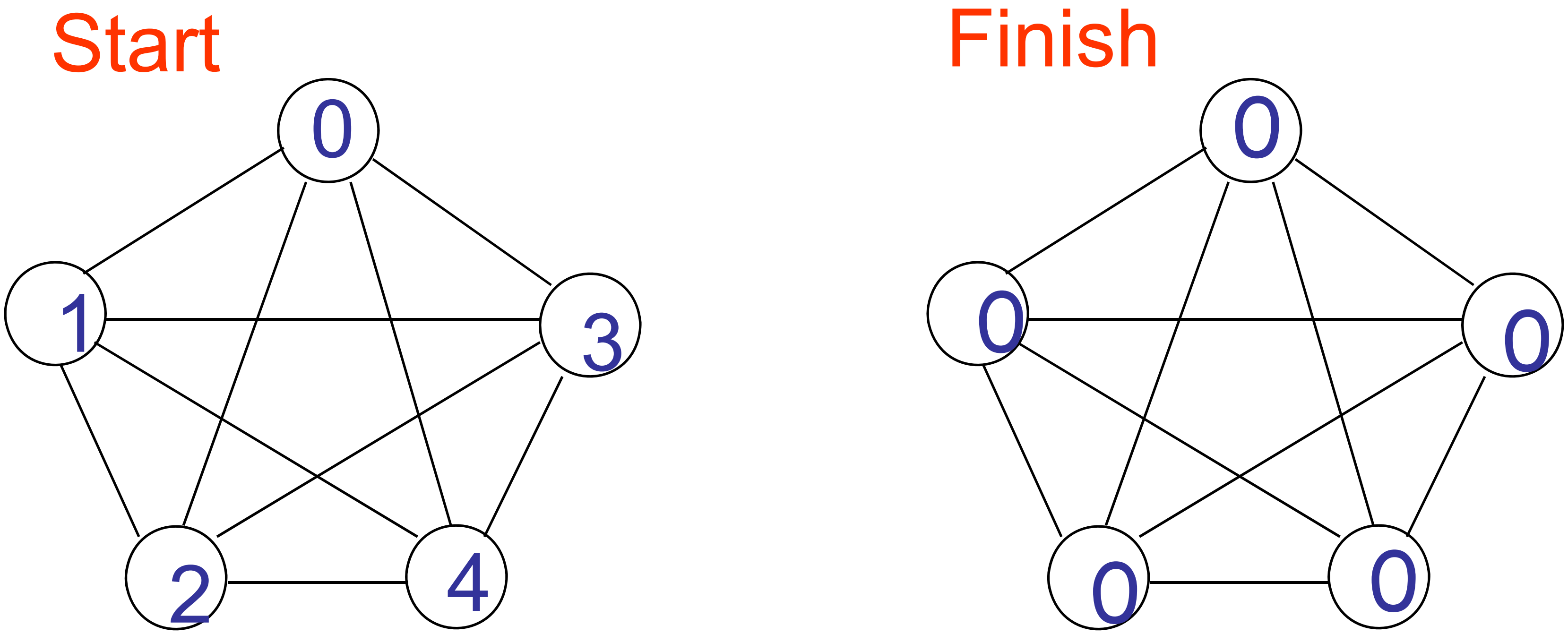


# An Algorithm?

Finish



# An Algorithm?



Without Failures, this algorithm gives consensus

If everybody starts with the same initial value, everybody decides on that value (minimum)

# Consensus w. Crash Failures

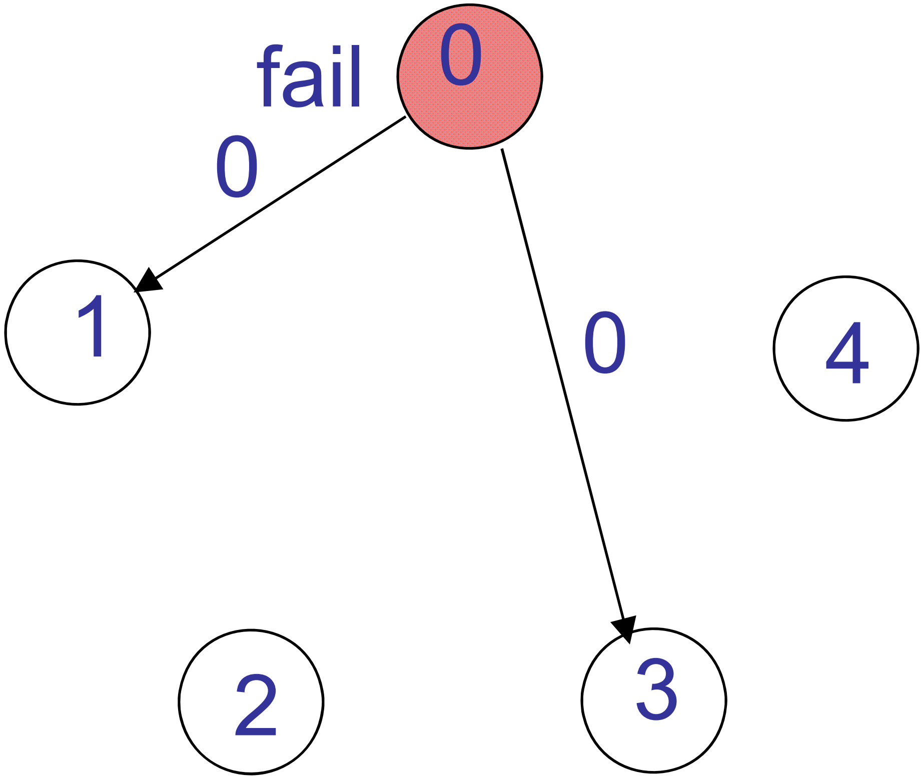
The simple algorithm doesn’t work

Each proces $p_i$ :

1. B-multicast value to all processors
2. Decide on the minimum

# Consensus w. Crash Failures

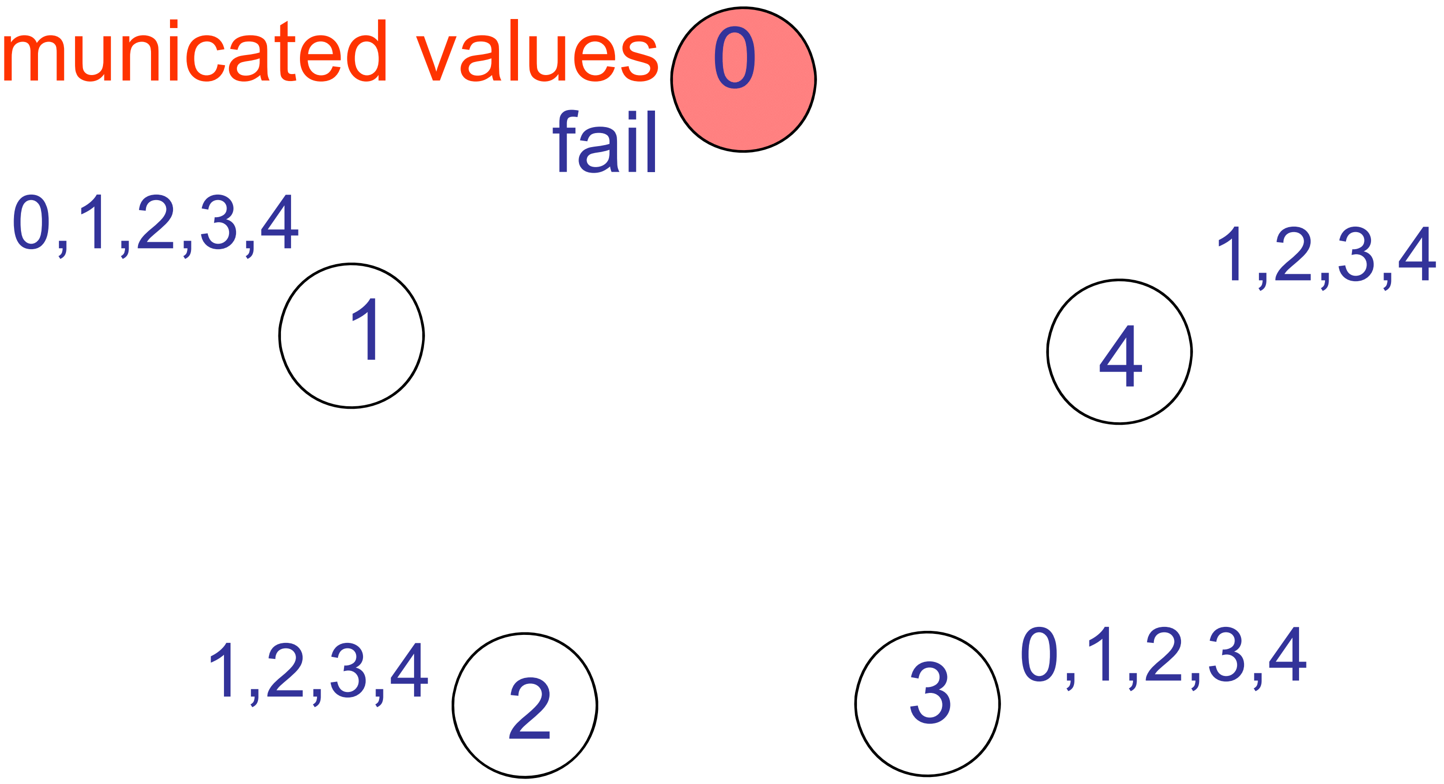
Start



Not all processes receives the proposed value from the failed process

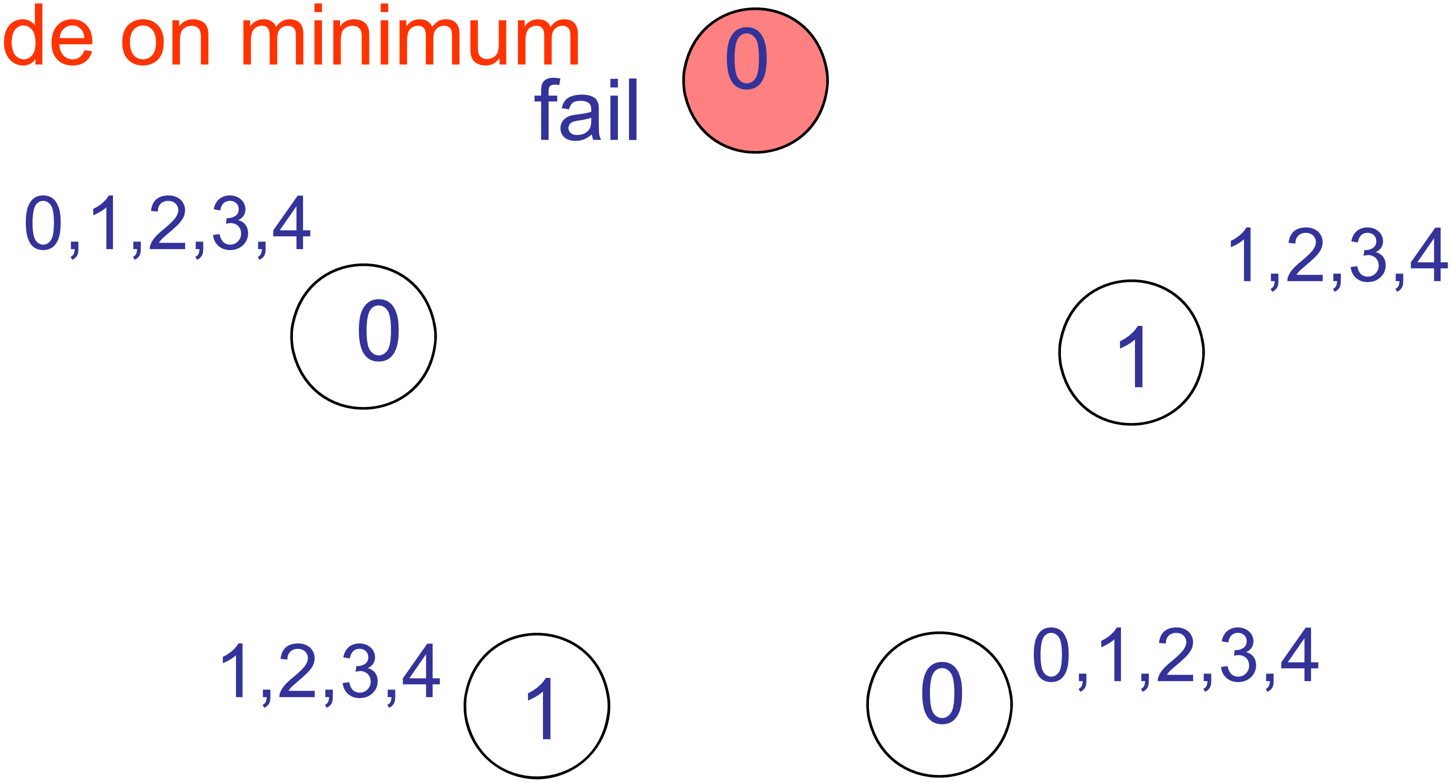
# Consensus w. Crash Failures

Communicated values



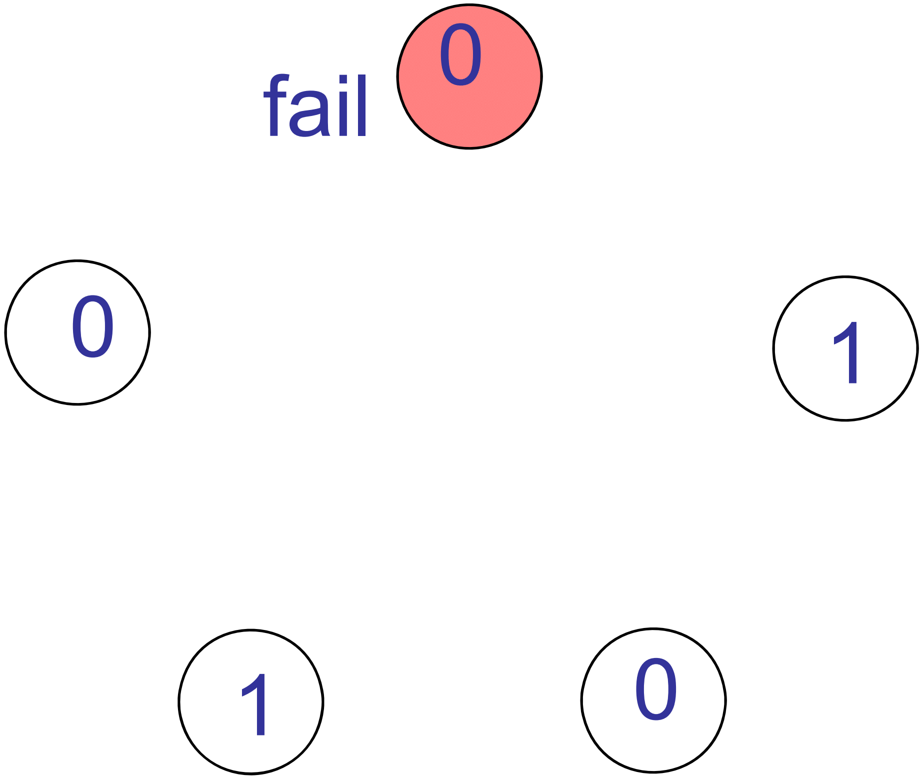
# Consensus w. Crash Failures

Decide on minimum



# Consensus w. Crash Failures

Finish



***No Consensus!!!***

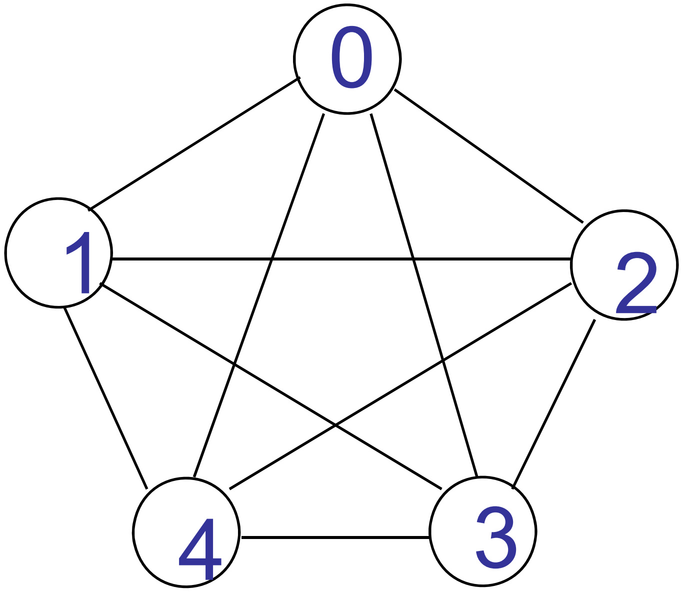
# f-resiliency

- ***f-resilient consensus algorithm***
  - Guarentees consensus with up to f failed process

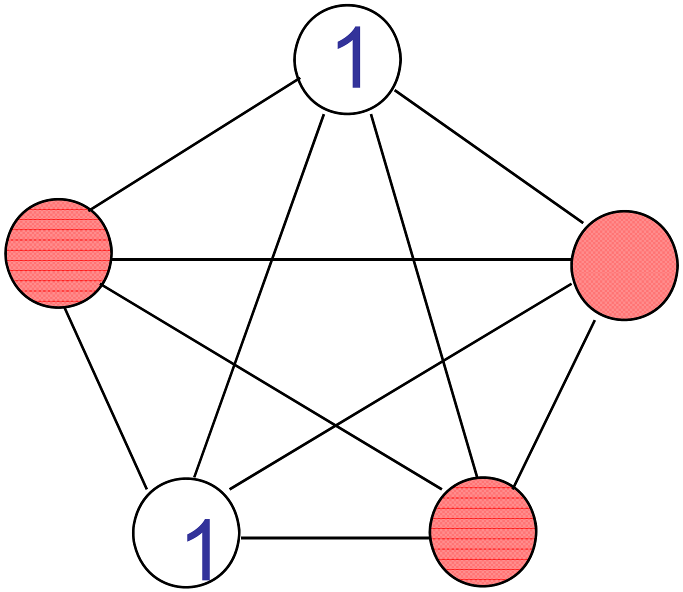
# Example 3-resiliency

Example: The input and output of a 3-resilient consensus algorithm

Start



Finish



# An f-resilient algorithm

Round 1:

Each process B-multicast its value

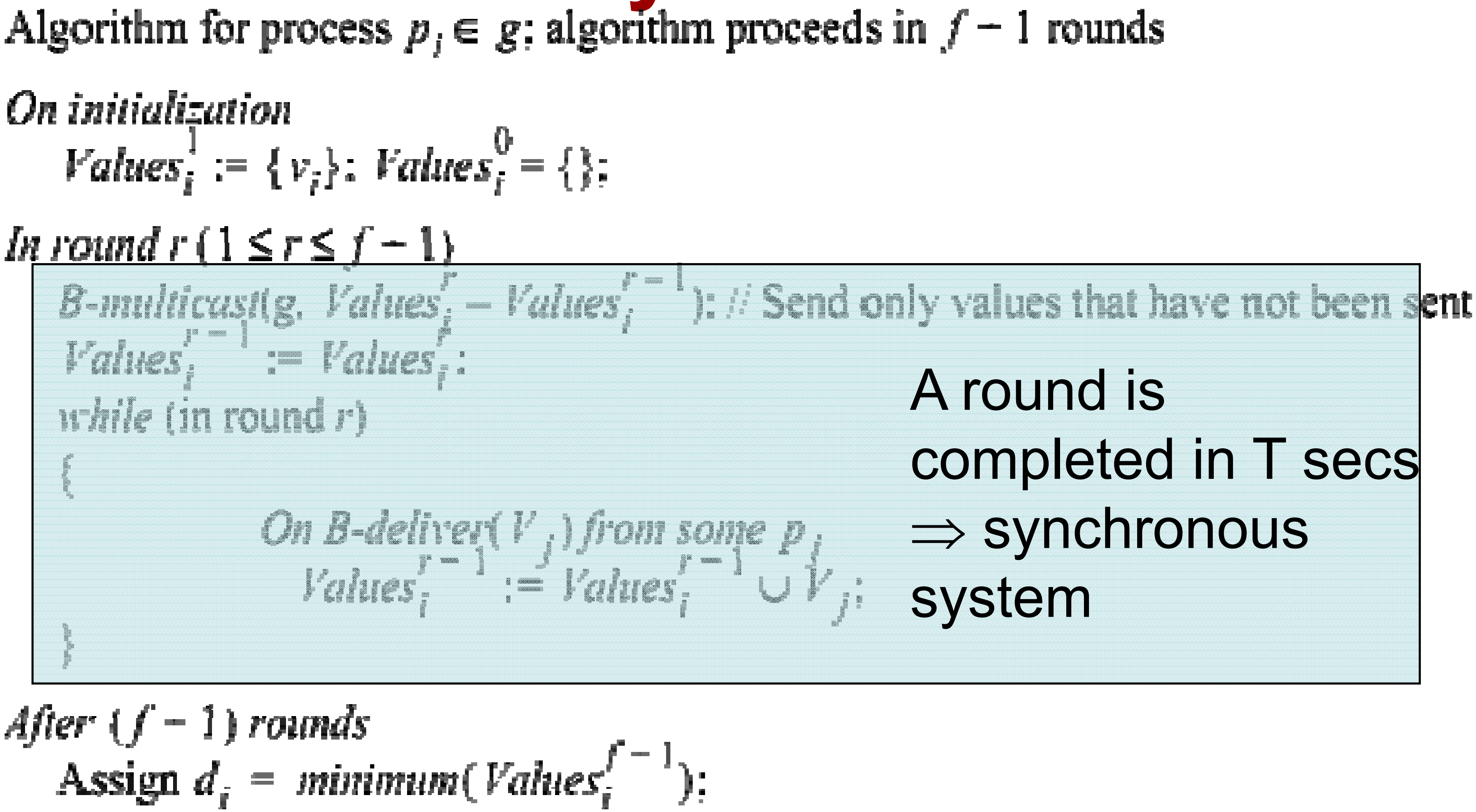
Round 2 to round f+1:

B-multicast any new received values

End of round f+1:

Decide on the minimum value received

# Consensus in a synchronous system

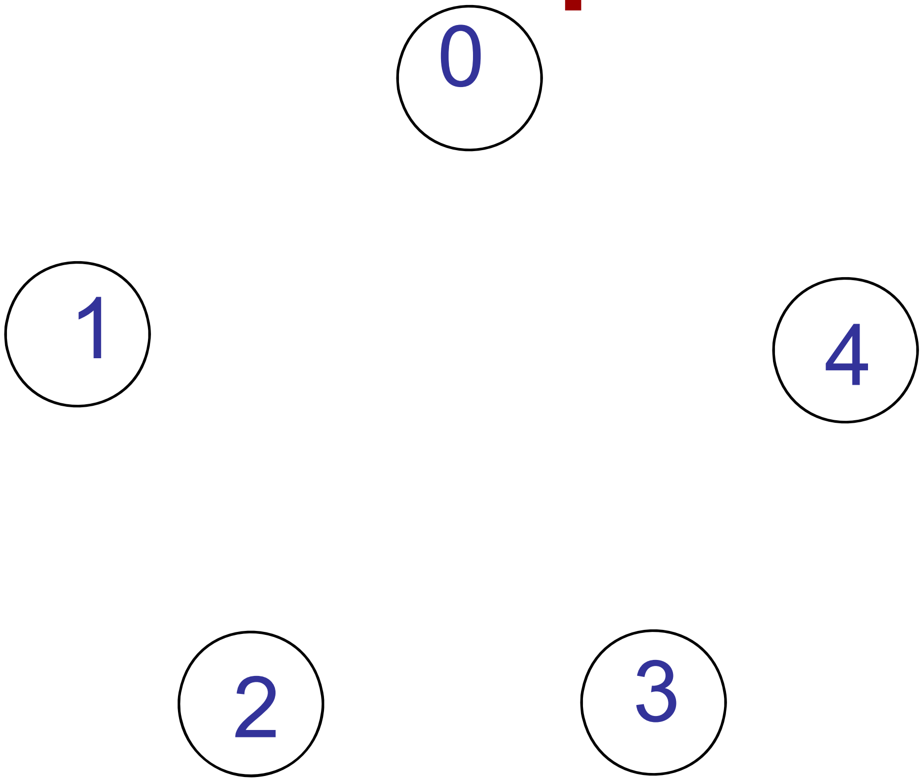


```
Algorithm for process $p_i \in g$; algorithm proceeds in $f - 1$ rounds

On initialization
    $Values_i^1 := \{v_i\}$; $Values_i^0 = \{\}$;

In round $r$ $(1 \le r \le f - 1)$
    B-multicast(g, $Values_i^r - Values_i^{r-1}$); // Send only values that have not been sent
    $Values_i^{r-1} := Values_i^r$;
    while (in round $r$)
    {
            On B-deliver($V_j$) from some $p_j$
                $Values_i^{r-1} := Values_i^{r-1} \cup V_j$;
    }

After $(f - 1)$ rounds
    Assign $d_i$ = minimum($Values_i^{f-1}$);
```

A round is completed in T secs ⇒ synchronous system

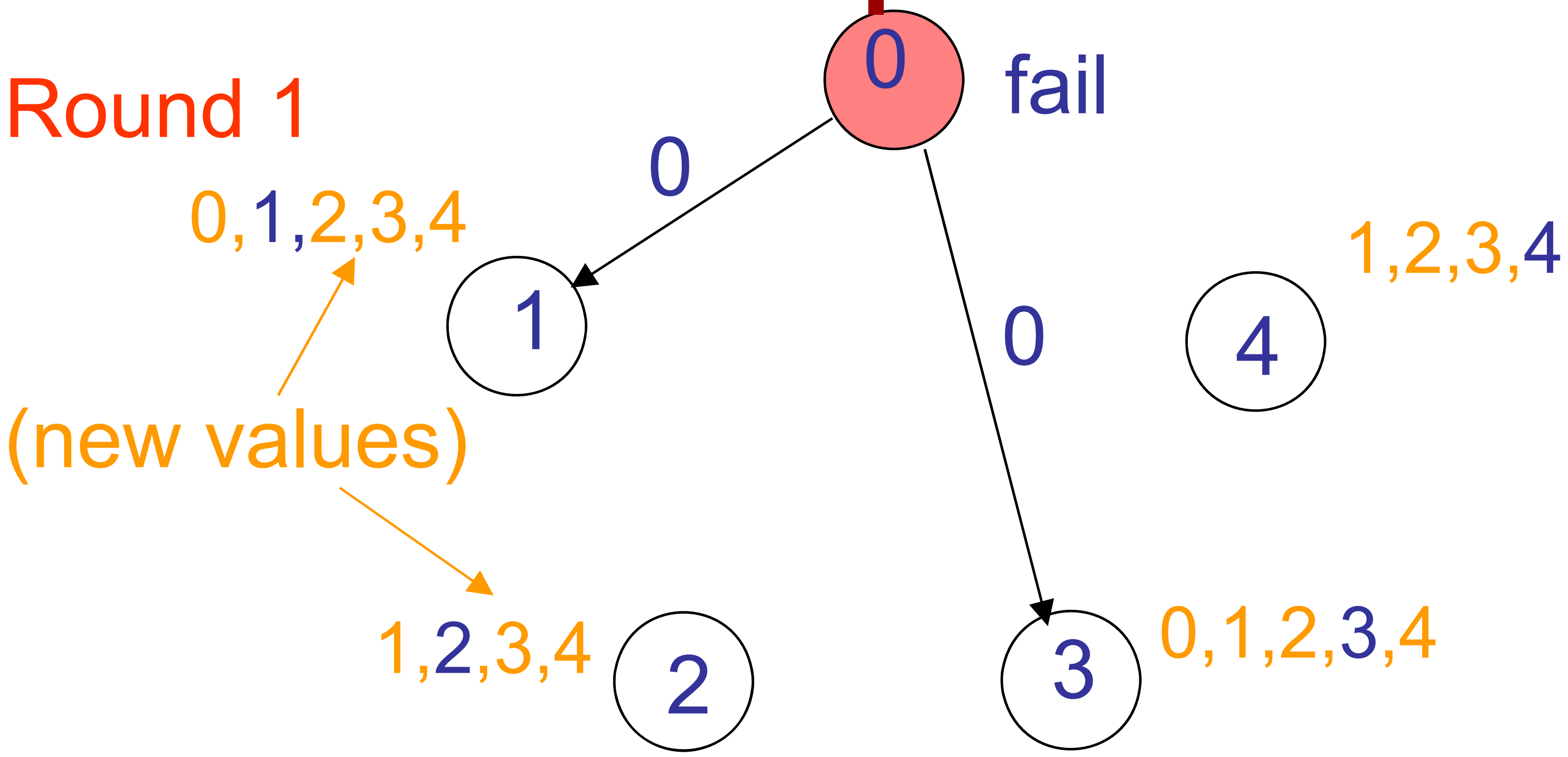
# Example

Start



f=1 failures, f+1 = 2 rounds needed

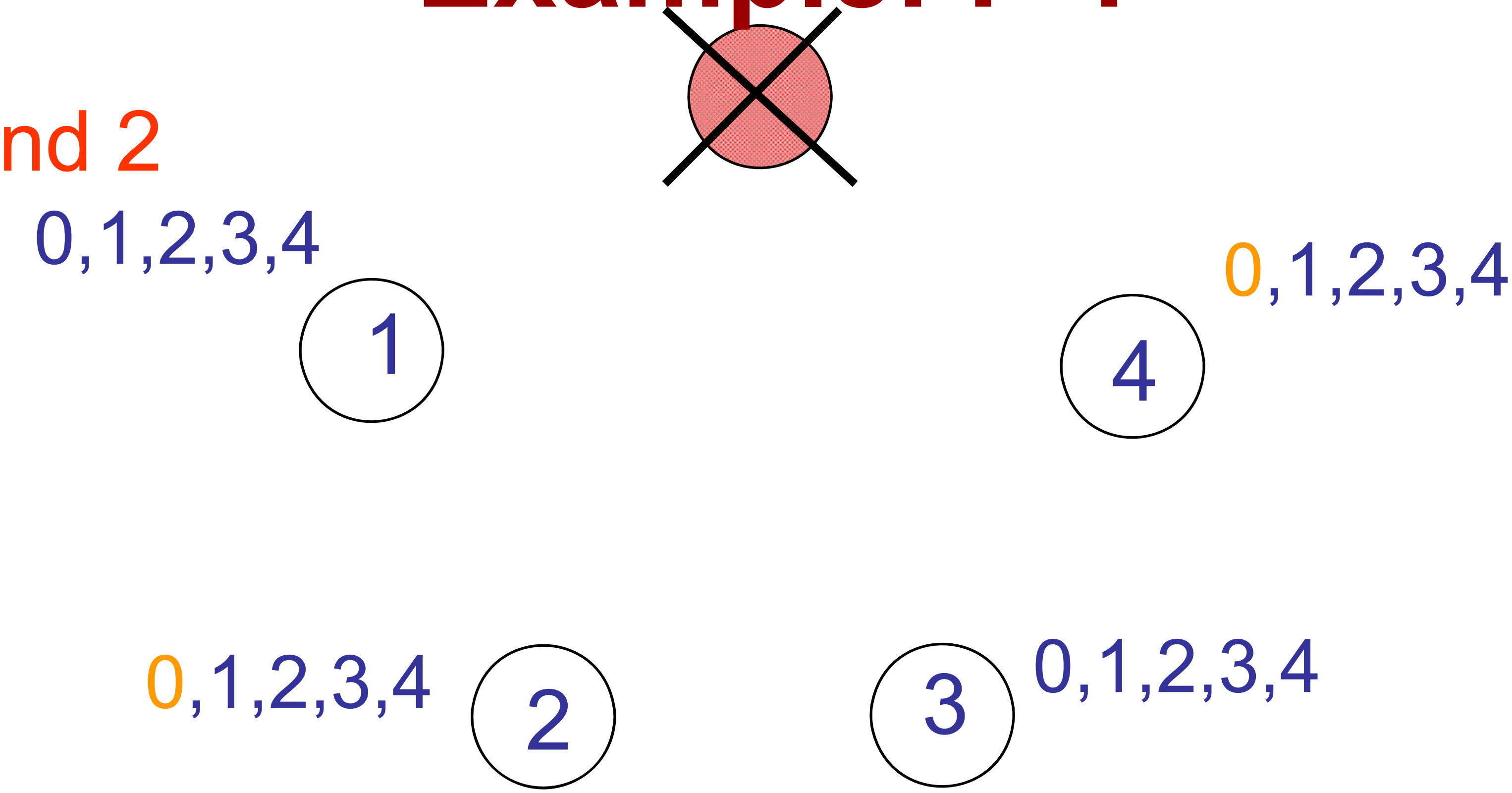
# Example: f=1



B-multicast all values to everybody

# Example: f=1

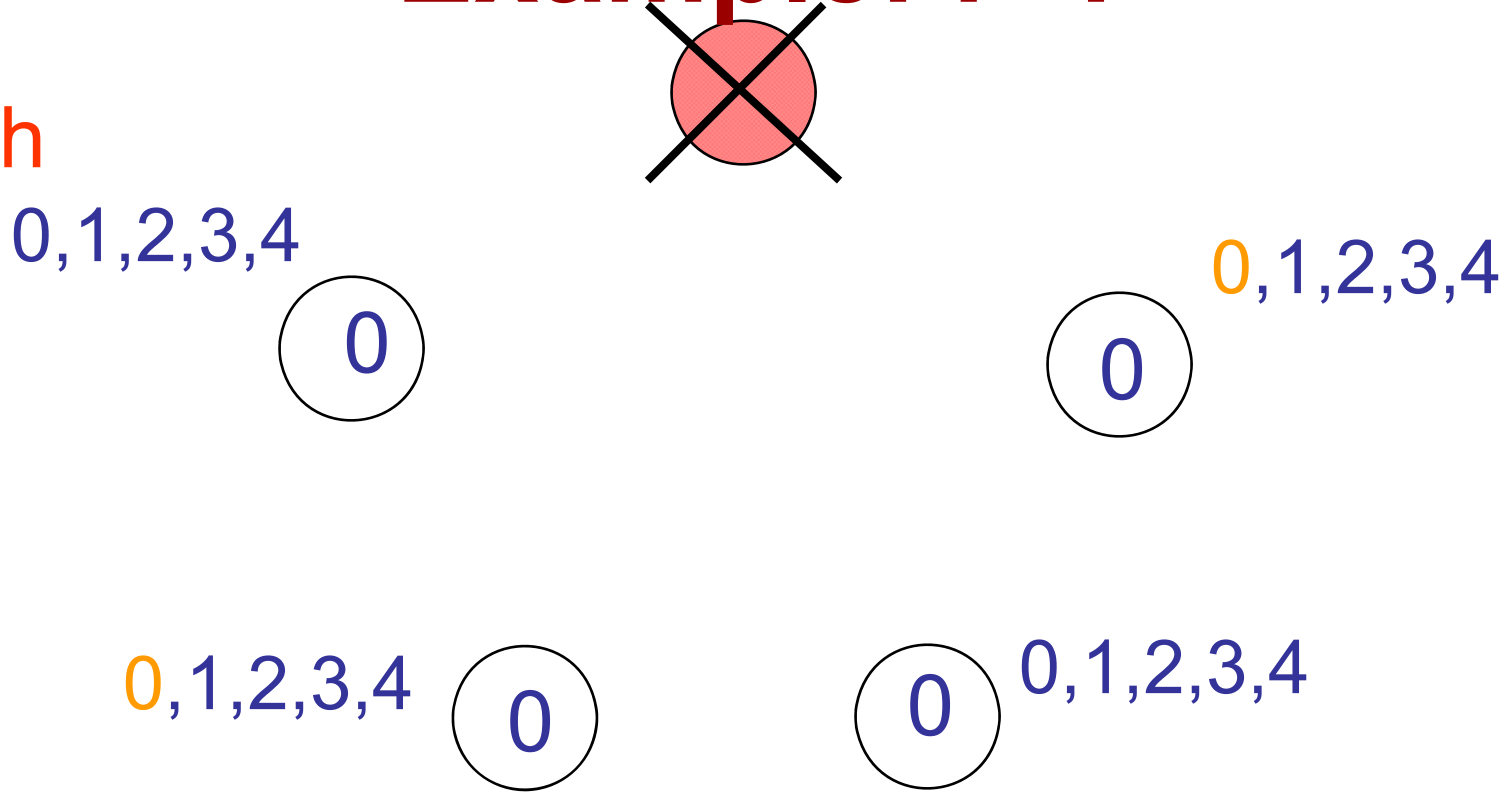
Round 2



B-multicast all new values to everybody

# Example: f=1

Finish



0,1,2,3,4

0

0,1,2,3,4

0

0,1,2,3,4

0

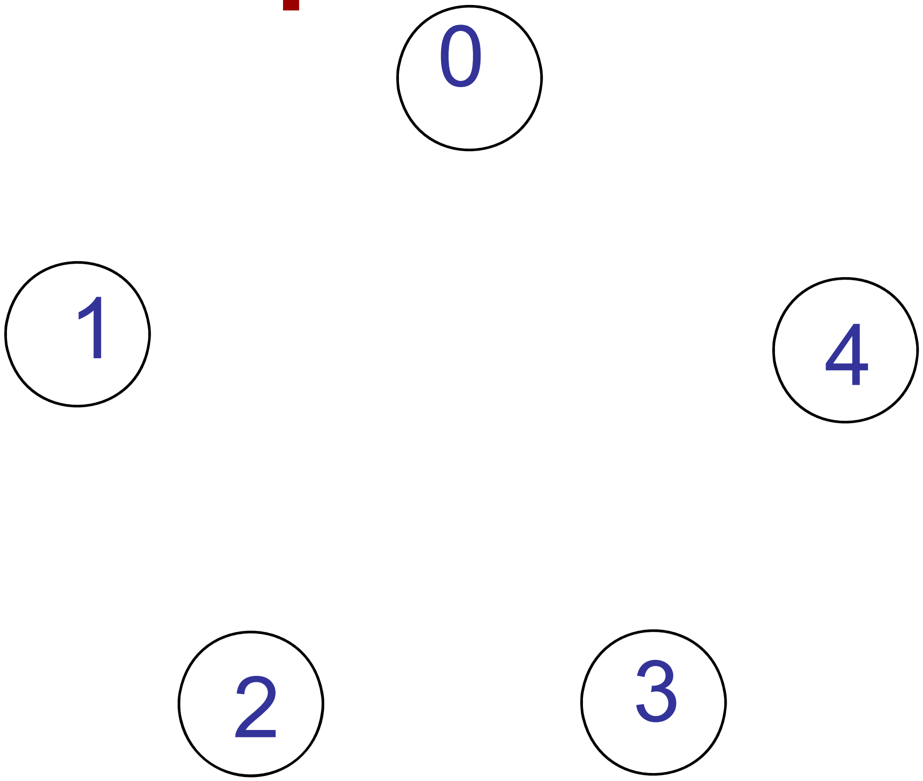
0

0,1,2,3,4

Decide on minimum value: forall i: $d_i$=0,

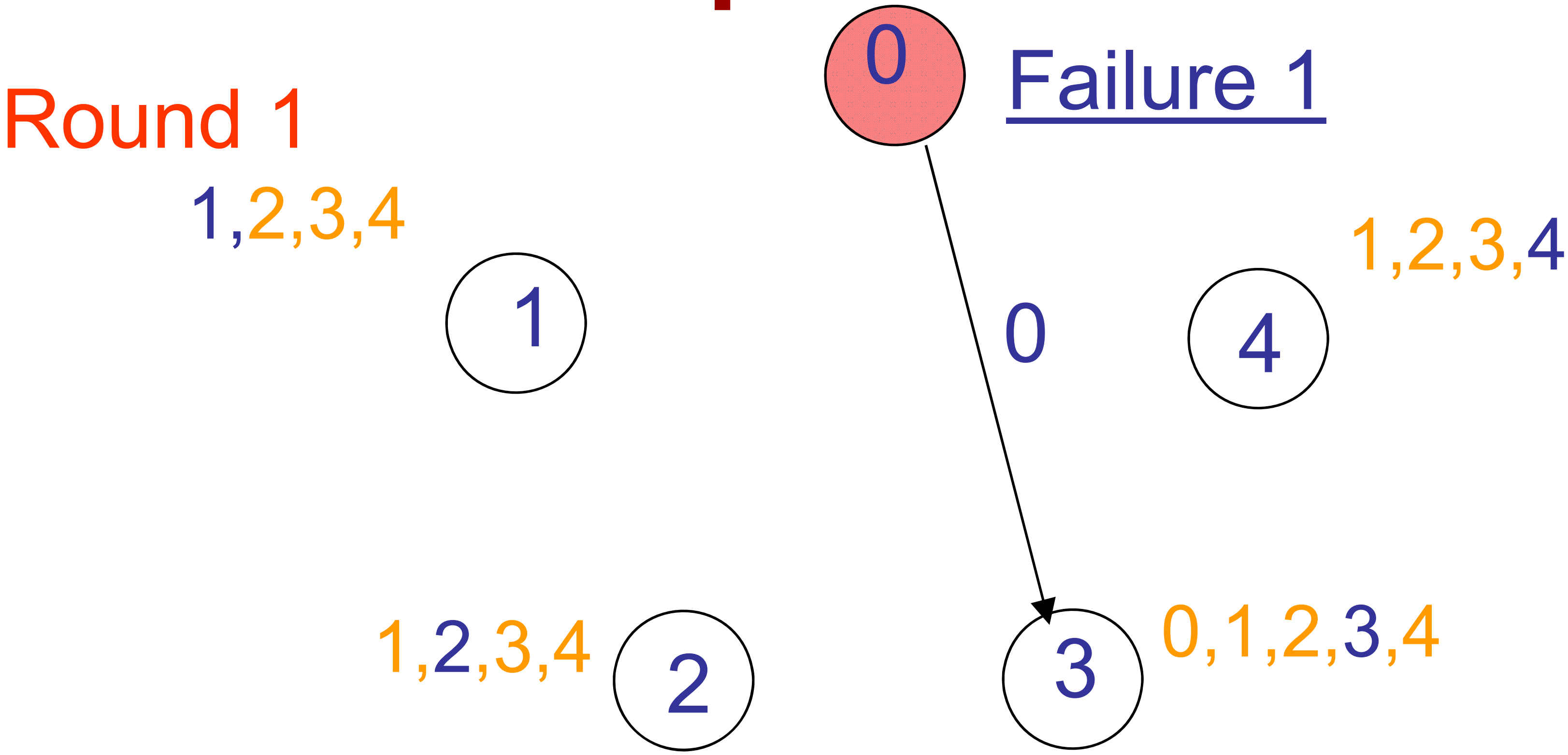
# Example run 1: f=2

Start



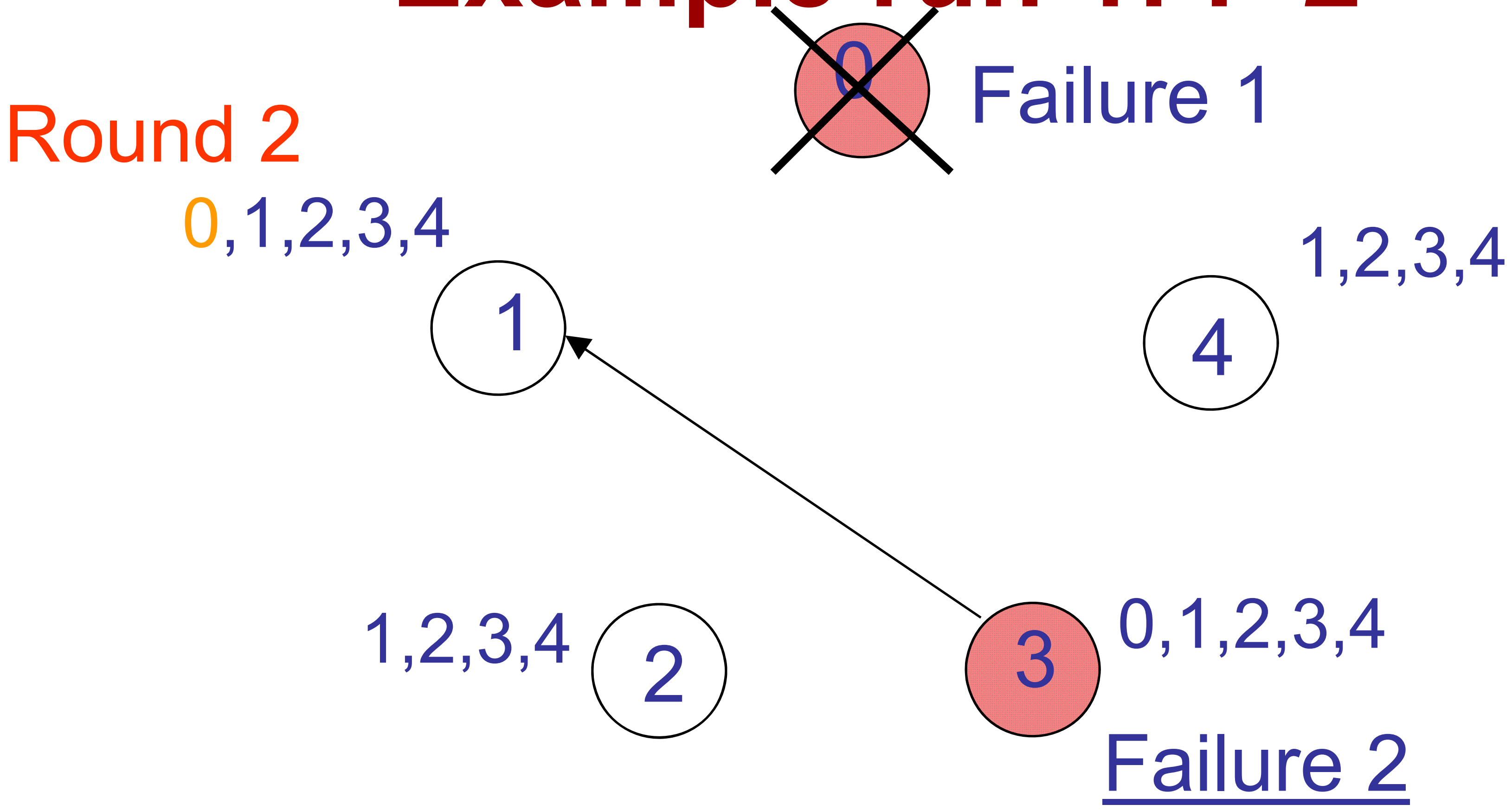
Example: f=2 failures, f+1 = 3 rounds needed

# Example run 1: f=2



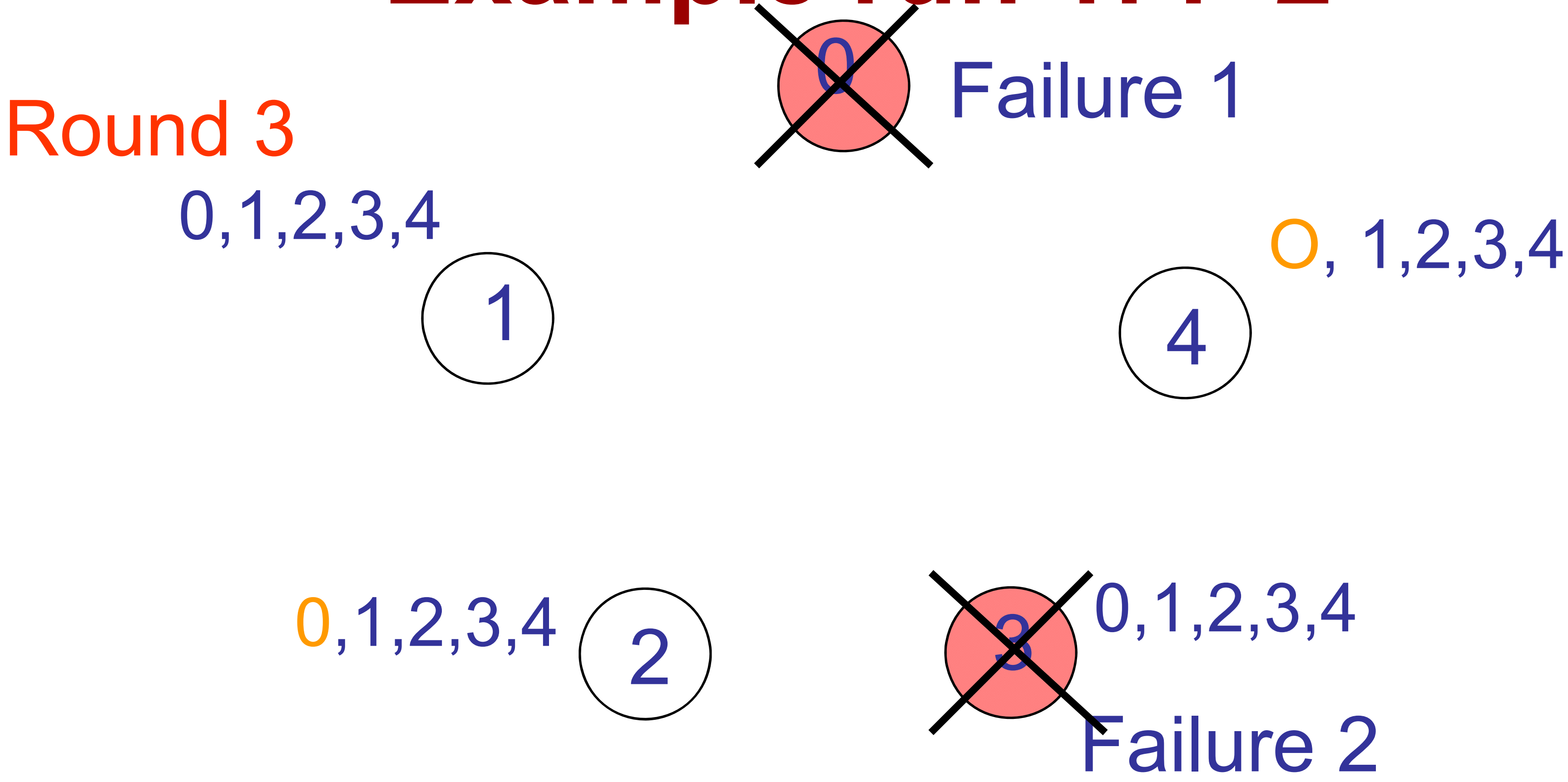
B-multicast all values to everybody

# Example run 1: f=2



B-multicast new values to everybody

# Example run 1: f=2



Round 3

0,1,2,3,4

Failure 1

O, 1,2,3,4

0,1,2,3,4

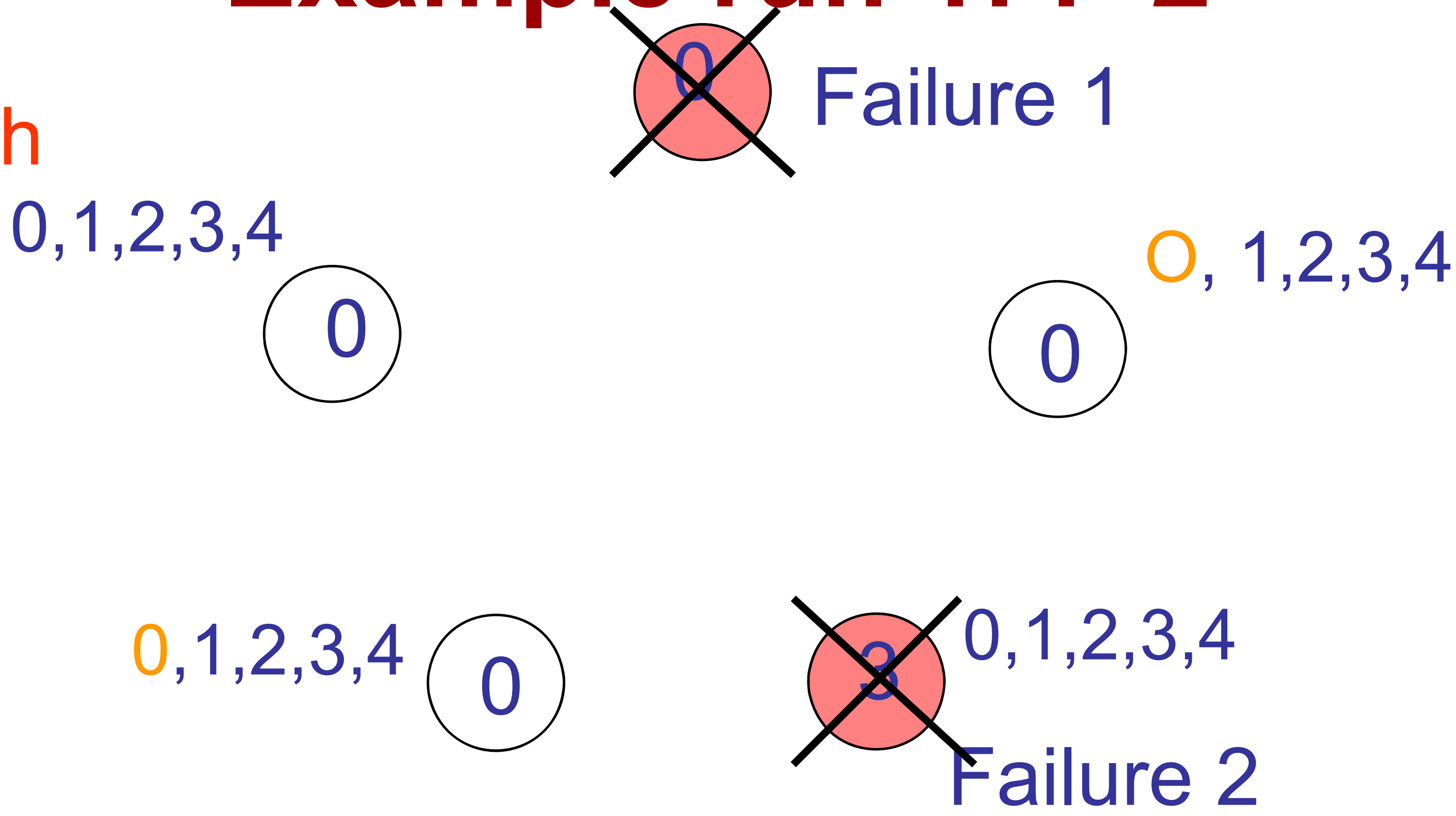
0,1,2,3,4

Failure 2

B-Multicast new values to everybody

# Example run 1: f=2

Finish



0

Failure 1

0,1,2,3,4

0

O, 1,2,3,4

0

0,1,2,3,4

0

3

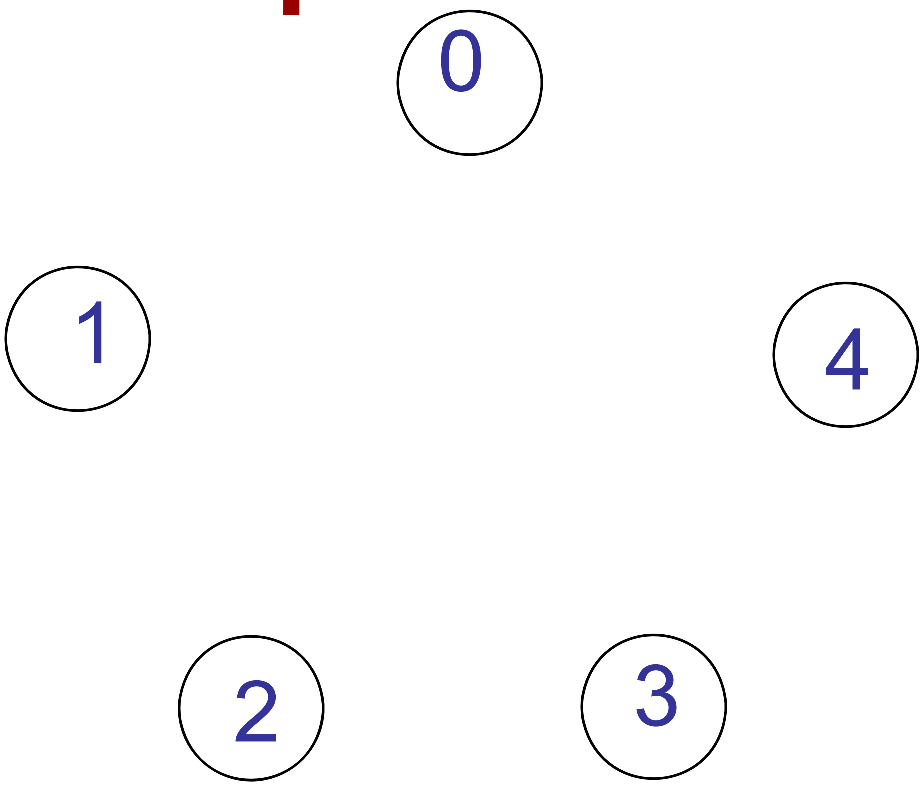
0,1,2,3,4

Failure 2

Decide on the minimum value

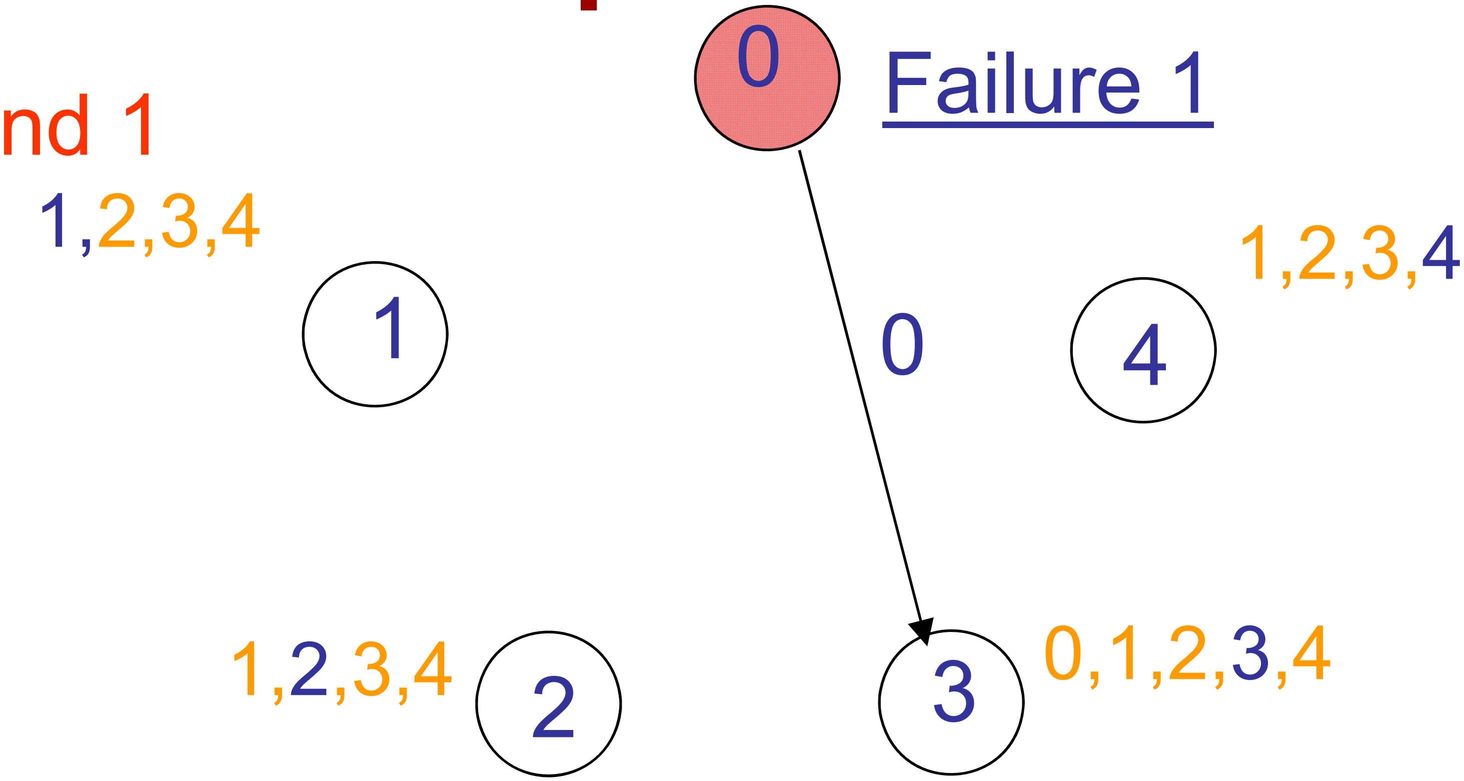
# Example run 2: f=2

Start



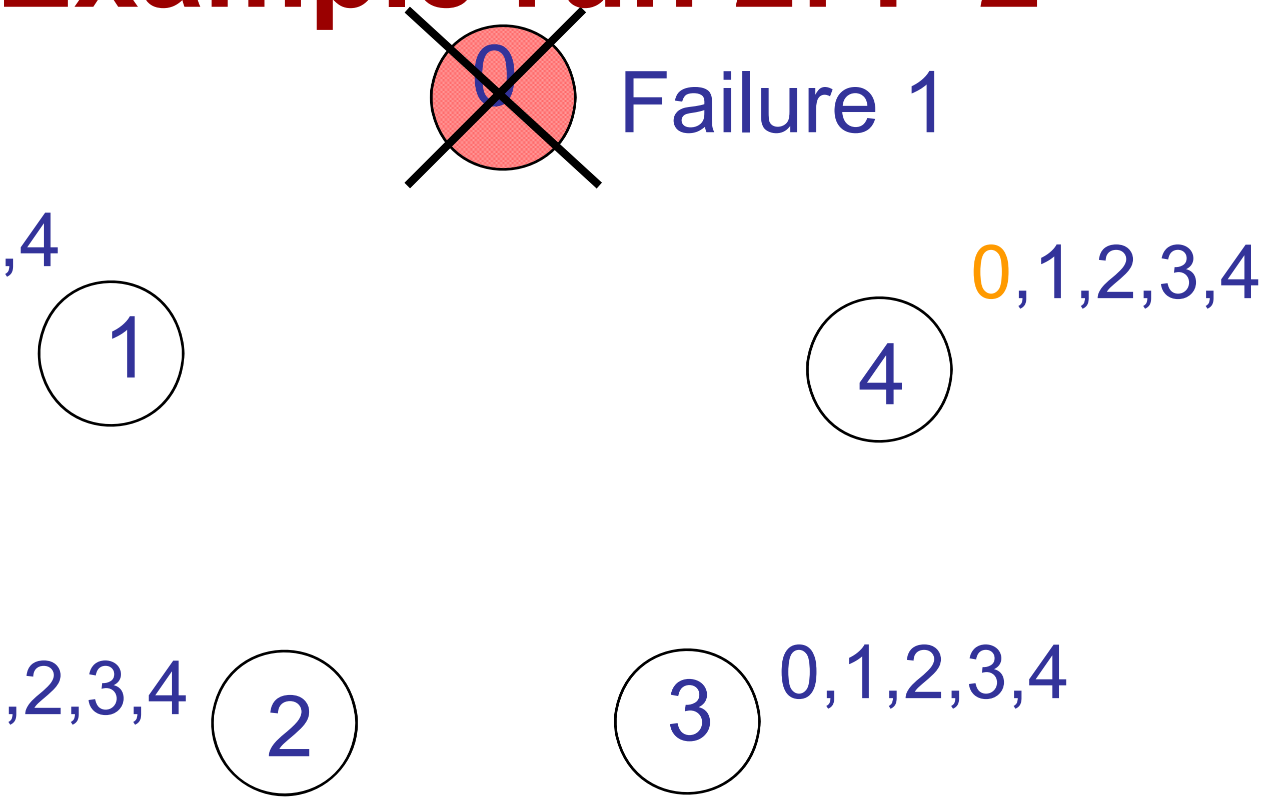
# Example run 2: f=2

Round 1



B-multicast all values to everybody

# Example run 2: f=2



Round 2

Failure 1

0,1,2,3,4

0,1,2,3,4

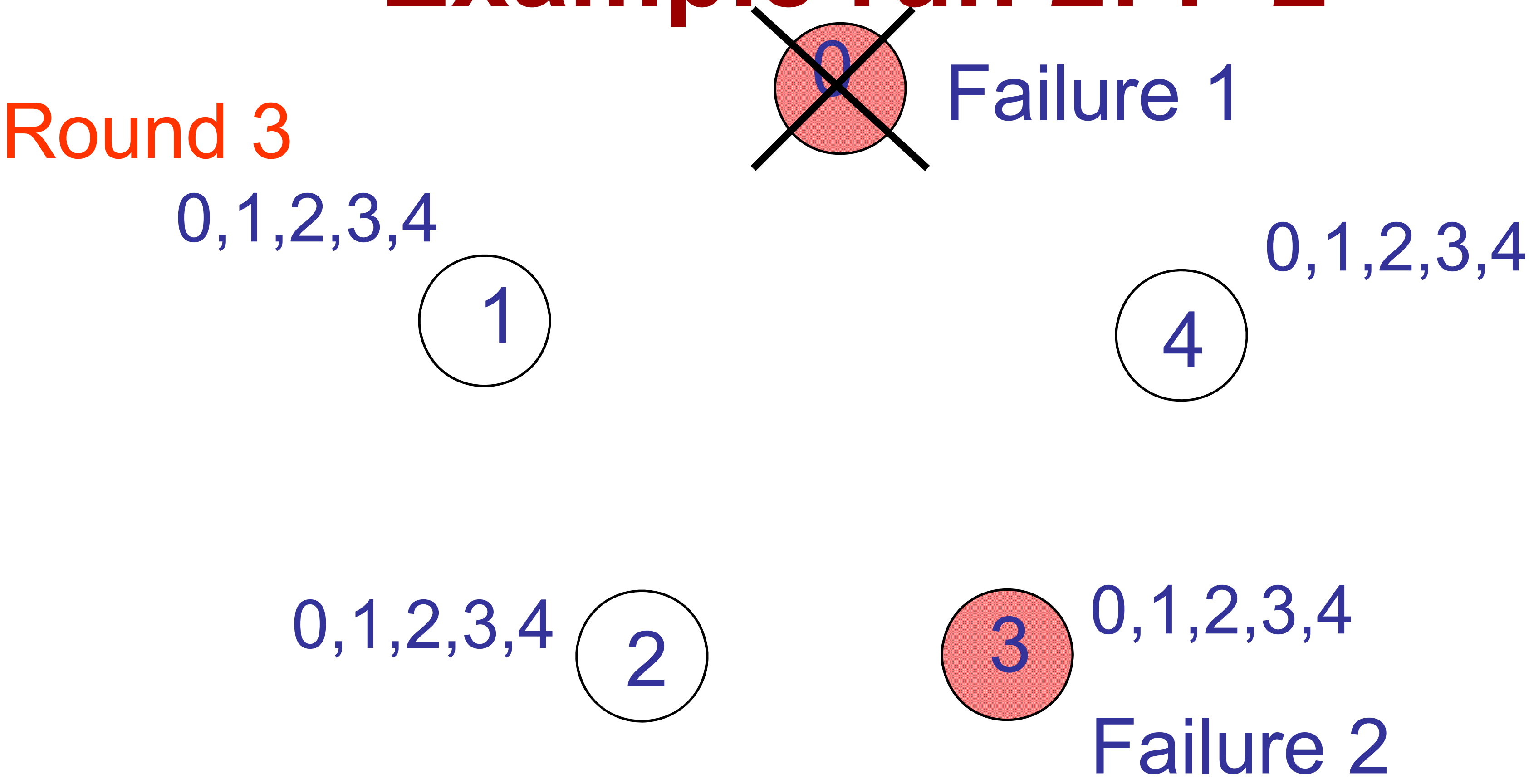
0,1,2,3,4

0,1,2,3,4

B-multicast new values to everybody

Remark: At the end of this round all processes know about all the other values

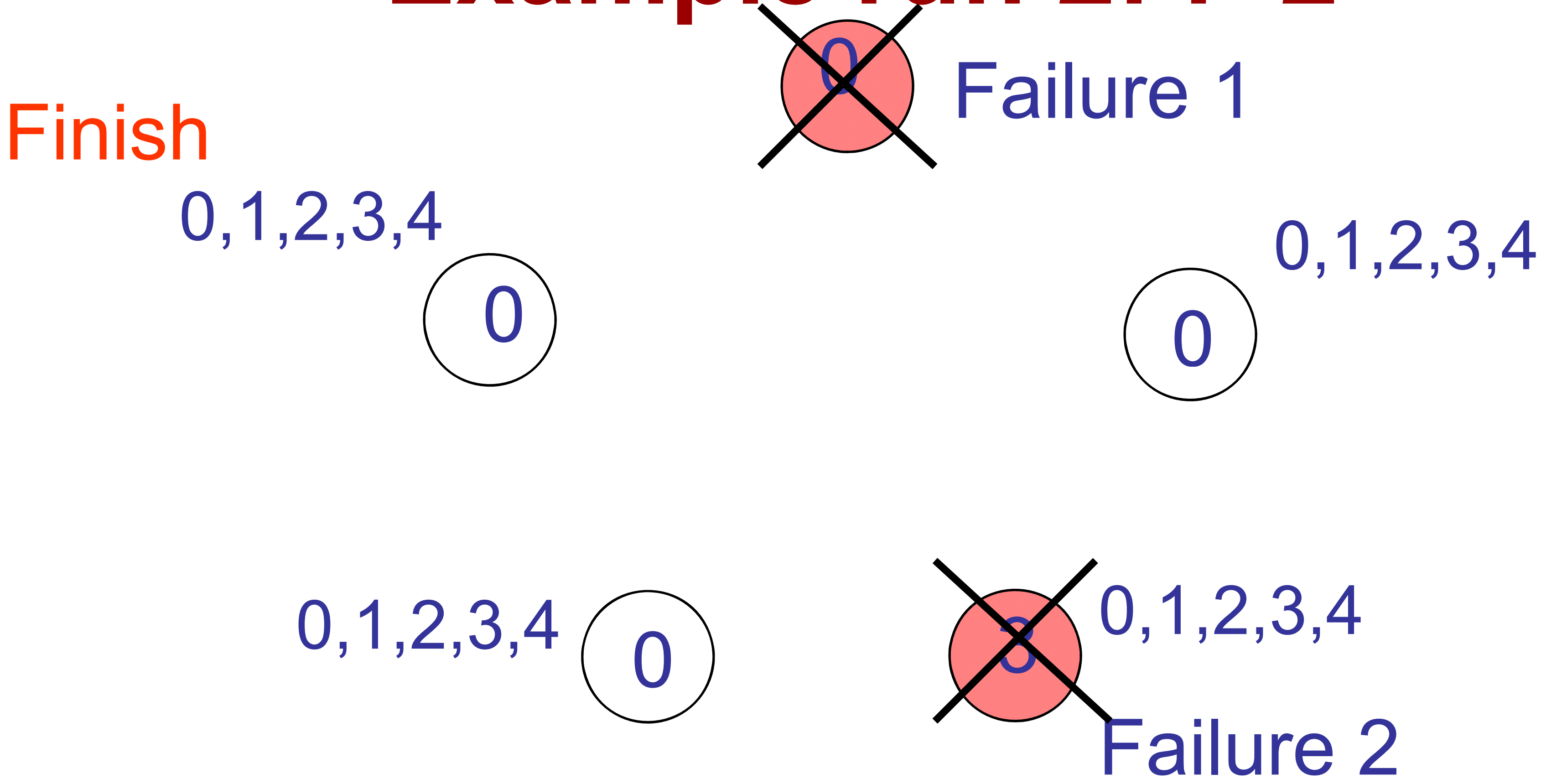
# Example run 2: f=2



B-multicast new values to everybody

(no new values are learned in this round)

# Example run 2: f=2



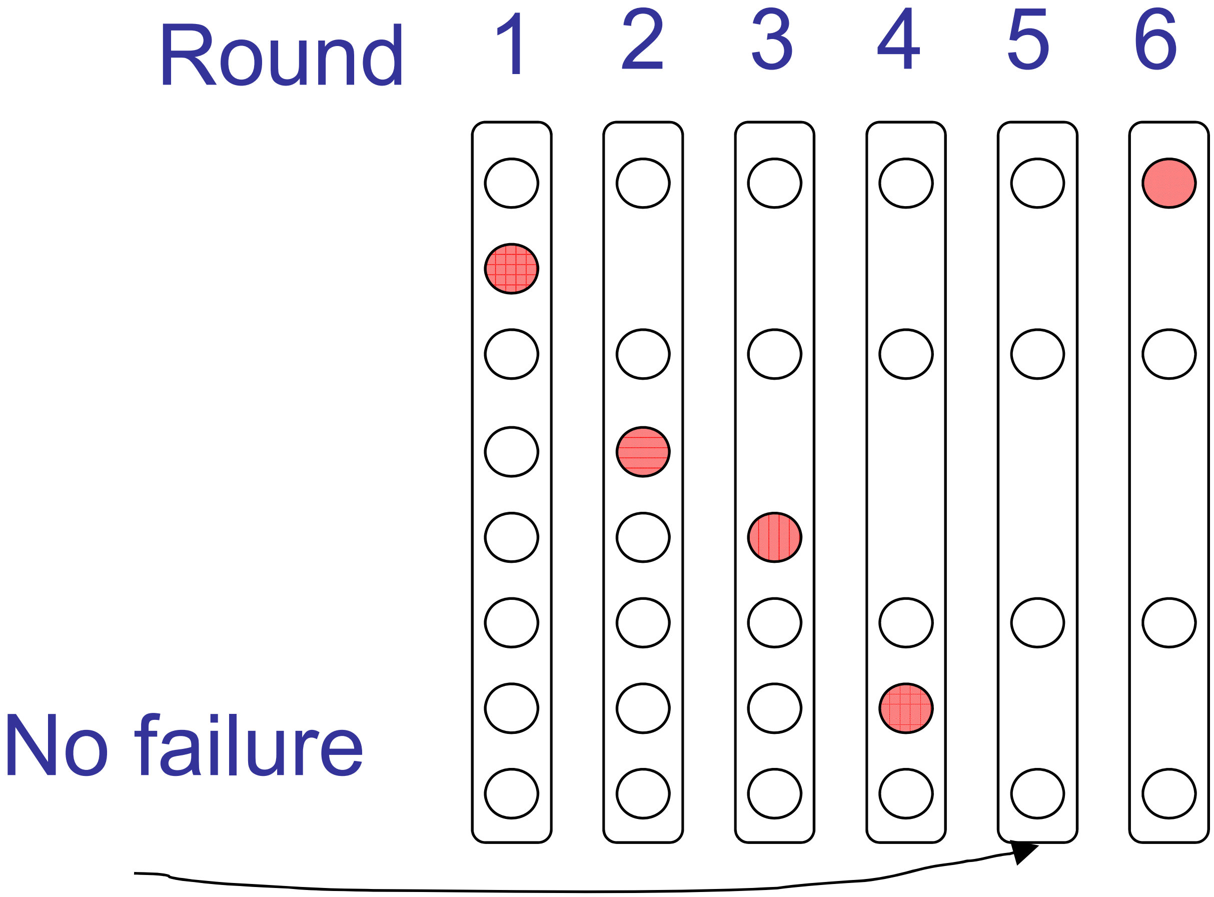
Decide on minimum value

# Observation

Example:

5 failures,

6 rounds



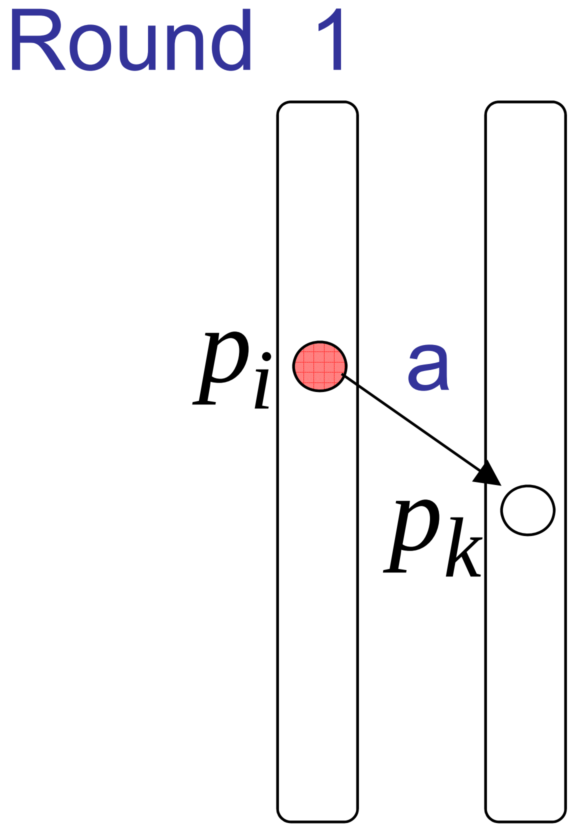
*If there are f failures and f+1 rounds then there is a round with no failed process*

# Need for f+1Rounds

- At the end of the round with no failure:
  - Every (non faulty) process knows about all the values of all other participating processes
  - This knowledge doesn't change until the end of the algorithm
- Therefore, at the end of the round with no failure:

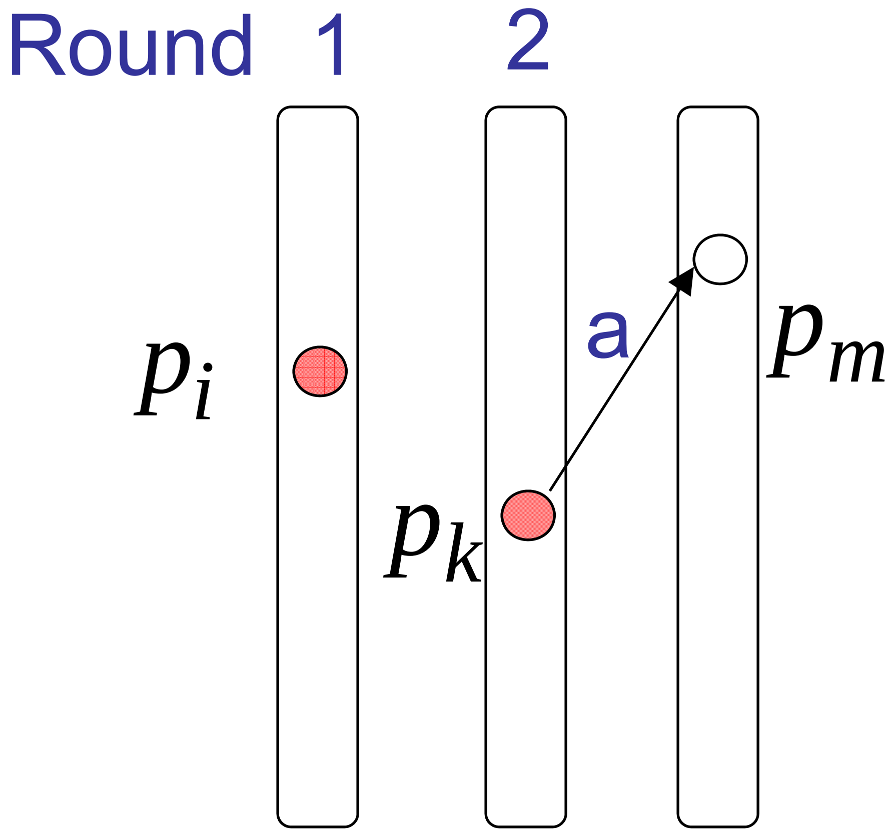
  everybody would decide the same value
- The exact position of this 'good' round is not known:
  - In worst-case we need f+1 rounds

# Worst-case Scenario



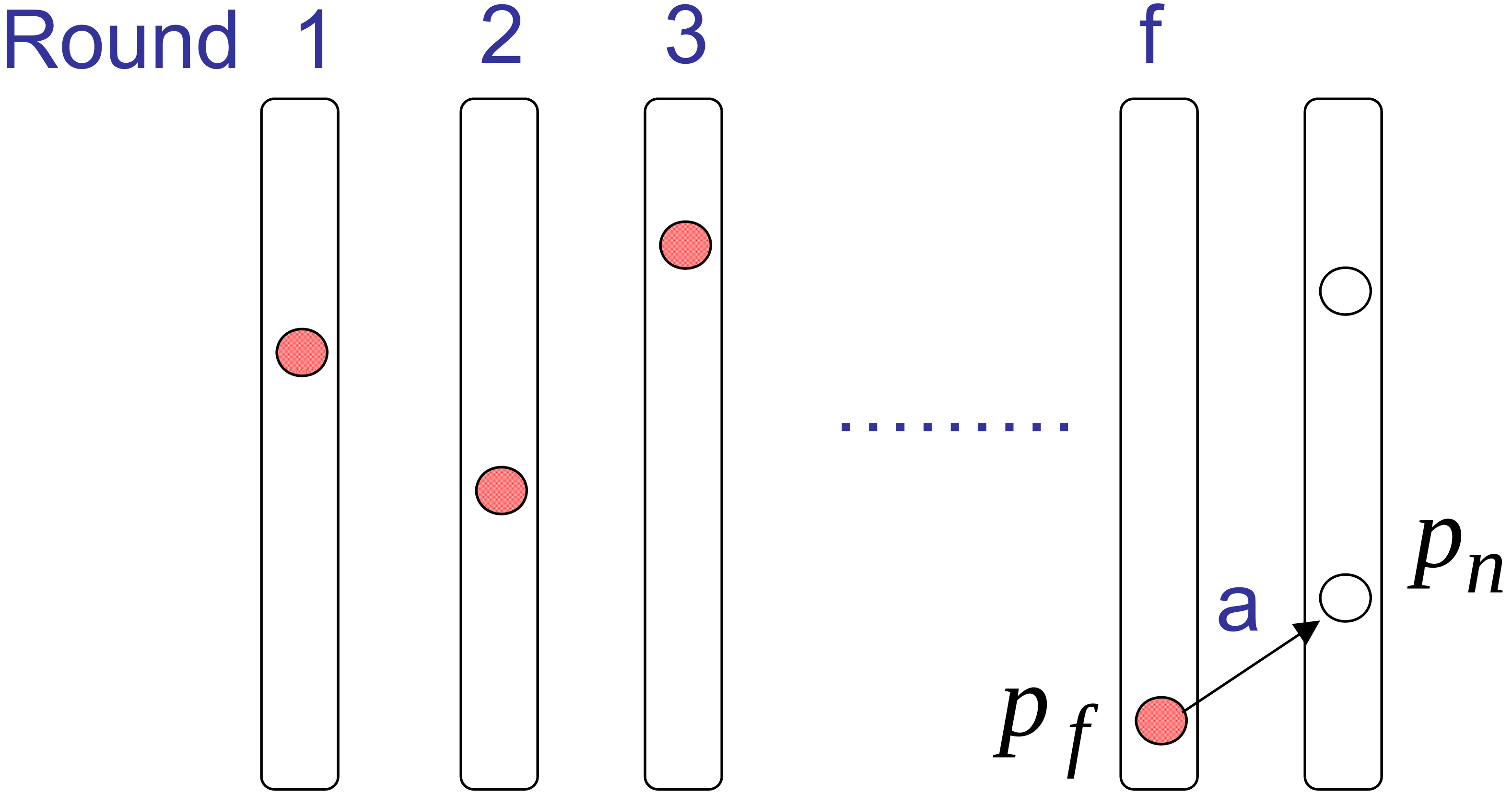
before process $p_i$ fails, it sends its value ***a*** to only one process $p_k$

# Worst-case Scenario



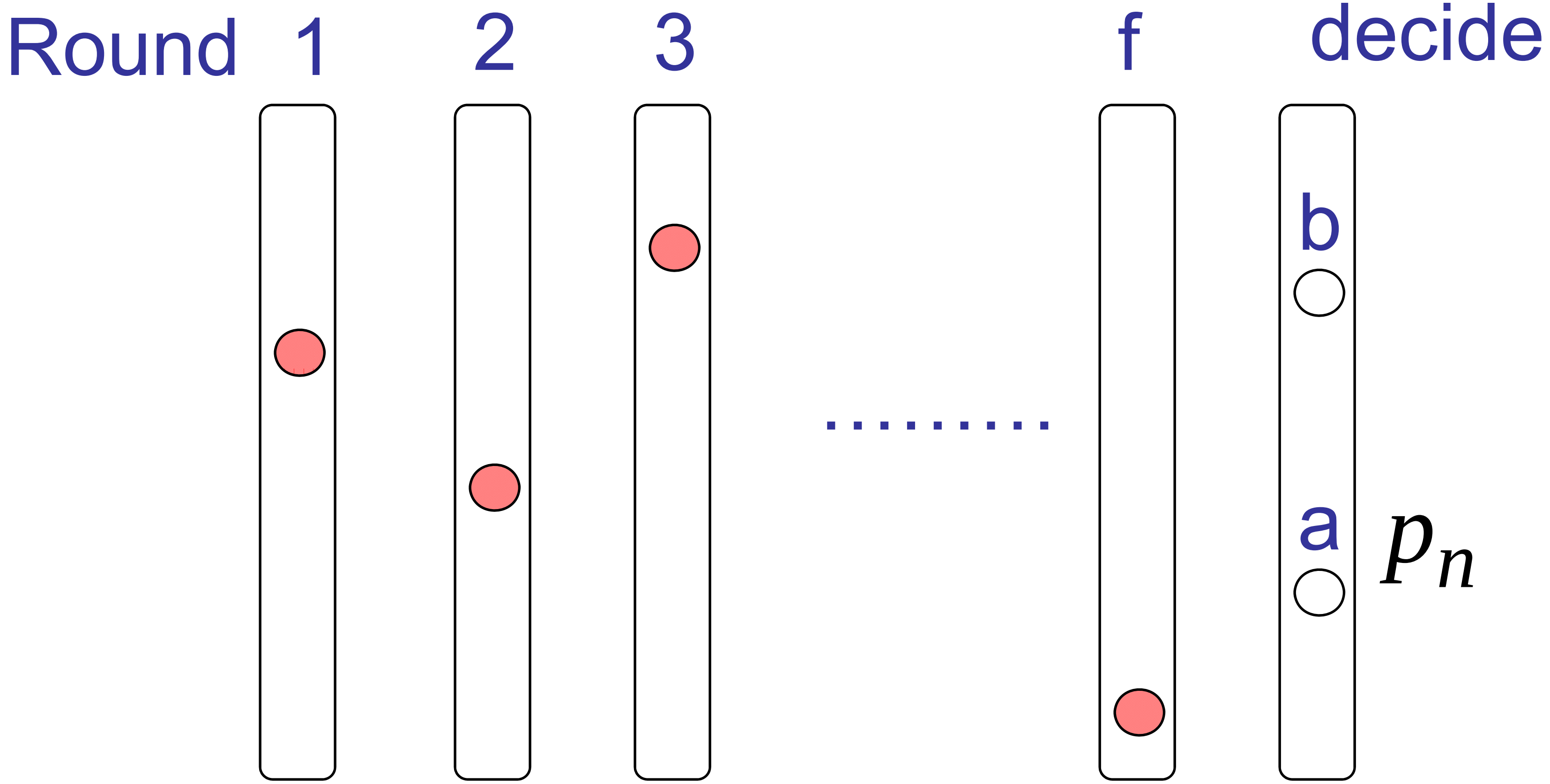
before process $p_k$ fails, it sends value **a** to only one process $p_m$

# Worst-case Scenario



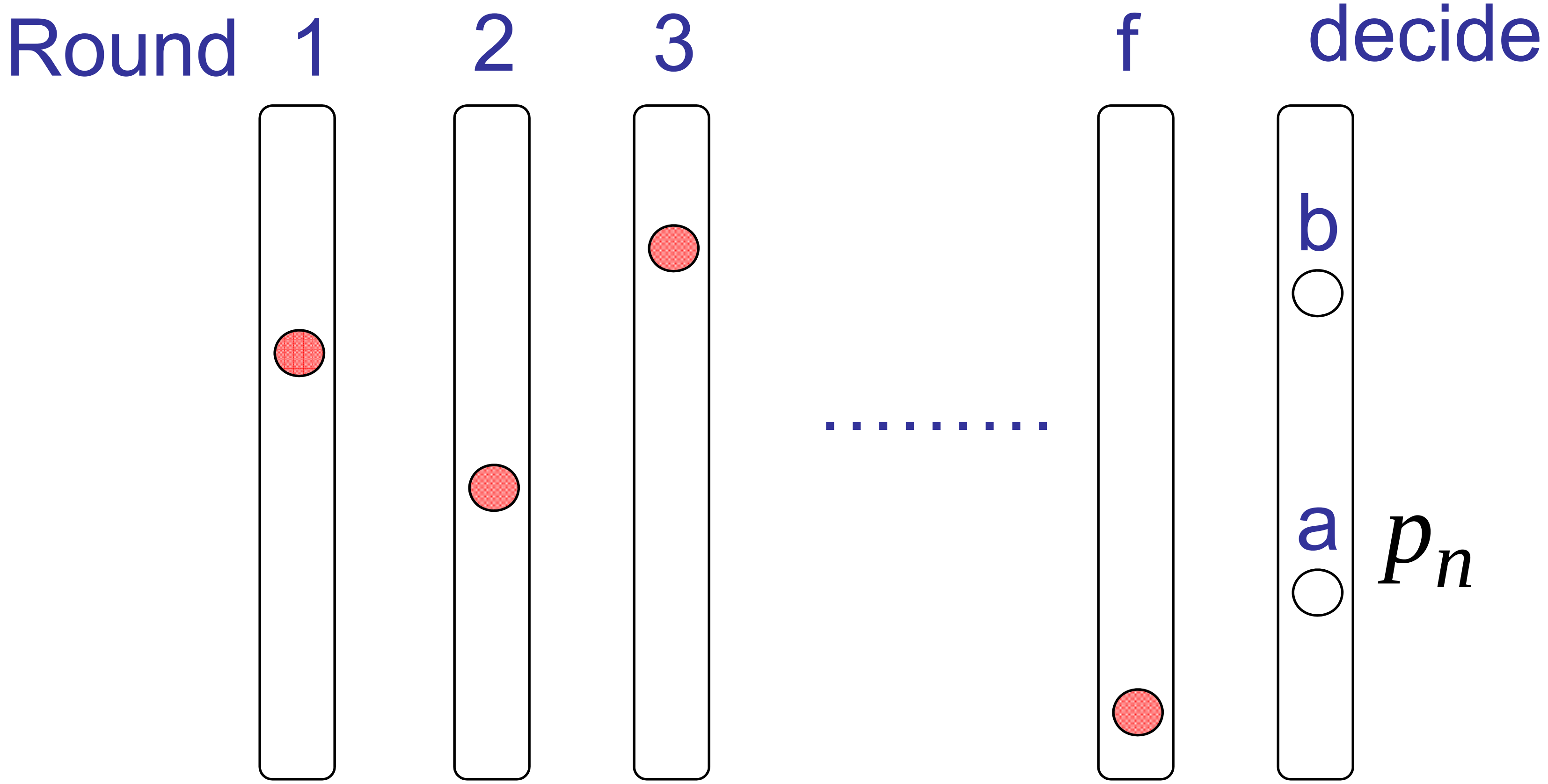
At the end of round **f** only one process $p_n$ knows about value **a**

# Worst-case Scenario



Process $p_n$ may decide **a**, and all other processes may decide another value **(b)**

# Worst-case Scenario



Therefore f rounds are not enough

At least f+1 rounds are needed

# A Lower Bound

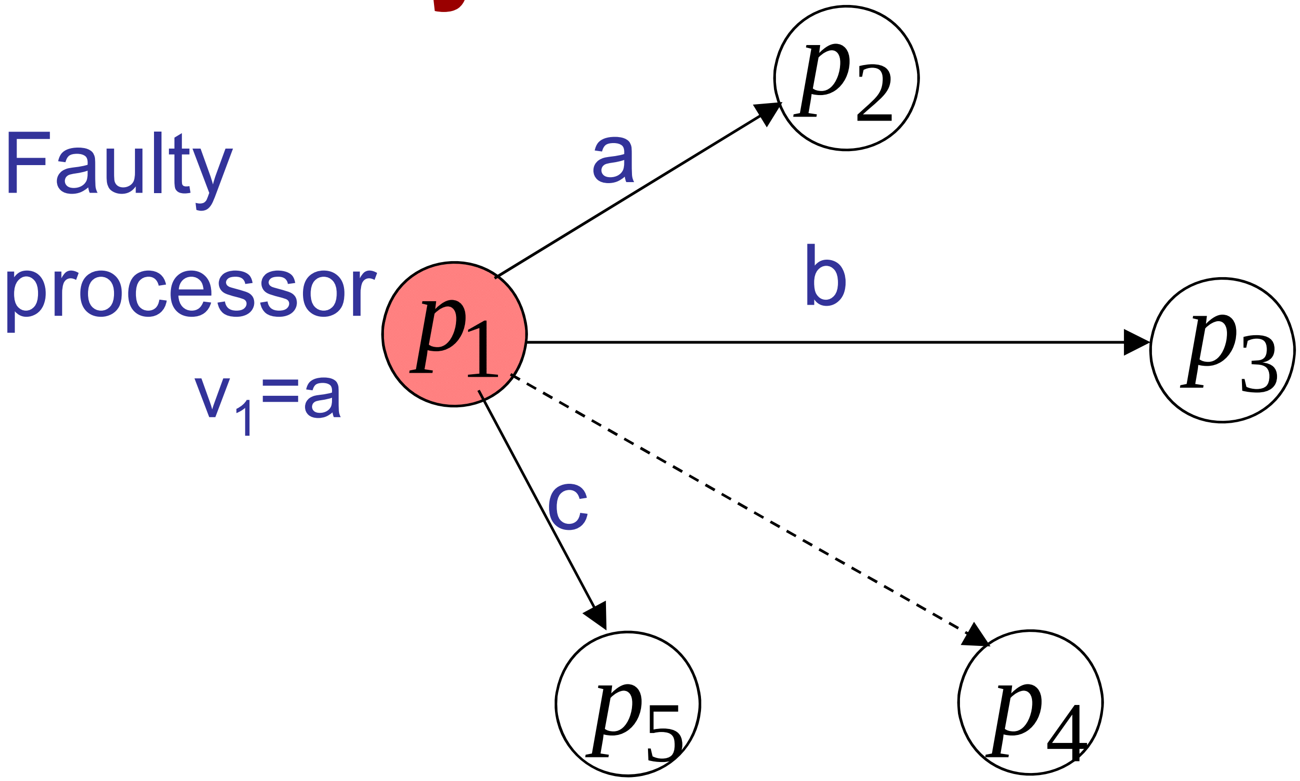
- Theorem
  - ***Any f-resilient consensus algorithm requires at least f+1 rounds***

# Byzantine Failures

# The Byzantine generals problem

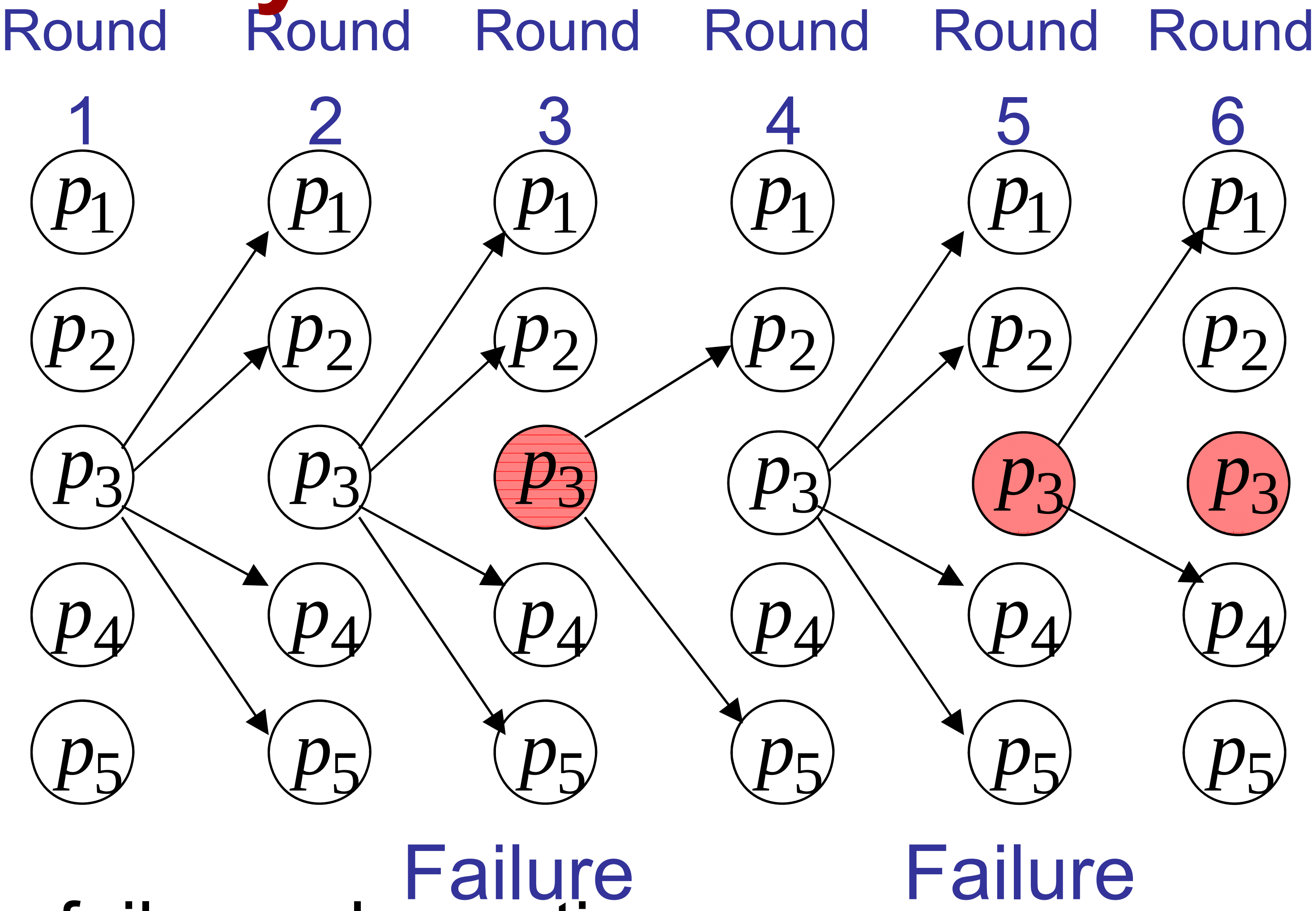
- Turkish invasion into Byzantium
  - Byzantine generals have to agree on attack or retreaval
  - The enemy works by corrupting the soldiers
  - Byzantine generals are notoriously treacherous ...
  - The loyal generals  have to prevent traitors from spoiling a coordinated attack
  - Messengers are sent to each other camps
  - Orders are distributed by exchange of messages, corrupt soldiers violate protocol at will
  - But corrupt soldiers can’t intercept and modify messages between loyal troops
  - The gong sounds slowly: there is ample time for loyal soldiers to exchange messages (all to all)

# Byzantine Failures



- Aka. Arbitrary Faults
  - Different processes receive different values
  - Ommision failures
  - Crash Failure

# Byzantine Failures

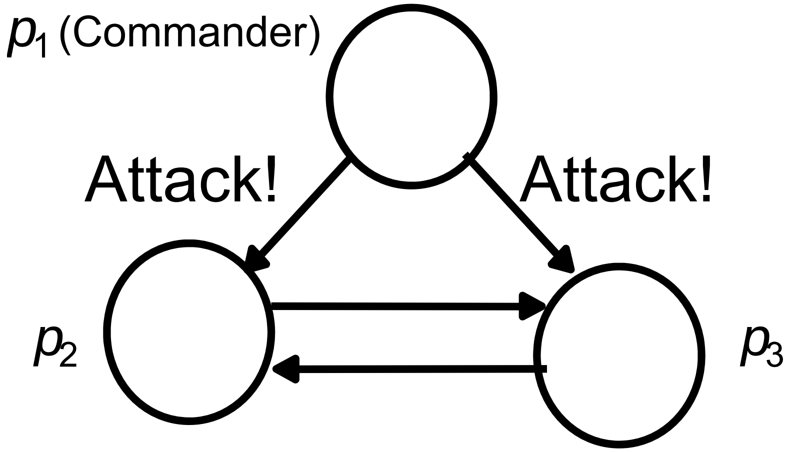


Round 1 Round 2 Round 3 Round 4 Round 5 Round 6

Failure Failure

After failure a byzantine process may continue functioning in the network

# Three byzantine generals



•Commanding general says attack or retreat!

•Processes may fail arbitrarily

•Processes must reach consensus

# Byzantine Generals

- ***Termination:*** Eventually each correct process sets its decision variable.
- ***Agreement:*** The decision value of all correct process is the same: if $p_i$ and $p_j$ are correct and have entered their *decided* state, then $d_i=d_j$ (for all $i,j2\ 1..N$).
- ***Integrity:*** If the *commander* is correct, then all correct processes decide on the value that the commander proposed.

# A Theorem

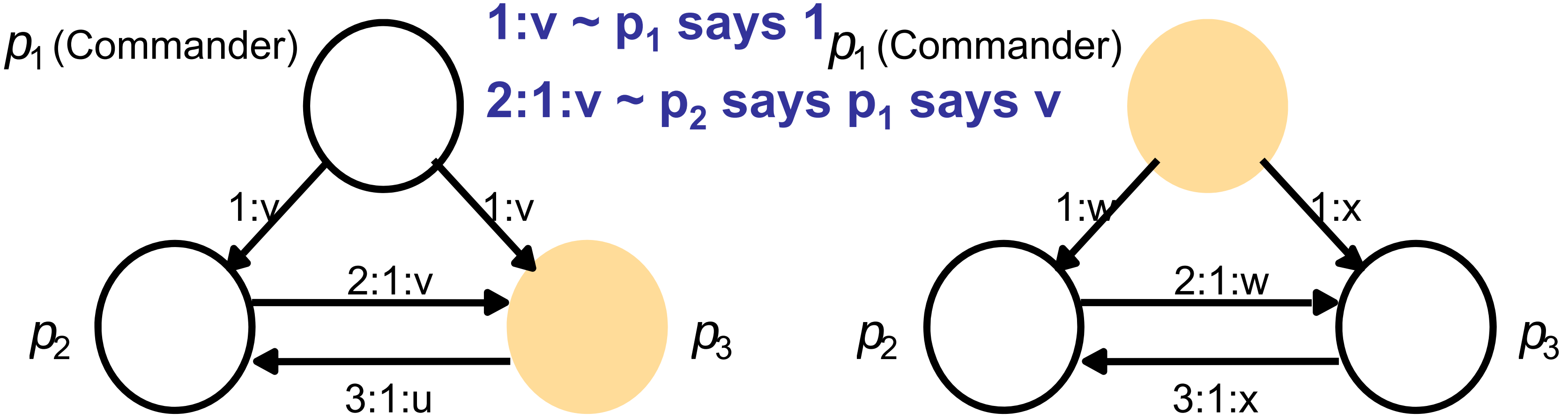
- N processes must tolerate f-faults
- ***There is no f-resilient algorithm if N≤3f***
- ***Outline***
  1. Impossibility with 3 processes case,
  2. Impossibility if N≤3f
  3. An algorithm for N≥3f+1 in synchronous systems
  4. Impossibility of consensus in asynchronous systems

# Impossibility of Three Byzantine Generals

**Notation:**

**1:v ~ $p_1$ says 1**

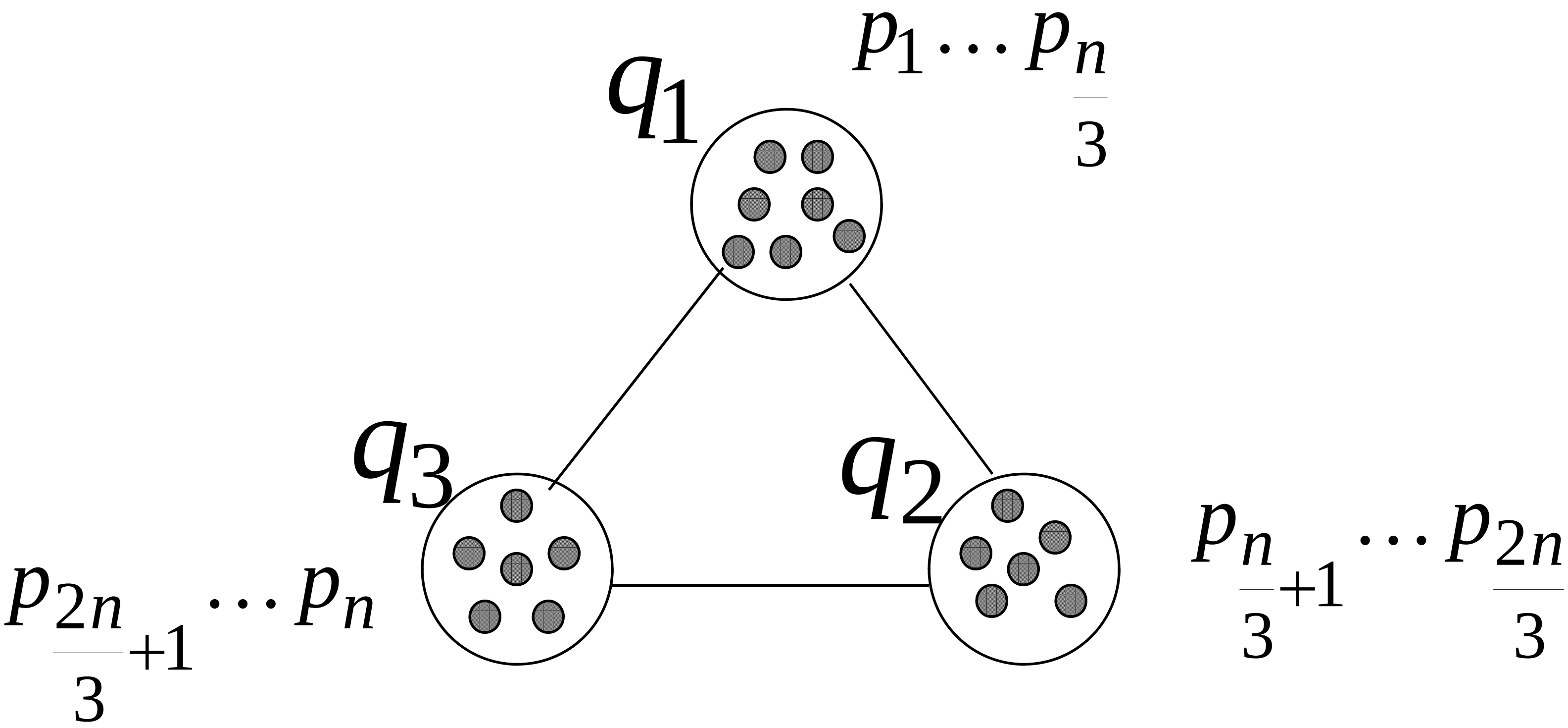
**2:1:v ~ $p_2$ says $p_1$ says v**



Faulty processes are shown shaded

1. Left: $p_2$ gets conflicting information. Which is correct?
2. If commander is correct $p_2$ and $p_3$ must decide ***v*** accordingly (integrity)
3. Right: Symmetrically, $p_2$ must decide ***w*** and $p_3$ must decide ***x***
4. An algorithm cannot distinguish scenarios: **No Agreement**

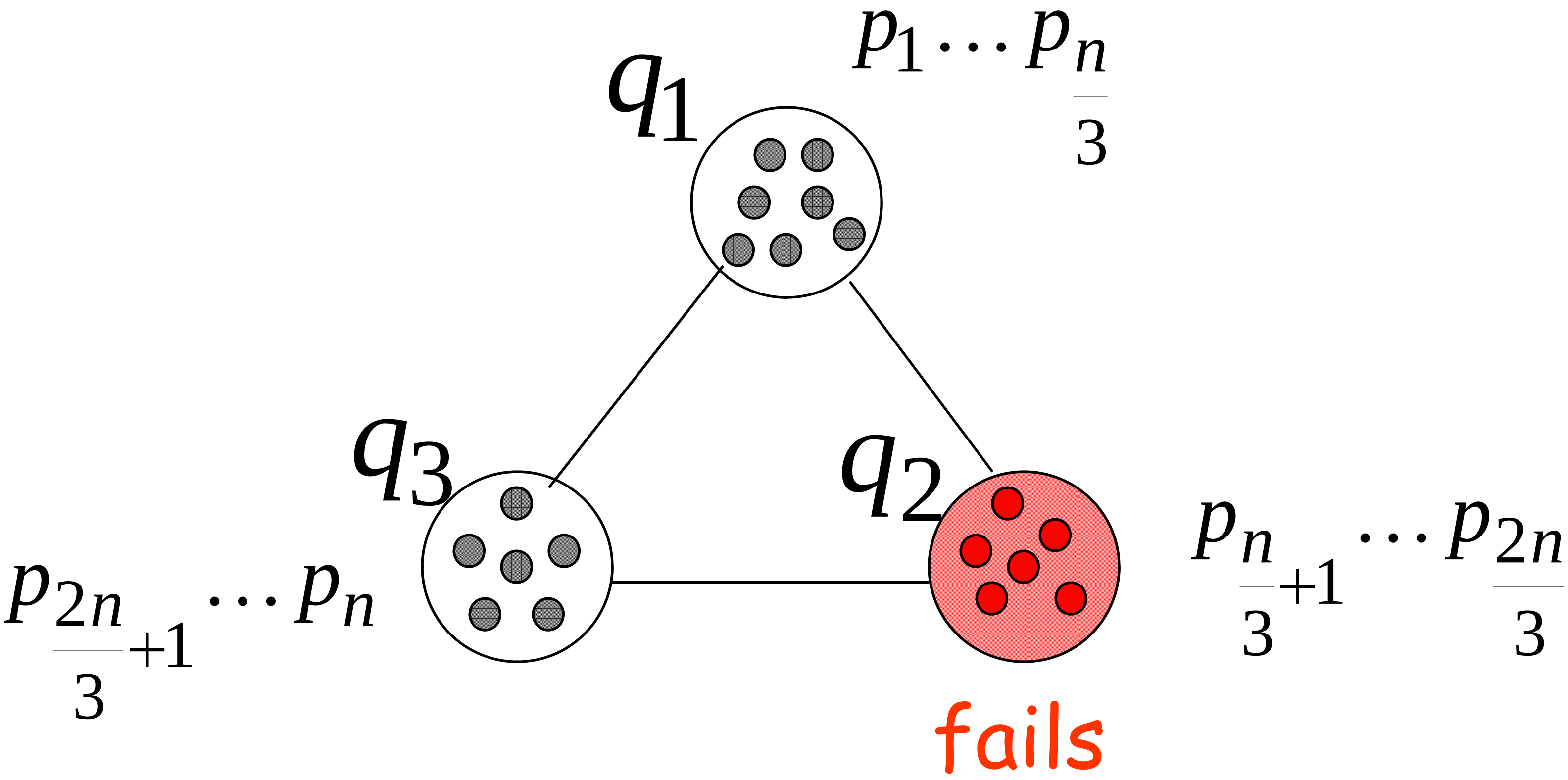
# Impossibility of
# N≤3f Byzantine Generals


$q_1$
$p_1 \cdots p_{\frac{n}{3}}$
$q_3$
$q_2$
$p_{\frac{2n}{3}+1} \cdots p_n$
$p_{\frac{n}{3}+1} \cdots p_{\frac{2n}{3}}$

Reduction:

Each process q simulates N/3 processes using **algorithm X**

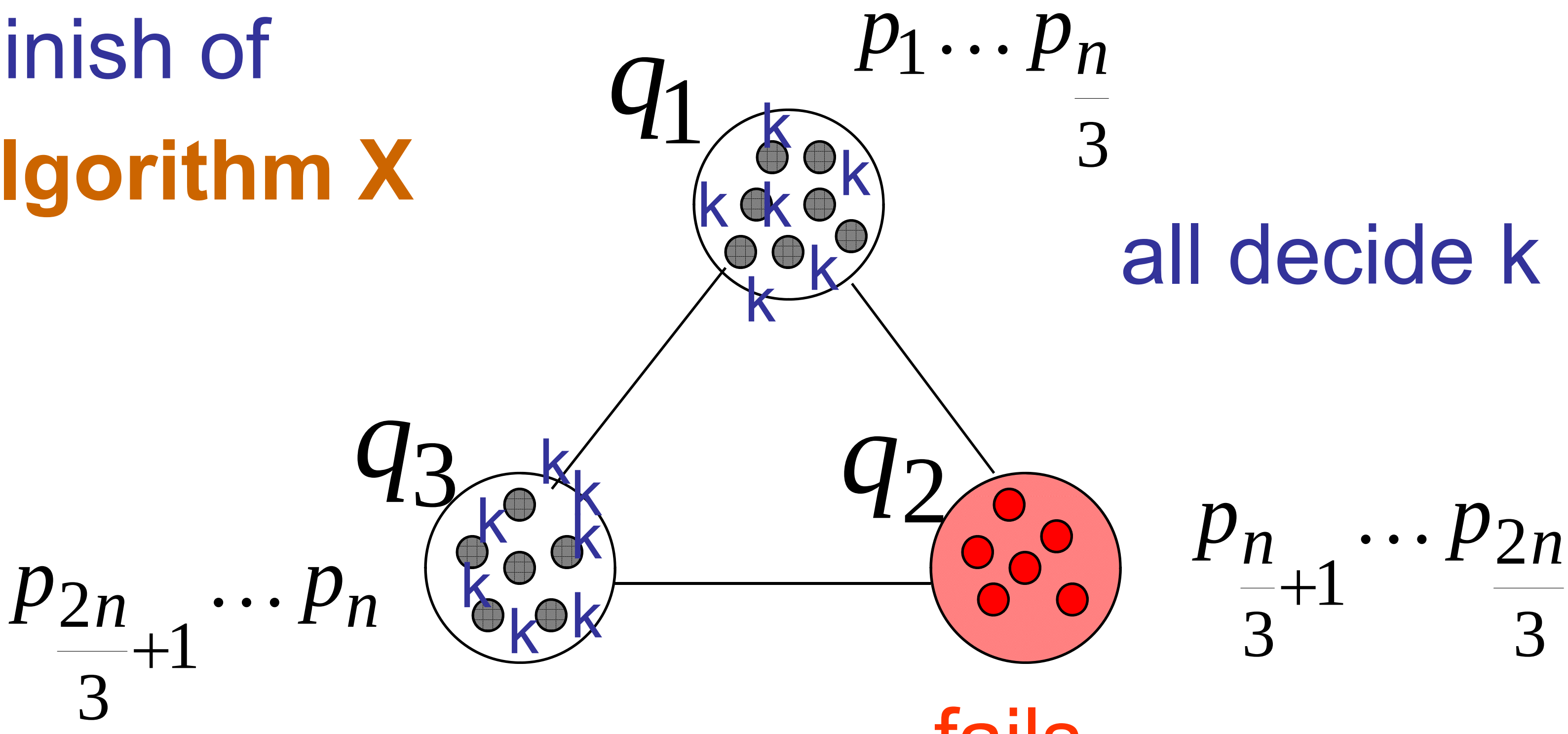
# Impossibility of N≤3f Byzantine Generals


$q_1$
$p_1 \cdots p_{\frac{n}{3}}$
$q_3$
$q_2$
$p_{\frac{2n}{3}+1} \cdots p_n$
$p_{\frac{n}{3}+1} \cdots p_{\frac{2n}{3}}$
fails

When a ‘q’ fails n/3 then processes fail too

# Impossibility of N≤3f Byzantine Generals

Finish of **algorithm X**


$q_1$
$p_1 \cdots p_{\frac{n}{3}}$
k
$q_2$
$p_{\frac{n}{3}+1} \cdots p_{\frac{2n}{3}}$
$q_3$
$p_{\frac{2n}{3}+1} \cdots p_n$

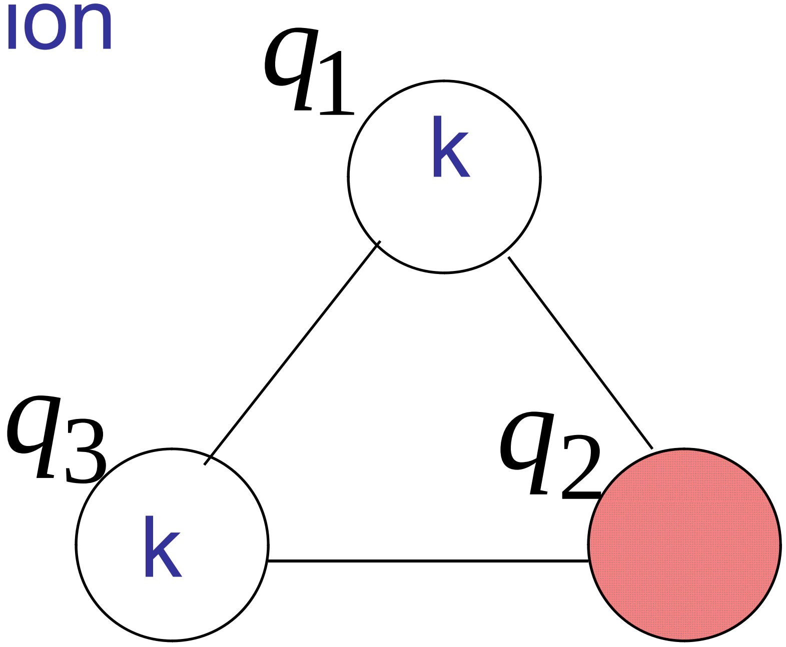
all decide k

fails

**algorithm X** tolerates n/3 failures

# Impossibility of
# N≤3f Byzantine Generals

Final decision



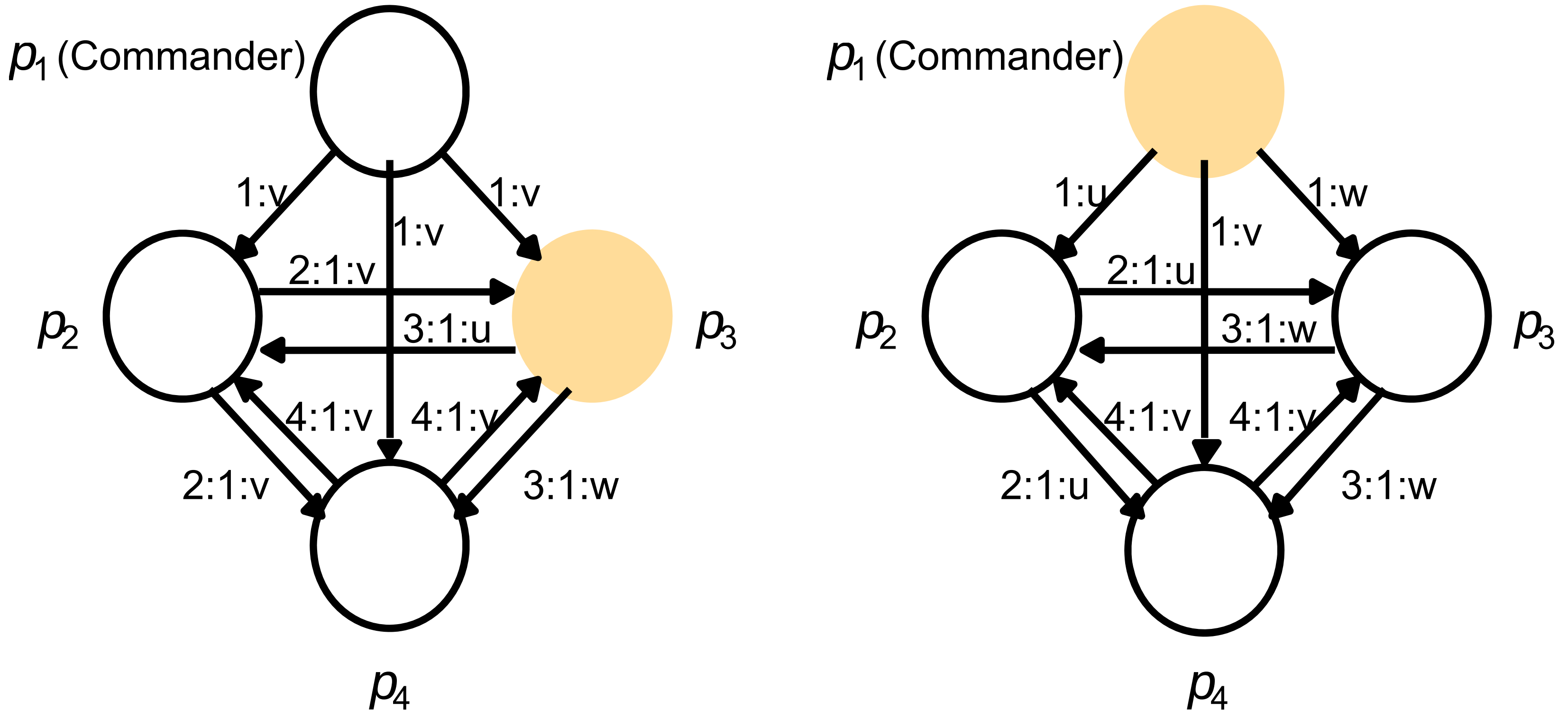
fails

We reached consensus with 1 failure

**Previously shown Impossible!!!**

**algorithm X cannot exist**

# Four byzantine generals



Faulty processes are shown shaded

$p_2$ and $p_4$ agrees:
$d_2$ =majority (v,v,u)=v

$d_4$ =majority (v,v,w)=v

$p_2$, $p_3$, and $p_4$ agrees:
$d_2$= $d_2$ = $d_4$ =majority (v,u,w)=$\perp$

$\Rightarrow$Use common default value

# Cost of Byzantine Generals

- Requires $f$+1 rounds,
- Sends $O(n^{\text{f+1}})$ messages
- If we use digital signatures a solution exist with $O(n^2)$ messages (f+1 rounds)
  - False claims not possible:
  - If ”p says v” other processes can detect if ”q says p says w”
- Truely arbitrary failures are rare.

# Impossibility of Consensus in *asynchronous* systems

- No algorithm exists to reach consensus
  - (Concensus may possibly (very often) be reached, but cannot always guaranteed)
  - Neither for crash or byzantine failues
- Eg. Two-army problem:
  - There is some program continutation that avoids consensus
- No guaranteed solution to
    - Byzantine generals problem
    - Interactive consistency
    - Totally ordered reliable multicast

# Two-Army Problem

Carthage
Bad guys
Sparta

The two-army problem:

1. Sparta and Carthage together can beat Bad guys but not individually. Therefore, they have to decide to attack at exactly the same time.
2. Sparta general sends a message to Carthage general to attack at noon
3. How does he know that Carthage general received the message?

Messenger (unreliable channel)

***Arbitrarily slow processes (or channels) are indistinguishable from crashed ones (omission)***

# Workarounds in an asynchronous system

- Masking faults:
  - restart crashed process and use persistent storage
  - Eg recovery files like in databases
- Use failure detectors:
  - make failure *fail-silent* by discarding messages
- Probabilistic algorithms:
  - conceal strategy for adversary

END