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Quantitative Models and Analysis for Reactive Systems

PhD Defence

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Distributed and Embedded Systems unit,
Department of Computer Science, Aalborg University

November 18, 2011

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- The big picture
- Specifications and Reactive Systems
- Research Hypothesis and Goals

2 Models, Specifications, and Verification
- Timed Automata and Semantics
- Specification Languages: CTL

3 Robustness and Quantitative Analysis
- From equivalences to distances
- Approximating properties of systems
- Approximating Specifications
- Robustness and Implementability

4 Conclusion & Final Remarks

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Understanding systems, and making useful predictions

- Errors persist in mission and safety critical systems.
- Failures are expensive or tragic.

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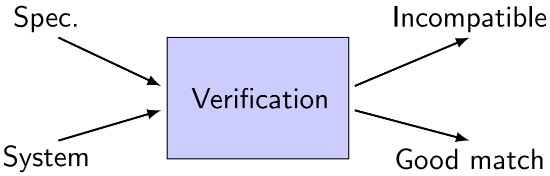
- Solution: *Formal methods*
  - Models of systems and requirements
  - Verification: manual or automated.



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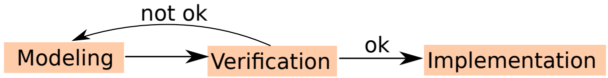

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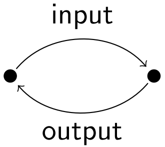
Reactive systems

- Reactive systems [Pnueli'85] & [Milner '89]
  - non-terminating, communicating



- Control systems.
- Embedded systems.
- Distributed and communicating systems.

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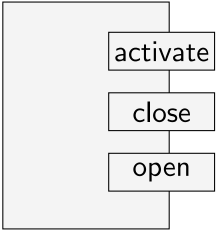
Specifications



Qualitative

1. If **open** occurs an **activate** must have been performed.
2. If **open** occurs **close** must follow.

Quantitative

3. **open** will occur at most $12ms$ after **activate**.
4. ...

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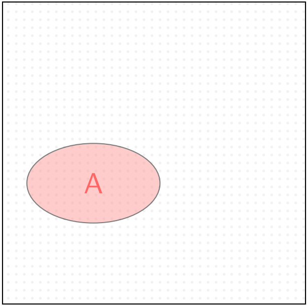
Specifications and Systems

- Systems are solutions to specifications



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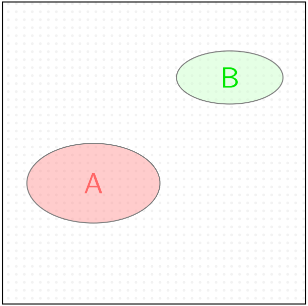

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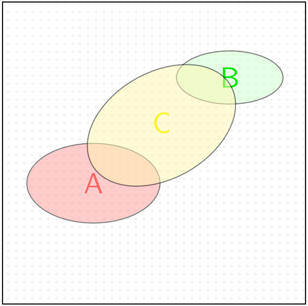

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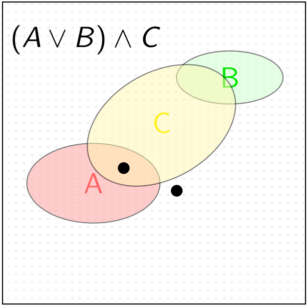

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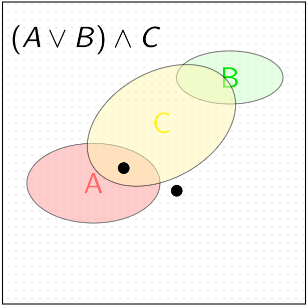

- What if $(A \vee B) \wedge C$ is empty?
- How can we choose between systems for a specification?
- **Idea:** Specifications rate systems e.g. $A(s) = 0.7$

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Thesis

Research Hypothesis

Using **quantitative techniques** and **game** theoretic approaches, it is possible to leverage the limitations of the Boolean framework for formal verification of reactive systems.

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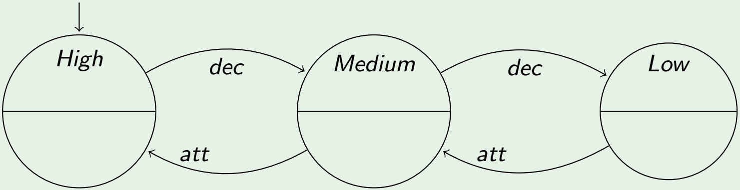
Real-Time Reactive Systems

- Timed automaton [Alur and Dill '94]
- Weighted timed automaton [Alur+, Behrmann+ '01]

A simple production system



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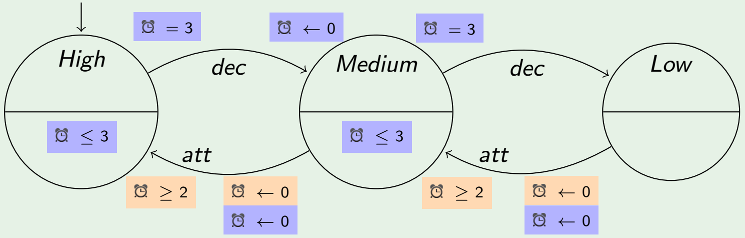

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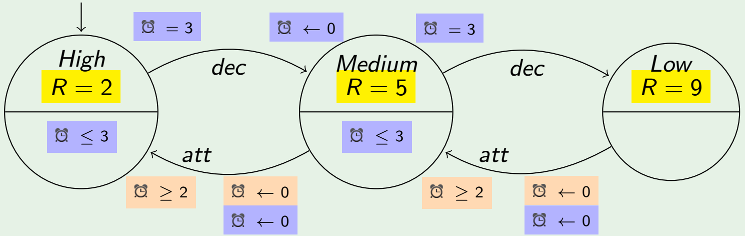

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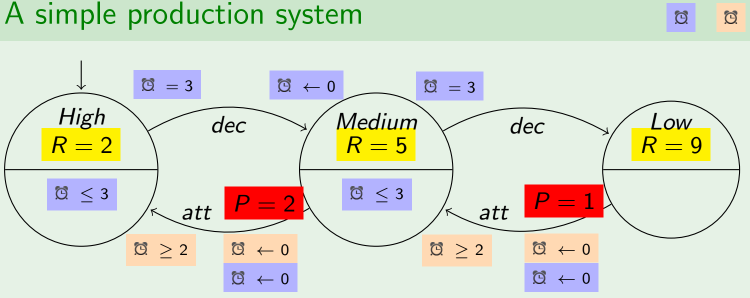

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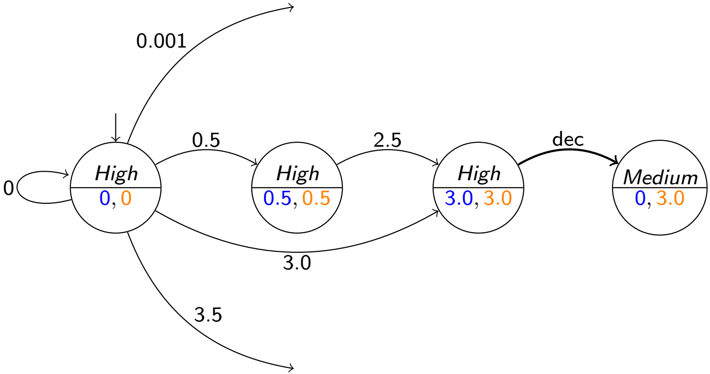
Semantics



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Ehrenfeucht-Fraïssé games

- Characterizes Bisimulation [Milner '89]
- A blind Attacker gives us trace equivalence [Hoare '85]
- Time abstract games!

Bisimulation game [Stirling '95]

An *Attacker* and *Defender* plays a round from s and t.

1. Attacker chooses s or t and a move, e.g. $s \xrightarrow{a} s'$
2. Defender proposes a matching move, e.g. $t \xrightarrow{a} t'$, from s or t opposite the attacker.

Another round is played from s' and t' if a match was found.

- $s \not\sim t$ if the attacker can *win*, and $s \sim t$ otherwise.

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Specification languages: CTL

- Safety properties & invariants

CTL Specifications [Clarke, Emerson '81]

- AGEX(*true*) — Non-termination.
- EF(*error*) — Reachability.
- AGEF(EX(*Medium*) $\vee$ EX(*Low*)) — "invariant choice".

TCTL Specifications [Alur, Courcoubetis Dill '93]

- $High \vee EF_{[0,4]} High$ — If the production level is not H, can it be obtained within 4 time units?

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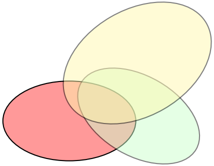
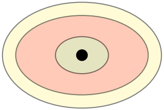
Properties of CTL

[Brown, Clarke, Grümberg '87]





- Adequacy: CTL can distinguish (only) inequivalent systems.
- Expressivity: can express specifications with exactly one solution (up to $\sim$)!

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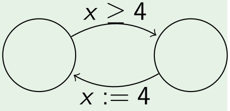
Towards Quantitative Analysis

Models ✓



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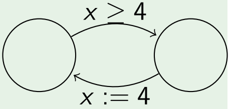

Specifications ✓

$\mathrm{AF}_{[0,4]} \mathit{High}$

Towards Quantitative Analysis

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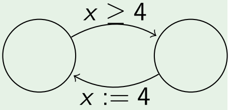
Models ✓



Specifications ✓

$AF_{[0,4]} High$

Verification

$[\![\phi]\!](s) = 3.14$

$d(s,t) = 42$

Boolean world	**"Quantification"**
Trace equivalence $\equiv$	Linear distance d_L
Bisimilarity $\sim$	Branching distance d_B
$s \sim t$ implies $s \equiv t$	$d_L(s,t) \leq d_B(s,t)$
Satisfaction $s \models \phi$	Multi-valued $[\![\phi]\!](s) \in \mathbb{R}$

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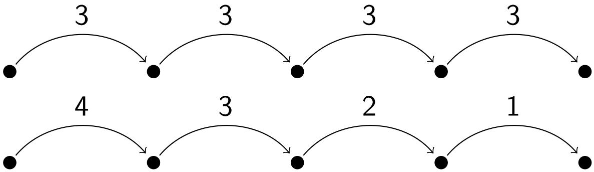
Behavior revisited

- Games are no longer win/loose but have values.
- Players try to optimize the value of the game.



- Point-wise distance 2
- Hamming distance 3
- Accumulating (discounted) distance 4
- Maximum-lead distance 2

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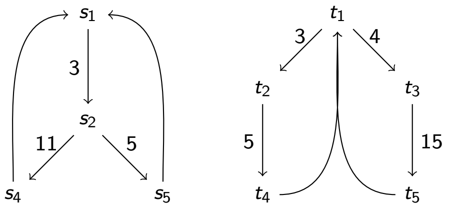
Measuring the dissimilarity between systems

The accumulating distance, discounted by $\lambda = 0.9$



$$d_L(s_1, t_1) = \sum_i (1 + 4\lambda)\lambda^{3i} \approx 17.0$$

$$d_B(s_1, t_1) = 1 + 10\lambda + \lambda^3 d_B(s_1, t_1) \approx 36.9$$

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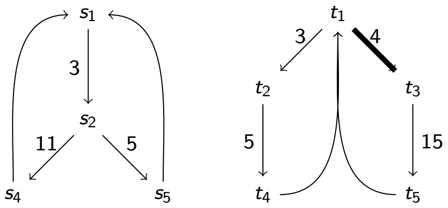

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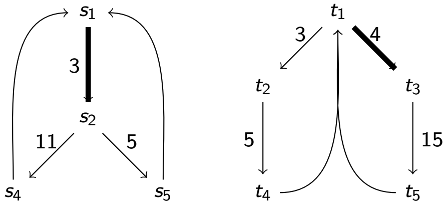

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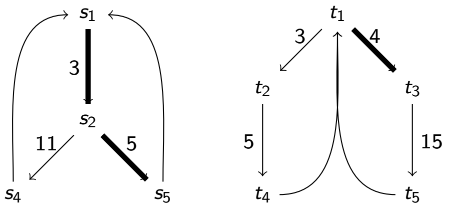

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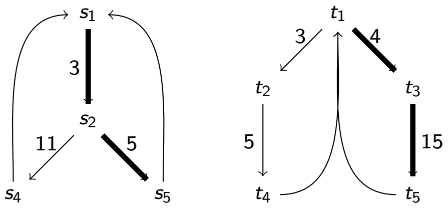

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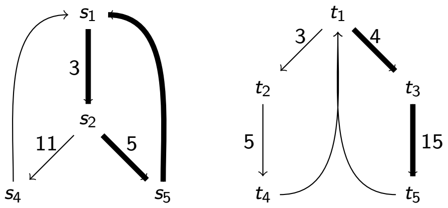

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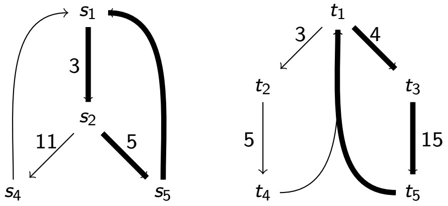

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Measuring the dissimilarity between systems

- In paper A

Theorem

Branching distances bound linear distances.

Theorem

For discounting factor $\lambda < 1$, accumulating branching distance from deterministic to non-deterministic weighted timed automata is computable.

- In paper B

Theorem

Computing accumulating distance is polynomial-time equivalent to computing the payoff for discounted games.

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Interpreting WCTL quantitatively

Syntax & Semantics

$$\Phi ::= \mathsf{p} \mid \neg \mathsf{p} \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid \mathsf{E}\Psi \mid \mathsf{A}\Psi$$

$$\Psi ::= \mathsf{X}_c \Phi \mid \mathsf{G}_c \Phi \mid \mathsf{F}_c \Phi \mid [\Phi_1 \mathsf{U}_c \Phi_2]$$

Every ϕ is interpreted $[\![\phi]\!]$: as a function in $[S \to \mathbb{R}_{\geq 0}]$

Example: $\phi = \mathsf{AG}(\mathit{High} \vee \mathsf{EF}_2 \mathit{High})$

$$[\![\phi]\!](s) = \sup_{\sigma \in P(s), k} \min \begin{cases} [\![\mathit{High}]\!](\sigma^k) \\ \inf_{\sigma' \in P(\sigma^k), k'} \sum_{j=0}^{k} |\sigma'(j)_w - 2| + [\![\mathit{High}]\!](\sigma'^{k'}) \end{cases}$$

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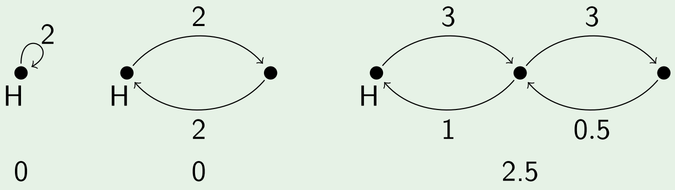
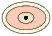


Properties of WCTL

- In paper D

Quantitative Adequacy [de Alfaro, Faella, Stoelinga'04]

For every S and T $d_B(S,T) \leq \epsilon$ if and only if, for every property ϕ in WCTL $|[\![\phi]\!](S) - [\![\phi]\!](T)| \leq \epsilon$



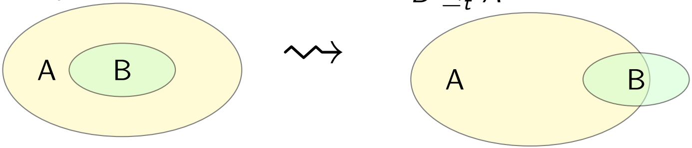
Quantitative Expressiveness

For each S, and every $\gamma > 0$, there is a (single) characteristic property ϕ_S^γ in WCTL, such that: $[\![\phi_S^\gamma]\!](T) \in [\epsilon - \gamma, \epsilon + \gamma]$ if and only if $d_B(S,T) \leq \epsilon$

Approximating Specifications: MTS & Quantitative Refinement

- Modal Transition Systems [Larsen & Thomsen '88]
  - (De)Composition of specifications: $A \parallel B$ and $A \setminus\!\!\setminus B$

$B \le_t A$ $\quad\leadsto\quad$ $B \le_t^{\epsilon} A$



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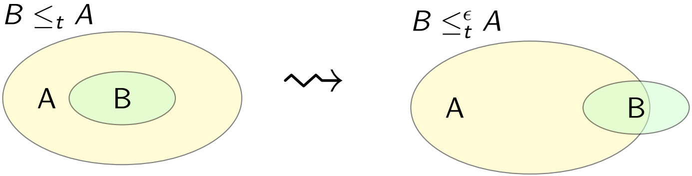

Results: Paper E

+ EXPTIME-hard to decide $B \leq_t^\epsilon A$, given $\epsilon > 0$
+ $B \leq_m^\epsilon A$ is decidable in $NP \cap coNP$, given $\epsilon > 0$
− No suitable conjunction operator ($\wedge$) is definable.

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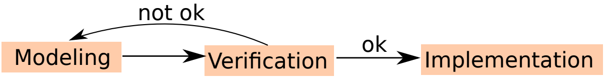

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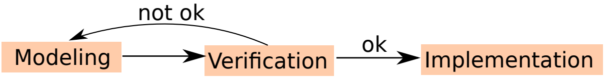

- Digital clock suffers from drift and finite precision.
- Digital hardware has finite execution speed.

Dynamical properties [Puri'98]

The effects of physical hardware corresponds to implicitly statically *enlarging* all constraints by some small $\Delta > 0$.

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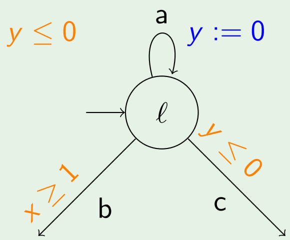
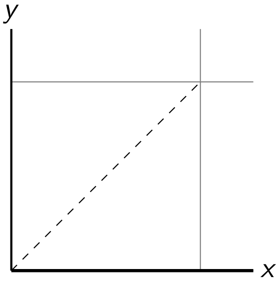
Example $\mathcal{B}$





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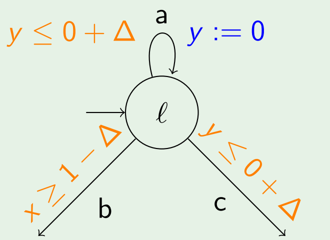
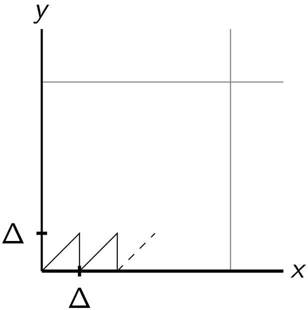
Example $\mathcal{B}_\Delta$





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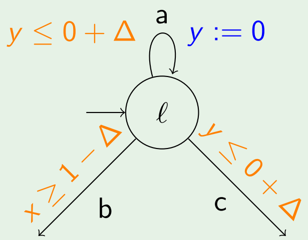
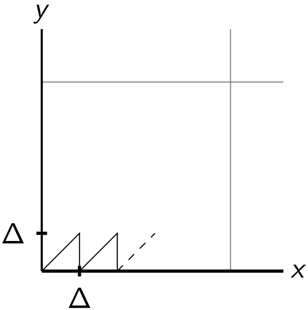
Example $\mathcal{B}$





- The Attacker wins the untimed game: **b** and **c** are available

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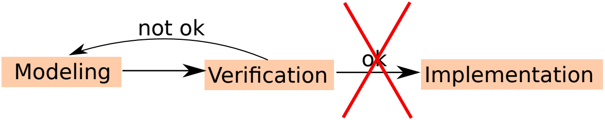

- Enlargement may induce extra (discrete) behavior
- Hence the formalism lacks **robustness**.

Even if models were *robust* what can we guarantee about the timing of implementations – in the presence of Δ?

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Resolving implementation issues

Safety robustness [Puri'98]

A timed automata $\mathcal{A}$ is safety robust w.r.t. locations B if there exist a $\Delta > 0$ such that $\mathcal{A}_\Delta$ is safe for B.

- in paper F we consider a stronger notion, capturing also *reactive* expectations.

ϵ-robustness

A timed automata $\mathcal{A}$ is ϵ-robust, for $\epsilon > 0$, if there exist a $\Delta > 0$ such that $d_B(\mathcal{A}, \mathcal{A}_\Delta) \leq \epsilon$.

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Resolving implementation issues

- in paper F

Theorem: Implementability of Timed Automata

Let $\mathcal{A} = (\mathcal{L}, \mathcal{C}, \Sigma, l_0, E)$ be a TA, safe w.r.t. $B \subseteq \Sigma$, then:

1. it has safety robust implementation with the same clocks and locations, and at most $|E| \cdot |Reg(\mathcal{A})|$ edges.
2. For all $\epsilon > 0$, it has a
   - ϵ-robust implementation w.r.t. $\sim_0$
   - ϵ-sampled and ϵ-robust implementation w.r.t. $\approx_{0^+}$.

- Meaning all WCTL properties are transferable between the intended design and the implementation.

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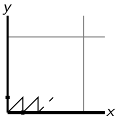
Robust implementations

Given a timed automaton $\mathcal{A}$, construct $\mathcal{A}'$ such that

- $\mathcal{A}$ has the same behaviour as $\mathcal{A}'$,
- $\mathcal{A}'$ is robust, i.e. $\mathcal{A}'$ has approximately the same behaviour as $\mathcal{A}'_\Delta$ for some $\Delta > 0$.

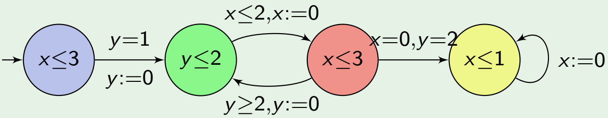
Notice: $\mathcal{B}_\Delta$ didn't respect the region automaton.

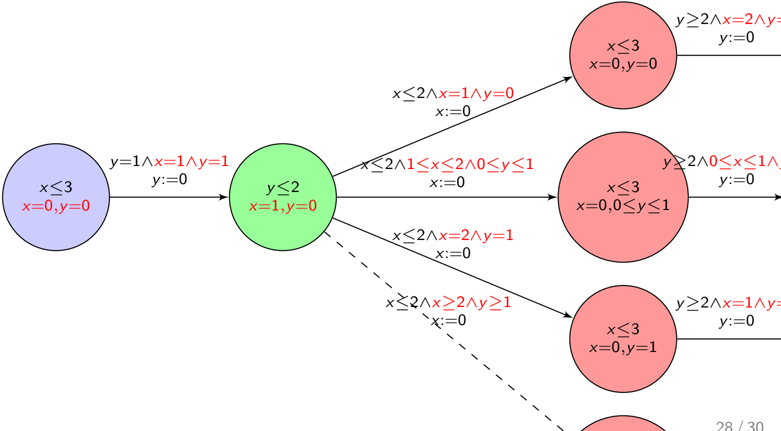


Basic idea: Enforce the region automaton: encoding regions in locations.

Robust implementations

Example $\mathcal{A}$





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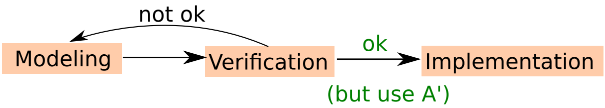
Consequences of (point-wise) ϵ-robustness



Reuse of tools

We need not rebuild existing tools providing automated verification. Rather the **code generation** step, will need to apply our construction.

Reliable fault detectors

A point-wise deviation may provide a upper bound in delays for each step of a communication protocol.

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Conclusion & Final Remarks

- Distances yield meaningful approximations of equality, system properties, and specifications by other specifications.
- Games turn out to be useful in defining distances.

What else?

- Quantitative analysis preserves expressivity, and the hierarchy of equivalences.
- What about other types of qualitative behavior?
- What about measuring Cost, Energy, Radiation?
- What about stochastic and probabilistic systems?

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Thank you!