On Decidability of Bigraphical Sortings

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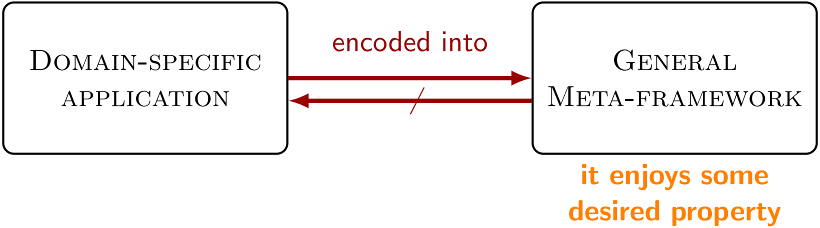
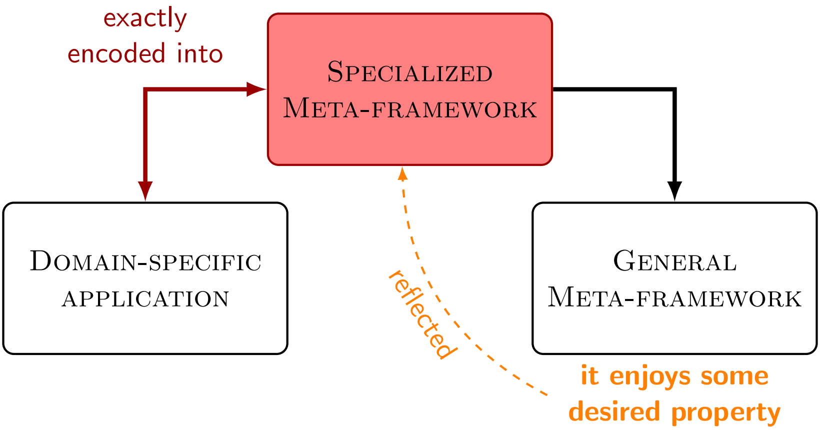


Often the meta-framework its "too general", and
an exact encoding requires to recast the theory



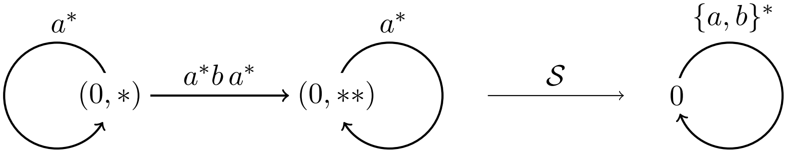
The specialized framework is
a **sorted version** of the general one

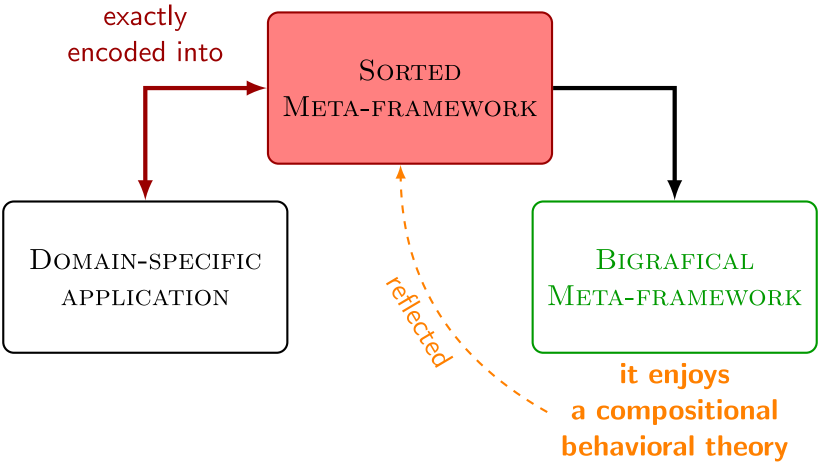
sorted category | base category

$$\mathcal{S} : \mathbf{X} \longrightarrow \mathbf{C}$$

+ **faithful** ($\mathrm{Hom}_{\mathbf{X}}(X, Y) \rightarrowtail \mathrm{Hom}_{\mathbf{C}}(\mathcal{S}X, \mathcal{S}Y)$ injective)
+ **surjective on objects**

Example: $M = (\{a, b\}^*, \cdot)$ monoid



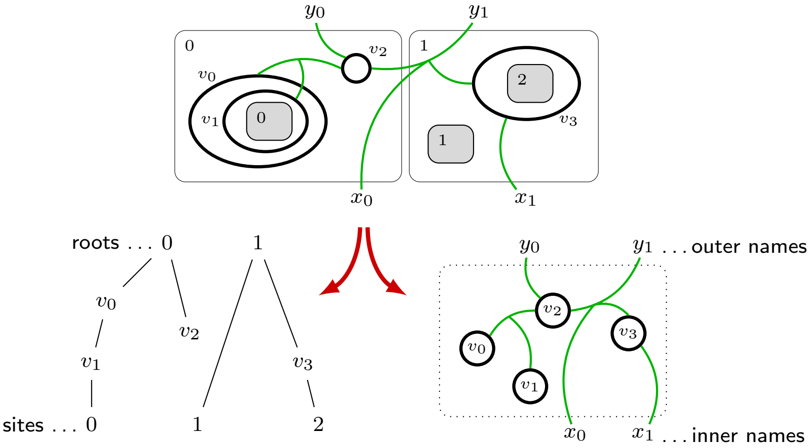


In this talk we focus our attention on
bigraphs and decidability issues on bigraphical sortings

Talk Outline

1. Introduction to Bigraphs
2. Bigraphical Sortings
3. Sortings and decidability
4. A decidable subclass of Sortings: Match Sorting
5. Expressiveness of Match Sorting
   + Homomorphic Sortings
   + Local Bigraphs

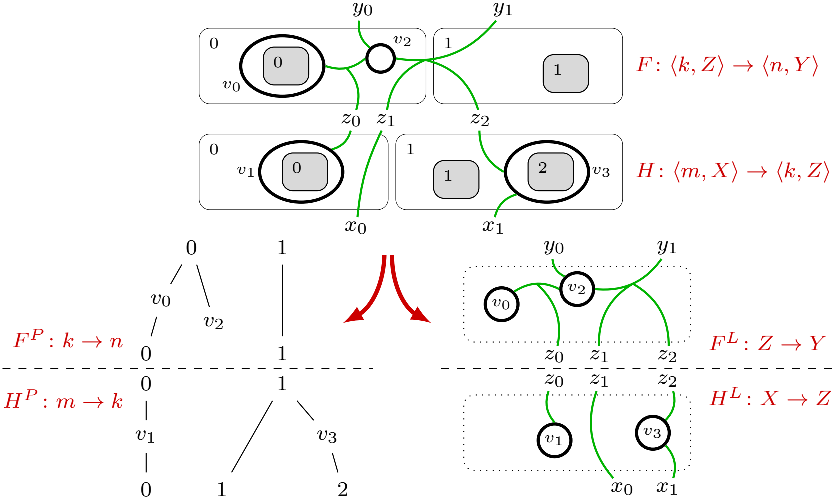
bigraph: $G = \langle G^P, G^L \rangle : \langle m, X \rangle \rightarrow \langle n, Y \rangle$



place graph: $G^P : m \rightarrow n$

link graph: $G^L : X \rightarrow Y$

composite: $F \circ H = \langle F^P \circ H^P, F^L \circ H^L \rangle \colon \langle m, X \rangle \to \langle n, Y \rangle$



$F^P \circ H^P : m \to n$ $\qquad$ $F^L \circ H^L \colon X \to Y$

A general and intuitive class of bigraphical sortings:

Predicate Sortings: $\mathcal{S}_P \colon \mathbf{X} \to \mathbf{C}$
sortings from decomposable predicates P over **C**-morphisms

- the image of $\mathcal{S}_P$ is precisely the set of morphisms satisfying P
- $\mathcal{S}_P$ transfers RPOs (if **C** has RPOs then **X** has RPOs too)

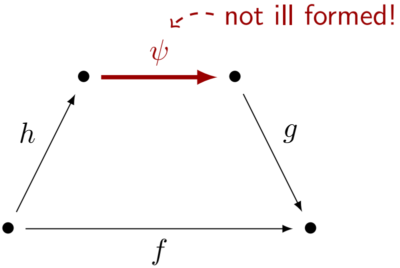
$$P(f \circ g) \implies P(f) \wedge P(g)$$

(decomposability)

necessary due to fuctoriality
$\mathcal{S}_P(f \circ g) = \mathcal{S}_P(f) \circ \mathcal{S}_P(g)$

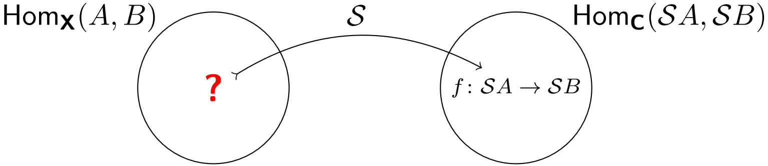
Theorem (Factorization): A predicate P on morphisms is decomposable iff there exists a set of morphism Φ such that

$$P(f) \quad \text{iff} \quad \forall g, \psi, h : f = g \circ \psi \circ h \implies \psi \notin \Phi$$



An exhaustive construction of the sorted category is unfeasible

Proposal: use the base category morphisms and check if they are well-sorted (hence, if they have a sorting pre-image)



for predicate sortings
it is enough to check $P(f)$

POST CORRISPONDENCE PROBLEM (UNDECIDABLE)

Instance: a finite set of pairs of words $\{(\alpha_1, \beta_1), \ldots, (\alpha_n, \beta_n)\}$ in $\{a, b\}^*$.

Question: there exist a sequence $i_0, i_1, \ldots, i_k$ $(1 \leq i_j \leq n)$ such that

$$\alpha_{i_0} \cdot \ldots \cdot \alpha_{i_k} \stackrel{?}{=} \beta_{i_0} \cdot \ldots \cdot \beta_{i_k}.$$

The reduction (sketch):

- define an encoding $[\![\cdot]\!]$ of PCP instances to **Big**-morphisms

$$\mathcal{P}_{\mathsf{fin}}(\{a, b\}^* \times \{a, b\}^*) \to \mathsf{Hom}_{\mathbf{Big}}(\epsilon, \epsilon)$$

- show that $U \subseteq \mathsf{Hom}_{\mathbf{Big}}(\epsilon, \epsilon)$ is decomposable and undecidable

$$U = \{f \in \mathsf{Hom}_{\mathbf{Big}}(\epsilon, \epsilon) \mid \forall g, \phi, h.\ f = g \circ \phi \circ h \Rightarrow \phi \notin \Phi_{PCP}\}$$
$$\Phi_{PCP} = \{[\![i]\!] \mid i \in \mathsf{PCP}\}$$

P decidable $\implies$ $\mathcal{S}_P$-pre-image existence decidable

Some general problems:

- not easy to define a predicate that is also decomposable
- usually predicates are complicated and not easy to be understood at first sight
- a new algorithm every time a new P is chosen

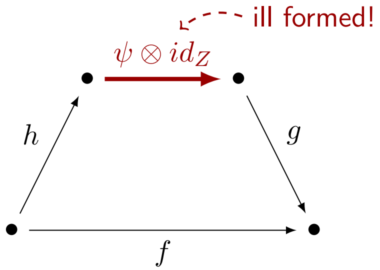
A possible solution:

1. define predicates from sets of ill-formed morphisms
2. provide a universal algorithm that checks for ill-formed occurrences

Construction:

1. define a Rec (possibly infinite) set Φ of ill-formed bigraphs
2. $M_\Phi = \{f : \forall \psi \in \Phi.\ f \neq g \circ (\psi \otimes id_Z) \circ h\}$ (decomposable)
3. $\mathcal{S}_{M_\Phi} : \mathbf{X} \to \mathbf{Big}$ (Debois' predicate sorting)

$\neg M_\Phi(f) \iff$



decidable:
matching algorithm - - - - - → $f = g \circ (\psi \otimes id_Z) \circ h$
(Damgaard et.al '07)

Decision algorithm for match predicates

Input: A finite bigraph G, and a Rec set Φ of ill-formed bigraphs
Question: Decide whether $M_\Phi(G)$ holds

`checkFin`(G, Φ) (Φ finite)

res $=$ `true`
for each $\psi \in \Phi$
  if `matchCheck`(ϕ, G)
    res $=$ `false`; `break`
endfor

`checkInf`(G, Φ) (Φ infinite)

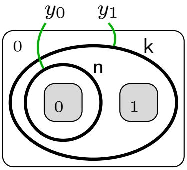
$M =$ `allMatchable`(G)
res $=$ `checkFin`$(G, M \cap \Phi)$

computes the set of
all bigraphs matchable in G

Expressiveness of match sortings

+ **Homomorphic sorting** (CCS, kind-Bigraphs, ...)

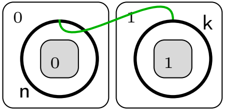
$\Phi_{\text{HOM}} = \Big\{$



$\mid \forall \mathsf{n}, \mathsf{k}.\ prnt_\theta(\phi(\mathsf{n})) \neq \phi(\mathsf{k}) \Big\}$

+ **Local bigraphs** (π-calculus, λ-calculus, ...)

$\Phi_{\text{LOC}} = \Big\{$



$\mid \forall \mathsf{n}, \mathsf{k}.$ n has a binding port $\Big\}$

Conclusions:

+ Investigated the decidability of sortings
+ Proposed an decidable subclass of sorting (+ algorithm)
+ Proposed an intuitive way to define sortings
+ Investigated the expressive power of the decidable subclass

Future work:

+ Applying the same approach to other categories?
+ Investigate for a better algorithm
+ Integration into tools? (e.g. BPL)

Thanks