The BisimDist Library

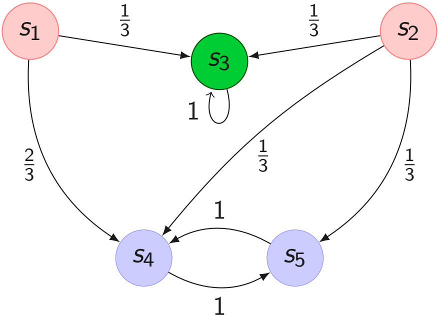
Efficient Computation of Bisimilarity Distances for Markovian Models

Giorgio Bacci, Giovanni Bacci, Kim G. Larsen, Radu Mardare

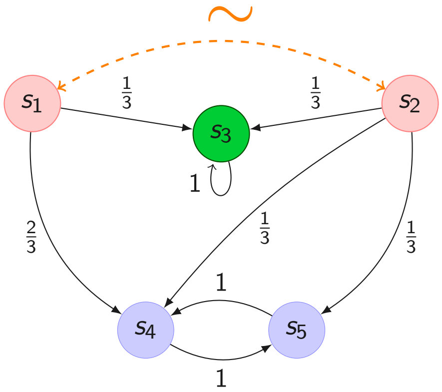
Dept. of Computer Science, Aalborg University

QEST 2013
27-30 August, Buenos Aires - Argentina

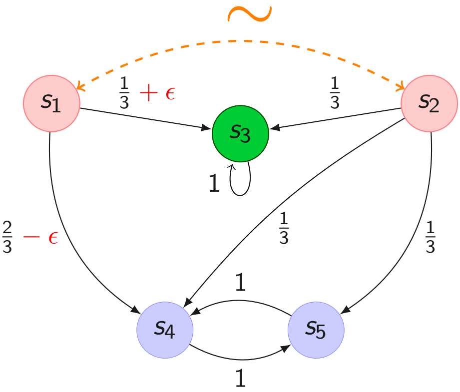
Motivations



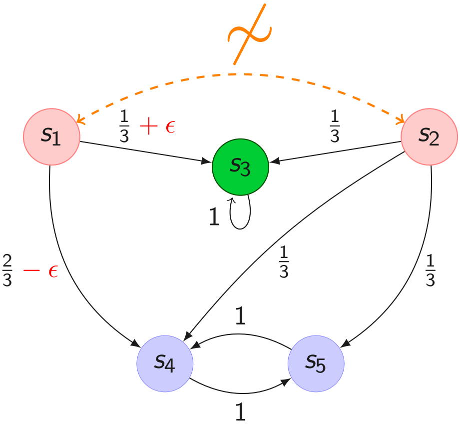
Motivations



Motivations



Motivations



From equivalences to distances [Giacalone-Jou-Smolka'90]

Pseudometrics $d\colon S \times S \to \mathbb{R}_{\geq 0}$ are the quantitative analogue of an equivalence relation

equivalence		pseudometric
$s \equiv s$	$\rightsquigarrow$	$d(s,s) = 0$
$s \equiv t \implies t \equiv s$	$\rightsquigarrow$	$d(s,t) = d(t,s)$
$s \cong u \wedge u \cong t \implies s \cong t$	$\rightsquigarrow$	$d(s,u) + d(u,t) \geq d(s,t)$

From equivalences to distances [Giacalone-Jou-Smolka'90]

Pseudometrics $d\colon S \times S \to \mathbb{R}_{\geq 0}$ are the quantitative analogue of an equivalence relation

equivalence		**pseudometric**
$s \equiv s$	$\rightsquigarrow$	$d(s,s) = 0$
$s \equiv t \implies t \equiv s$	$\rightsquigarrow$	$d(s,t) = d(t,s)$
$s \cong u \wedge u \cong t \implies s \cong t$	$\rightsquigarrow$	$d(s,u) + d(u,t) \geq d(s,t)$

Bisimilarity Pseudometrics

$$d(s,t) = 0 \iff s \sim t$$

Pseudometrics on Markovian Models

Markov Chains:

- pseudometrics of Desharnais et al. [TCS'04]
- fixed point def. by van Breugel and Worrell [LMCS'08]

Remarkable properties Chen et al. [FoSSaCS'12]

$$\sup_{\varphi \in \mathsf{LTL}} |Pr(s \vDash \varphi) - Pr(t \vDash \varphi)| \leq d^{\mathsf{MC}}(s,t)$$

Markov Decision Processes:

- pseudometrics of Ferns et al. [UAI'04] (fixed point def.)

Remarkable properties Ferns et al. [UAI'04]

$$|V^*(s) - V^*(t)| \leq d^{\mathsf{MDP}}(s,t)$$

Pseudometrics on Markovian Models

Markov Chains:

- pseudometrics of Desharnais et al. [TCS'04]
- fixed point def. by van Breugel and Worrell [LMCS'08]

Remarkable properties Chen et al. [FoSSaCS'12]

$$\sup_{\varphi \in \mathrm{LTL}} |Pr(s \vDash \varphi) - Pr(t \vDash \varphi)| \leq d^{\mathrm{MC}}(s,t)$$

Markov Decision Processes:

- pseudometrics of Ferns et al. [UAI'04] (fixed point def.)

Remarkable properties Ferns et al. [UAI'04]

$$|V^*(s) - V^*(t)| \leq d^{\mathrm{MDP}}(s,t)$$

Applications of the pseudometrics

Model Reduction: clustering states which are close enough

Abstraction Testing: analytical testing of model abstractions

Parameters Extimation: baricentrum as the optimal

Model Prediction: closest to the 'optimal' (usually, not sound)

Bisimilarity pseudometrics have been extensively used in AI

- Policy tranfer — Castro, Precup [AAAI'10]
- Basis function discovery — Comanici, Precup [AAAI'11]
- Automatic inference of temporally extended actions — Castro, Precup [RL'11]

Applications of the pseudometrics

Model Reduction: clustering states which are close enough

Abstraction Testing: analytical testing of model abstractions

Parameters Extimation: baricentrum as the optimal

Model Prediction: closest to the 'optimal' (usually, not sound)

Bisimilarity pseudometrics have been extensively used in AI

- Policy tranfer — Castro, Precup [AAAI'10]
- Basis function discovery — Comanici, Precup [AAAI'11]
- Automatic inference of temporally extended actions — Castro, Precup [RL'11]

Applications of the pseudometrics

Model Reduction: clustering states which are close enough

Abstraction Testing: analytical testing of model abstractions

Parameters Extimation: baricentrum as the optimal

Model Prediction: closest to the 'optimal' (usually, not sound)

Bisimilarity pseudometrics have been extensively used in AI

- Policy tranfer — Castro, Precup [AAAI'10]
- Basis function discovery — Comanici, Precup [AAAI'11]
- Automatic inference of temporally extended actions — Castro, Precup [RL'11]

Applications of the pseudometrics

Model Reduction: clustering states which are close enough

Abstraction Testing: analytical testing of model abstractions

Parameters Extimation: baricentrum as the optimal

Model Prediction: closest to the 'optimal' (usually, not sound)

Bisimilarity pseudometrics have been extensively used in AI

+ Policy tranfer — Castro, Precup [AAAI'10]
+ Basis function discovery — Comanici, Precup [AAAI'11]
+ Automatic inference of temporally extended actions — Castro, Precup [RL'11]

Applications of the pseudometrics

Model Reduction: clustering states which are close enough

Abstraction Testing: analytical testing of model abstractions

Parameters Extimation: baricentrum as the optimal

Model Prediction: closest to the 'optimal' (usually, not sound)

Bisimilarity pseudometrics have been extensively used in AI

+ Policy tranfer — Castro, Precup [AAAI'10]
+ Basis function discovery — Comanici, Precup [AAAI'11]
+ Automatic inference of temporally extended actions — Castro, Precup [RL'11]

Applications of the pseudometrics

Model Reduction: clustering states which are close enough

Abstraction Testing: analytical testing of model abstractions

Parameters Extimation: baricentrum as the optimal

Model Prediction: closest to the ‘optimal’ (usually, not sound)

Bisimilarity pseudometrics have been extensively used in AI

- Policy tranfer — Castro, Precup [AAAI’10]
- Basis function discovery — Comanici, Precup [AAAI’11]
- Automatic inference of temporally extended actions — Castro, Precup [RL’11]

Existing methods for computing the distance

Iterative Methods (approximated)

+ based on a fixed point characterization of the pseudometric
+ **Markov Chains** — van Breugel, Worrell [LMCS'08]
+ **Markov Decision Processes** — Ferns et al. [UAI'04]

Iterative + Heuristics — Comanici et al. [QEST'12] (approximated)

+ focus on states where the impact is expected to be greater
+ (similar to asynchronous dynamic programming)

Linear Programming — Chen et al. [FoSSaCS'12] (exact)

+ solution of a linear program with exponentially many constraints
+ ellipsoid method $\implies$ polynomial

Existing methods for computing the distance

Iterative Methods (approximated)

- based on a fixed point characterization of the pseudometric
- **Markov Chains** — van Breugel, Worrell [LMCS'08]
- **Markov Decision Processes** — Ferns et al. [UAI'04]

Iterative + Heuristics — Comanici et al. [QEST'12] (approximated)

- focus on states where the impact is expected to be greater
- (similar to asynchronous dynamic programming)

Linear Programming — Chen et al. [FoSSaCS'12] (exact)

- solution of a linear program with exponentially many constraints
- ellipsoid method $\Longrightarrow$ polynomial

Existing methods for computing the distance

Iterative Methods (approximated)

- based on a fixed point characterization of the pseudometric
- **Markov Chains** — van Breugel, Worrell [LMCS'08]
- **Markov Decision Processes** — Ferns et al. [UAI'04]

Iterative + Heuristics — Comanici et al. [QEST'12] (approximated)

- focus on states where the impact is expected to be greater
- (similar to asynchronous dynamic programming)

Linear Programming — Chen et al. [FoSSaCS'12] (exact)

- solution of a linear program with exponentially many constraints
- ellipsoid method $\Longrightarrow$ polynomial

All existing methods require to explore the entire state space

What if we only need $d(s, t)$?

(can we avoid computation of d on all pairs of states?)

On-the-fly Algorithms

All existing methods require to explore the entire state space

What if we only need $d(s, t)$?

(can we avoid computation of d on all pairs of states?)

we proposed an **on-the-fly** algorithm:

- + lazy exploration of $\mathcal{M}$ (only where and when is needed)
- + save computational cost (time & space)

On-the-fly Algorithms

All existing methods require to explore the entire state space

What if we only need $d(s,t)$?

(can we avoid computation of d on all pairs of states?)

we proposed an **on-the-fly** algorithm:

- + lazy exploration of $\mathcal{M}$ (only where and when is needed)
- + save computational cost (time & space)

Markov Chains
[TACAS'13]

Markov Decision Processes
[MFCS'13]

Empirical Results (all-pairs)

# States	On-the-Fly (exact)		Iterating (approximated)			Approx. Error*
	Time (s)	# TPs	Time (s)	# Iterations	# TPs	
5	0.019	1.191	0.0389	1.733	26.733	0.139
6	0.059	3.046	0.092	1.826	38.133	0.146
7	0.138	6.011	0.204	2.194	61.728	0.122
8	0.255	8.561	0.364	2.304	83.028	0.117
9	0.499	12.042	0.673	2.579	114.729	0.111
10	1.003	18.733	1.272	3.111	174.363	0.094
11	2.159	25.973	2.661	3.556	239.557	0.096
12	4.642	34.797	5.522	4.042	318.606	0.086
13	6.735	39.958	8.061	4.633	421.675	0.097
14	6.336	38.005	7.188	4.914	593.981	0.118
17	11.261	47.014	12.805	5.885	908.61	0.132
19	26.635	61.171	29.654	6.961	1328.60	0.140
20	34.379	66.457	38.206	7.538	1597.92	0.142

(*) $\epsilon = \max_{s,t \in S} \delta_\lambda(s,t) - d(s,t)$

Empirical Results (single-pair)

# States	out-degree = 3		2 ≤ out-degree ≤ # States	
	Time (s)	# TPs	Time (s)	# TPs
5	0.006	0.273	0.012	0.657
6	0.012	0.549	0.031	1.667
7	0.017	0.981	0.088	3.677
8	0.025	1.346	0.164	5.301
9	0.026	1.291	0.394	8.169
10	0.058	2.038	1.112	13.096
11	0.077	1.827	2.220	18.723
12	0.043	1.620	4.940	26.096
13	0.060	1.882	10.360	35.174
14	0.089	2.794	20.123	46.077

BisimDist Library

BisimDist is a Mathematica® library that provides two packages:

MCDist MDPDist

- Data structures (model definition)
- Data structure manipulators & visualizers
- Procedure for computing bisimilarity distances (on-the-fly!)
  - approximated methods (from known upper-bounds)
  - future-discount
- bisimilarity classes / quotient by bisimilarity

Library + Tutorials

http://people.cs.aau.dk/giovbacci/tools.html