Quantitative Simulations of Weighted Transition Systems

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True or False?

In formal methods, we typically use

- models of systems and their specification,
- A binary notion to describe whether models meet their specification.

A classical example is CCS and equivalencies; *bisimulation*, *weak bisimulation* and *language equivalence* ($\sim$, $\approx$ and $=_L$ resp.) where model and specification are either related – or not.
Also *reachability* and *safety* tends to be considered true or false.

Finally, when model-checking of logical formulae, properties are satisfied or not.

Introducing quantifiable properties in model – such as weights and time

We ask
Can we use metrics to compare models and specifications, more liberally? *e.g.* w.r.t simulation, we would like to know if are nearly equal, or far from it.

In case of reachability and safety, these have been addressed by Bouyere *et al.* in [2] at FORMATS'08 and Fahrenberg and Larsen in [1] at INFINITY'08

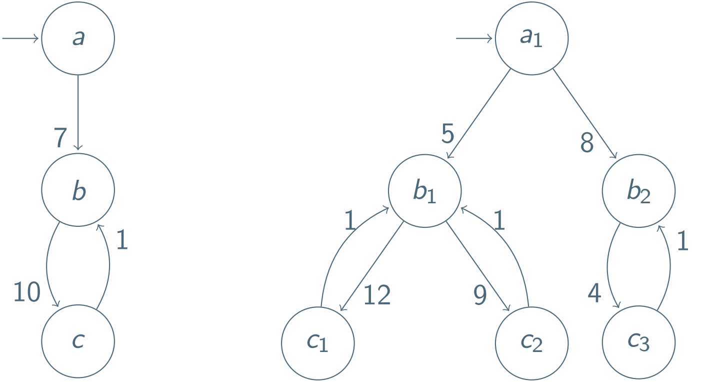
Why is this interesting?

Assuming our ultimate goal is to "push" formal methods of verification in to main-stream industry, quantitative analysis supports:

- Iterative development
- Progress estimation
- Estimate benefits of further development

Example w. weights

Classic simulation clearly wouldn't relate these. So how should we compare these? (assuming all are labeled identically)



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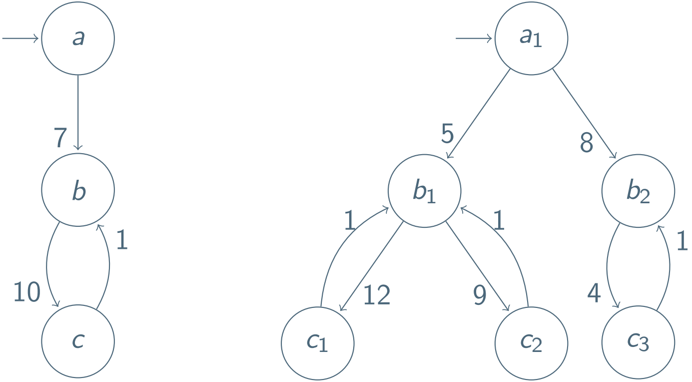
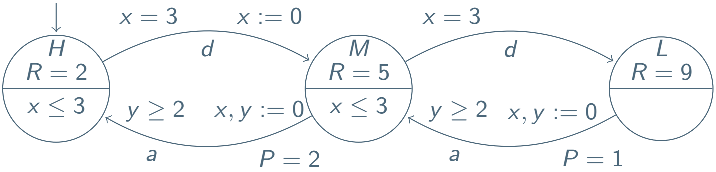


- Consider edges separately or the total sum over traces?
- Using traces (words) or simulations?

Weighted Timed automata

Introduced by Behrmann *et al.* [5] and Alur *et al.* [6] at HSCC'01.

A system with modes: *High*, *Medium*, and *Low*. After 3 time units, the mode degrades (action d). In Medium or Low mode, the system can be attended to (action a), which advances it to a higher mode.



the following cyclic behaviour provides an infinite run:

$$(H,0,0) \xrightarrow{3} (H,3,3) \xrightarrow{d} (M,0,3) \xrightarrow{3} (M,3,6) \xrightarrow{d} (L,3,6) \xrightarrow{1}$$
$$(L,4,7) \xrightarrow{a} (M,0,0) \xrightarrow{3} (M,3,3) \xrightarrow{a} (H,0,0) \rightarrow \cdots$$

Weighted Transition Systems

Definition (WTS)

A weighted transition system is a triple $(\mathcal{S}, w, lg)$ where

- $\mathcal{S} = \langle S, s_0, \Gamma, R \rangle$ is a labeled transition system, with states S, initial state s_0, alphabet Γ, and transitions $R \subseteq S \times \Gamma \times S$,
- $w : R \to \mathbb{R}_{\geq 0}$ assigns weights to transitions, and
- $lg : \Gamma \to \mathbb{R}_{\geq 0}$ assigns lengths to labels.

The cost c : $R \to \mathbb{N}$ is the product of the transition weight w and length of the label lg – observe the “distance” of the WTS transitions:

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$$\frac{4 \cdot x - 2 \cdot x}{x} = 2$$

Semantics of WTA as WTS

The semantics of a WTA $\mathcal{A}$ is given by a WTS $\mathbf{W} = (\mathcal{S}, w, lg)$, where $\mathcal{S} = (S, (l_0, v_0), \{\star\} \cup \mathbb{R}_{\geq 0}, T)$ is the (usual) labeled transition system associated with the underlying TA of $\mathcal{A}$, $lg(\star) = 1$, $lg(\delta) = \delta$ for $\delta \in \mathbb{R}_{\geq 0}$, and for $t \in T$,

$$w(t) = \begin{cases} price(e) & \text{if } t = (l, v) \xrightarrow{\star} (l', v') \text{ and } e = l \xrightarrow{\psi, \star, C} l' \in E \\ rate(l) & \text{if } t = (l, v) \xrightarrow{\delta} (l, v + \delta) \end{cases}$$

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Observe that:

- Switch transitions labeled $\star$ are given lengths 1
- Delay transitions labeled δ are given length δ.

Trace equivalence

Trace (or language) equivalence, for WTS, is the comparison of sets of all traces, such that for states $s, t \in S$:

$$\mathrm{Tr}(s) = \mathrm{Tr}(t)$$

We write $s =_L t$ to denote that s and t are un-weighted trace equivalent.

Next. we use both the standard alphabet and cost values. Traces which are not un-weighted equivalent. are assigned the distance ∞.

Quantifying Trace Distances

The *point-wise trace distance* between states $s, t \in S$ is:

$$\|s,t\|_\bullet = \max \begin{cases} \sup\limits_{\sigma \in \mathrm{Tr}(s)} \inf\limits_{\sigma' \in \mathrm{Tr}(t)} \{\sup\limits_{i} & |\mathrm{c}(\sigma(i)) - \mathrm{c}(\sigma'(i))|\} \\ \sup\limits_{\sigma \in \mathrm{Tr}(t)} \inf\limits_{\sigma' \in \mathrm{Tr}(s)} \{\sup\limits_{i} & |\mathrm{c}(\sigma(i)) - \mathrm{c}(\sigma'(i))|\} \end{cases}$$

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$s_i(\sigma) = \sum_{j=0}^{i} lg(\sigma(j))$ and $0 < \lambda < 1$

we use the length for discounting the accumulated length...

(future)

Introduction
Formalisms
Relations
properties
Logic
Characterisation
Final Remarks

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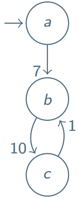
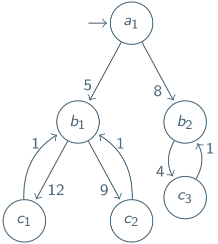
Trace examples





$$\sigma = a \xrightarrow{7} b \xrightarrow{10} c \xrightarrow{1} b \xrightarrow{10} \cdots$$

$$\sigma_1 = a_1 \xrightarrow{5} b_3 \xrightarrow{12} c_1 \xrightarrow{1} b_1 \xrightarrow{12} \cdot\cdot$$

$$\sigma_2 = a_1 \xrightarrow{5} b_3 \xrightarrow{9} c_1 \xrightarrow{1} b_1 \xrightarrow{9} \cdots$$

$$\sigma_3 = a_1 \xrightarrow{8} b_2 \xrightarrow{4} c_3 \xrightarrow{1} b_2 \xrightarrow{4} \cdots$$

$$\|\sigma, \sigma'\|_\bullet = \sup_i \{|\mathrm{c}(\sigma(i)) - \mathrm{c}(\sigma'(i))|\}$$

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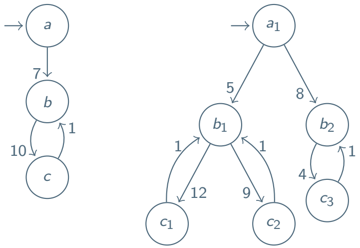

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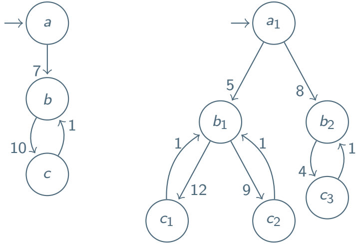

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For the finite traces ..

$$\|\sigma, \sigma_1\|_\bullet = 2 \quad \|\sigma, \sigma_2\|_\bullet = 2 \quad \|\sigma, \sigma_3\|_\bullet = 6$$

$$\|\sigma, \sigma_1\|_+ = 4 \quad \|\sigma, \sigma_2\|_+ = 3 \quad \|\sigma, \sigma_3\|_+ = 7$$

Additional trace distance

Observe that we now have 4 (distinct) trace metrics.

An additional two *maximum-lead* distances may be defined (w. an w.o. discounting) such that traces σ and σ' have distance 2.

$$\sigma = a \xrightarrow{7} b \xrightarrow{10} c$$

$$\sigma' = a_1 \xrightarrow{5} b_3 \xrightarrow{12} c_1$$

"Standard" (Bi)simulation for WTS

Definition (Weighted Simulation)

A binary relation $\mathcal{R} \subseteq S \times S$ is a simulation if and only if, whenever $(s, t) \in \mathcal{R}$ and $\alpha \in \Gamma$ and $c \in \mathbb{R}_{\geq 0}$ then

- if $s \xrightarrow{\alpha,c} s'$ then $t \xrightarrow{\alpha,c} t'$ with $(s', t') \in \mathcal{R}$ for some $t' \in S$

We say that t simulates s and write $s \preccurlyeq t$ whenever $(s, t) \in \mathcal{R}$ for some simulation $\mathcal{R}$.

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Define $s \sim t$ and $s \sim_{uw} t$ as usual.

Quantifying Simulation

As for language equivalence, we define quantitative simulation relations, in order to capture branching properties *i.e.* the behaviour of system models.

- Point-wise (bi)simulation
- Accumulated (bi)simulation
- Max-lead (bi)simulation (shown in [4] to be poly-time[1] decidable for timed automata)

[1] in the size of the region graph, which in turn is exponential in the size of clocks

Point-wise (bi)simulation

A family of relations $\mathbf{R} = \{\dot{\mathcal{R}}_\varepsilon \subseteq S \times S \mid \varepsilon \geq 0\}$ is a *point-wise bisimulation family* if $(s,t) \in \dot{\mathcal{R}}_\varepsilon \in \mathbf{R}$ and $\alpha \in \Gamma$ imply that

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Properties

We have the following properties for the defined relations:

1. the relation $\dot{\sim}_\varepsilon$ is:
   - The largest point-wise bisimulation family.
   - For all $\dot{\mathcal{R}}_\varepsilon$ and $\dot{\mathcal{R}}_{\varepsilon'} \in \dot{\sim}_\varepsilon$ where $\varepsilon \leq \varepsilon'$ then $\dot{\mathcal{R}}_\varepsilon \subseteq \dot{\mathcal{R}}_{\varepsilon'}$ and
   - For $\varepsilon \geq 0 : \sim \subseteq \dot{\sim}_\varepsilon \subseteq \sim_{uw}$
2. $r \dot{\sim}_\varepsilon s \wedge s \dot{\sim}_{\varepsilon'} t \implies r \dot{\sim}_{\varepsilon+\varepsilon'} t$
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Weighted HML

Weighted extension of HML with recursion [7]

Definition (Point-wise Logic)

Let $\mathcal{X}$ be a set of identifiers. Then the set $\mathcal{L}_w$ of formulae over $\mathcal{X}$ is the smallest set of formulae constructed according to the following abstract syntax:

$$\varphi ::= \mathbf{tt} \mid \langle\alpha\rangle_c\varphi \mid \varphi \wedge \varphi \mid [\alpha]_c\varphi \mid \varphi \vee \varphi \mid X \mid \nu X.\varphi \tag{1}$$

Intuitively $\langle\alpha\rangle_c$ and $[\alpha]_c$ denotes the availability of an α labeled transition with weight c.

The semantics are given as a valuation $[\![\varphi]\!]_{\mathscr{E}} : S \rightarrow \mathbb{R}_{\geq 0} \cup \{\infty\}$.

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Characterisation

Theorem (Logical Characterisation)

Given states s and t of a WTS, then

$$s \dot{\sim}_\varepsilon t \iff |[\![\varphi]\!]_{\mathscr{E}}(s) - [\![\varphi]\!]_{\mathscr{E}}(t)| \leq \varepsilon \textit{ for all } \varphi \in \mathcal{L}_w$$

For the non-disounted $\dot{\sim}_\varepsilon$

Where are we?

So far, we have identified:

- 6 relevant trace metrics.
- 6 relevant simulations (branching metrics).
- Established basic properties of the above metrics.
- A characterising (point-wise) logic.

Future work

Logics:

- Accumulating logic.
- Max-lead logic.
- Discounted versions.

Tool support:

- Prototype impl.

Metrics:

- Computability and Complexity
- Continuity w.r.t composition.

Questions, should we:

- add a metric on Γ?
- compare finite traces of unequal length?

Summary

Thank you

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- Extension of LTS to WTS.
- Semantics of WTA as WTS.
- Extension of $=_L$ to:
  - Point-wise distance.
  - Accumulated distance.
  - Max-lead distance.
- Extensions of $\sim$ to:
  - Point-wise distance.
  - Accumulated distance.
  - Max-lead distance.
- Point-wise Weighed-HML extending HML.
- Characterisation theorem.

Future work (highlights):

- Decidability of metrics.
- Logical characterisations.
- Prototype impl.

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Definition (Weighted Timed Automata)

A weighted timed automaton is a tuple $(L, l_0, \mathcal{C}, I, E, p, r)$ where

- L is a finite set of locations, with l_0 initial,
- $\mathcal{C}$ is a finite set of real-valued clocks,
- $I : L \to \Psi(\mathcal{C})$ assigns invariants to locations,
- $E \subseteq L \times \Psi(\mathcal{C}) \times 2^{\mathcal{C}} \times L$ is a set of edges,
- $p : E \to \mathbb{N}$ is a edge price function, and
- $r : L \to \mathbb{N}$ is a location rate function.

The set $\Psi(\mathcal{C})$ of clock constraints is generated by the grammar

$$\psi ::= x \bowtie k \mid x - y \bowtie k \mid \psi_1 \wedge \psi_2 \qquad \bowtie \in \{\leq, <, =, \geq, >\}$$

for $x, y \in \mathcal{C}$, $k \in \mathbb{R}$.

Semantics for logic

Definition
Given a WLTS $\mathbf{W} = (\mathcal{S}, w, lg)$ over a LTS $\mathcal{S} = (S, s_0, \Gamma, R)$ a declaration $\mathscr{D}$ and an interpretation $\mathscr{E}$, every formula $\varphi \in \mathcal{L}_w$ defines a valuation $[\![\varphi]\!]_{\mathscr{E}} : S \to \mathbb{R}_{\geq 0} \cup \{\infty\}$:

$$[\![\mathtt{tt}]\!]_{\mathscr{E}}(s) = 0 \tag{R1}$$

$$[\![\varphi_1 \wedge \varphi_2]\!]_{\mathscr{E}}(s) = max\{[\![\varphi_1]\!]_{\mathscr{E}}(s), [\![\varphi_2]\!]_{\mathscr{E}}(s)\} \tag{R2}$$

$$[\![\varphi_1 \vee \varphi_2]\!]_{\mathscr{E}}(s) = min\{[\![\varphi_1]\!]_{\mathscr{E}}(s), [\![\varphi_2]\!]_{\mathscr{E}}(s)\} \tag{R3}$$

$$[\![\langle\alpha\rangle_c\varphi]\!]_{\mathscr{E}}(s) = \begin{cases} min\Big\{max\{\frac{|c-d|}{lg(\alpha)}, [\![\varphi]\!]_{\mathscr{E}}(s')\} \mid s \xrightarrow{\alpha,d} s'\Big\} \\ \text{or } \infty \text{ whenever } s \stackrel{\alpha}{\not\to} \end{cases} \tag{R4}$$

$$[\![[\alpha]_c\varphi]\!]_{\mathscr{E}}(s) = \begin{cases} max\Big\{max\{\frac{|c-d|}{lg(\alpha)}, [\![\varphi]\!]_{\mathscr{E}}(s')\} \mid s \xrightarrow{\alpha,d} s'\Big\} \\ \text{or } 0 \text{ whenever } s \stackrel{\alpha}{\not\to} \end{cases} \tag{R5}$$

$$[\![X]\!]_{\mathscr{E}} = \mathscr{E}(X) \tag{R6}$$

$$[\![\nu X.\varphi]\!]_{\mathscr{E}} = sup\{\rho \in \Delta \mid \rho = [\![\varphi]\!]_{\mathscr{E}[X:=\rho]}\} \tag{R8}$$

Where $s \not\to$ denotes the fact that $\nexists s'$ such that $(s, \alpha, s') \in R$.
We will write $s \models_c \varphi$ whenever $[\![\varphi]\!]_{\mathscr{E}}(s) = c$.