

Decidability and Symbolic Verification

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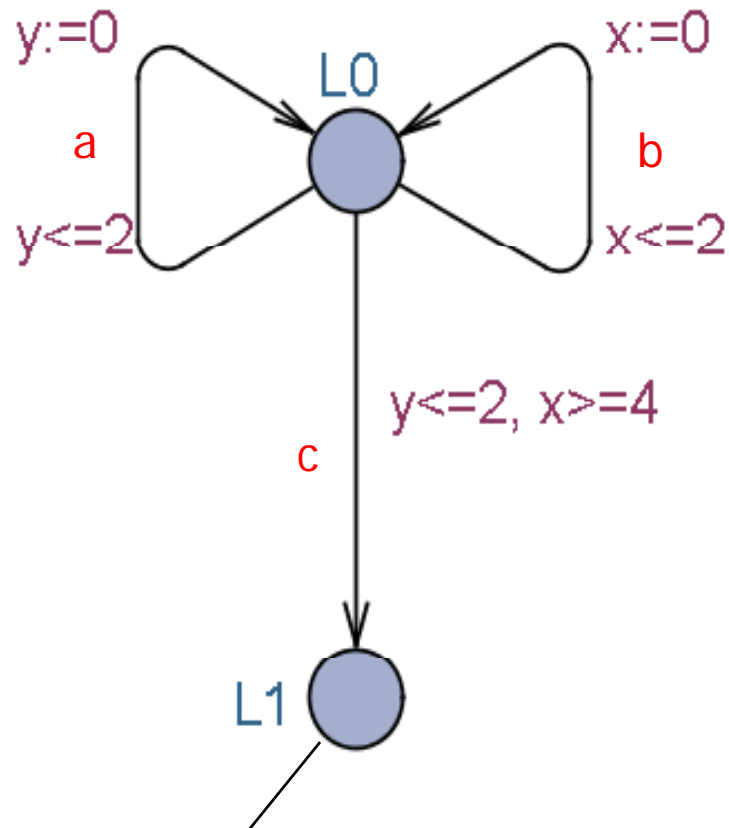


Overview

- Decidability
 - Region Construction
 - Reachability & Bisimulation Checking
- Symbolic Verification
 - On-the-fly Exploration
 - Zones and Difference Bounded Matrices (DBM)
 - Clock Difference Diagrams (CDD)
- Verification Options



Reachability ?



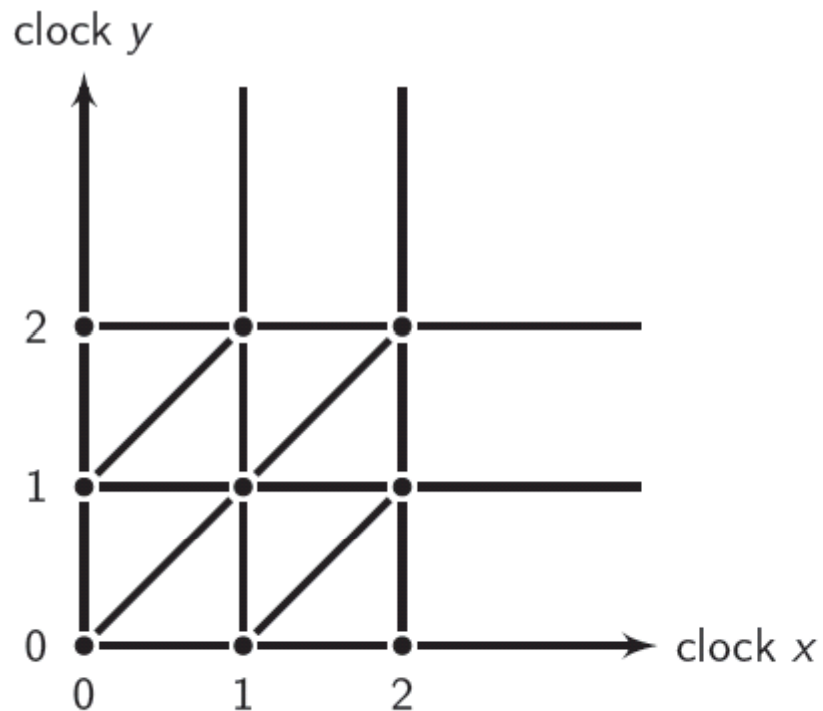
OBSTACLE:
Uncountably infinite
state space

$L \times \mathbb{R}^C$
locations clock-valuations

Reachable from initial state $(L_0, x=0, y=0)$?



The Region Abstraction



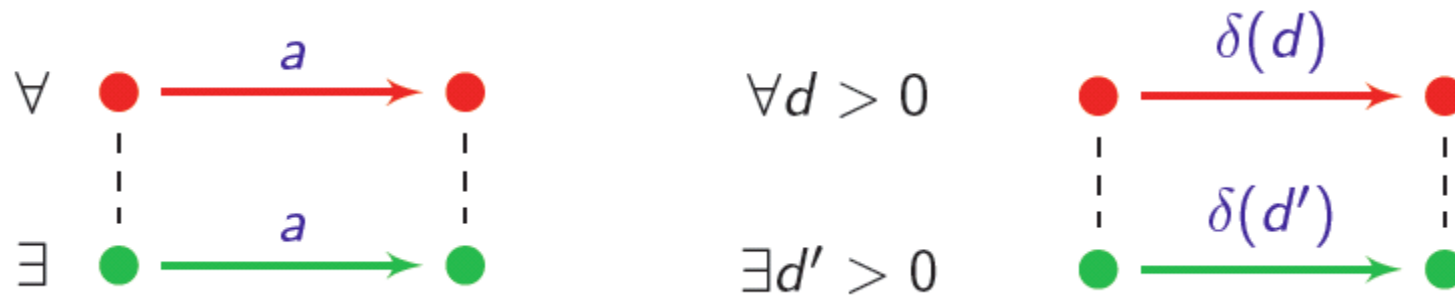
- “compatibility” between regions and constraints
- “compatibility” between regions and time elapsing

↪ an equivalence of finite index
a time-abstract bisimulation



Time Abstracted Bisimulation

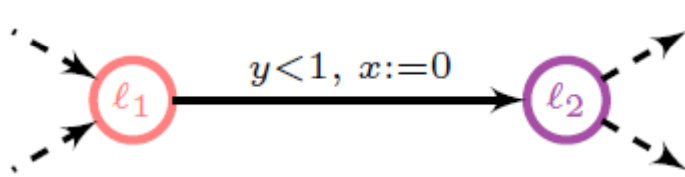
This is a relation between \bullet and \bullet such that:



... and vice-versa (swap \bullet and \bullet).



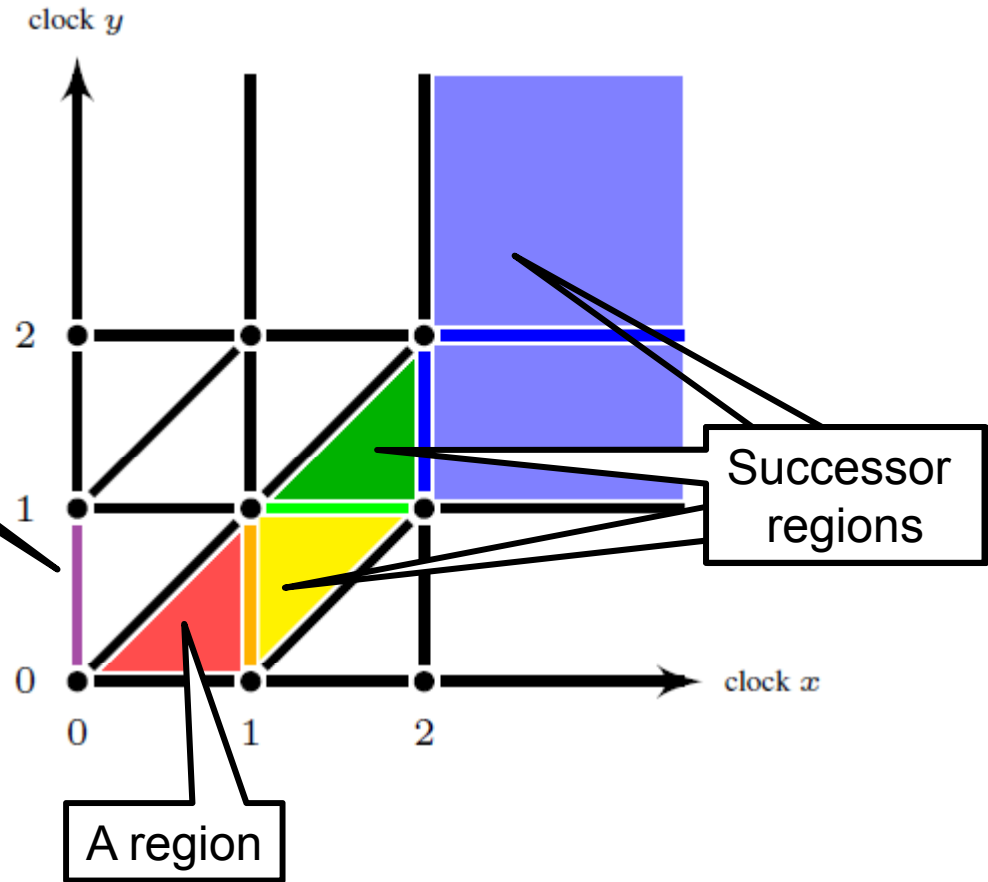
Regions – From Infinite to Finite



Reset region

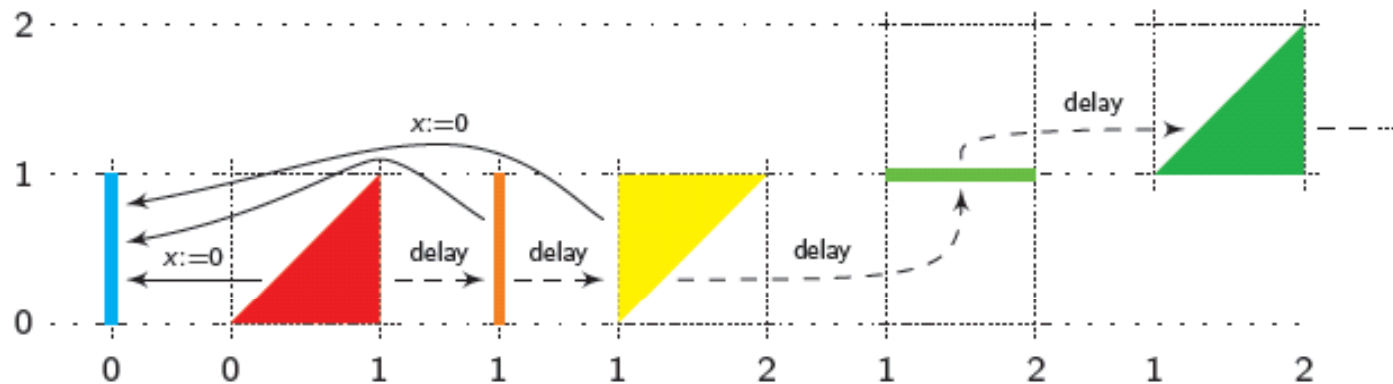
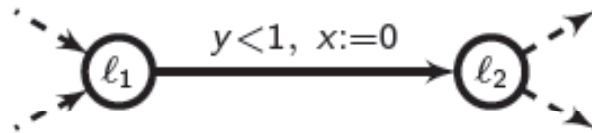
THM [AD90]
Reachability is decidable (and PSPACE-complete) for timed automata

THM [CY90]
Time-optimal reachability is decidable (and PSPACE-complete) for timed automata

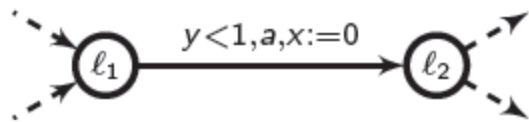


Region Graph

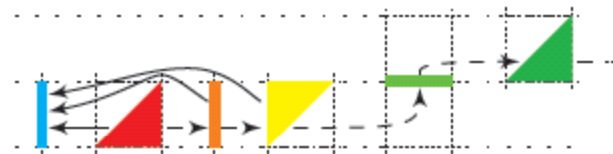
It “mimicks” the behaviours of the clocks.



Region Automaton = Finite Bisimulation Quotient

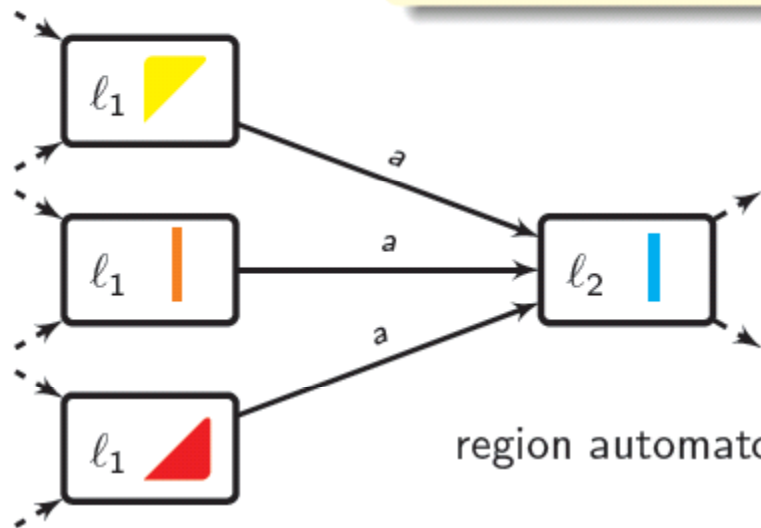


timed automaton



region graph

$$\mathcal{L}(\text{reg. aut.}) = \text{UNTIME}(\mathcal{L}(\text{timed aut.}))$$

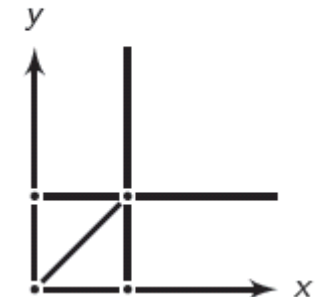
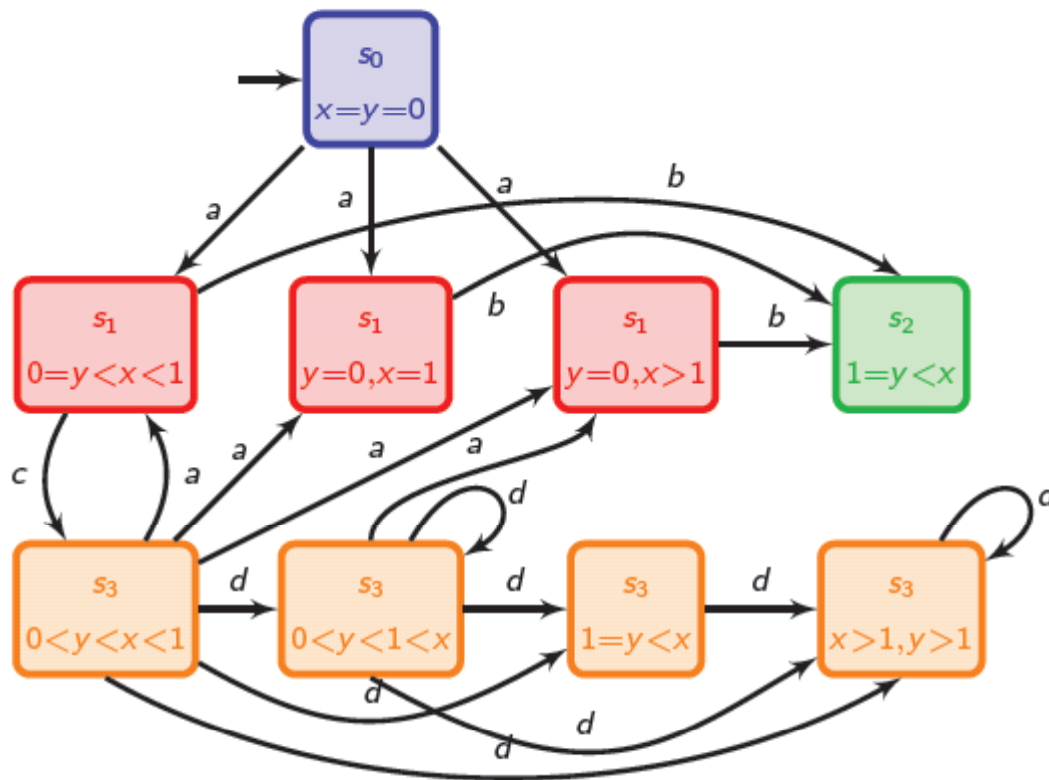
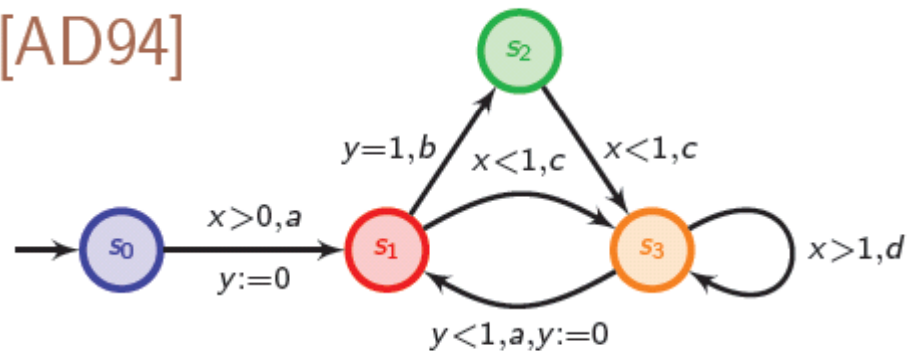


region automaton

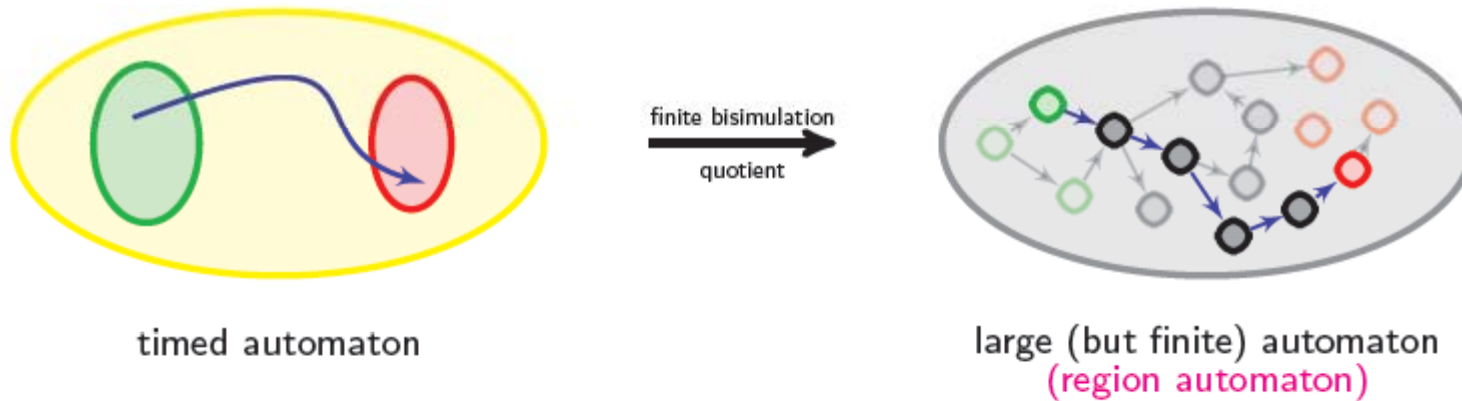


An Example

[AD94]



Region Automaton



LARGE: exponential in the number of clocks and in the constants (if encoded in binary). The number of regions is

$$\prod_{x \in X} (2M_x + 2) \cdot |X|! \cdot 2^{|X|}$$



Fundamental Results

- Reachability ☺ PSPACE-c
- Model-checking
 - TCTL ☺ PSPACE-c ; MTL ☹ UNDECIDABLE ; MITL ☺ EXPSPACE-c
- Bisimulation, Simulation
 - Timed ☺ EXPTIME-c ; Untimed ☺
- Trace-inclusion
 - Timed ☹ UNDECIDABLE ; Untimed ☺ PSPACE-c

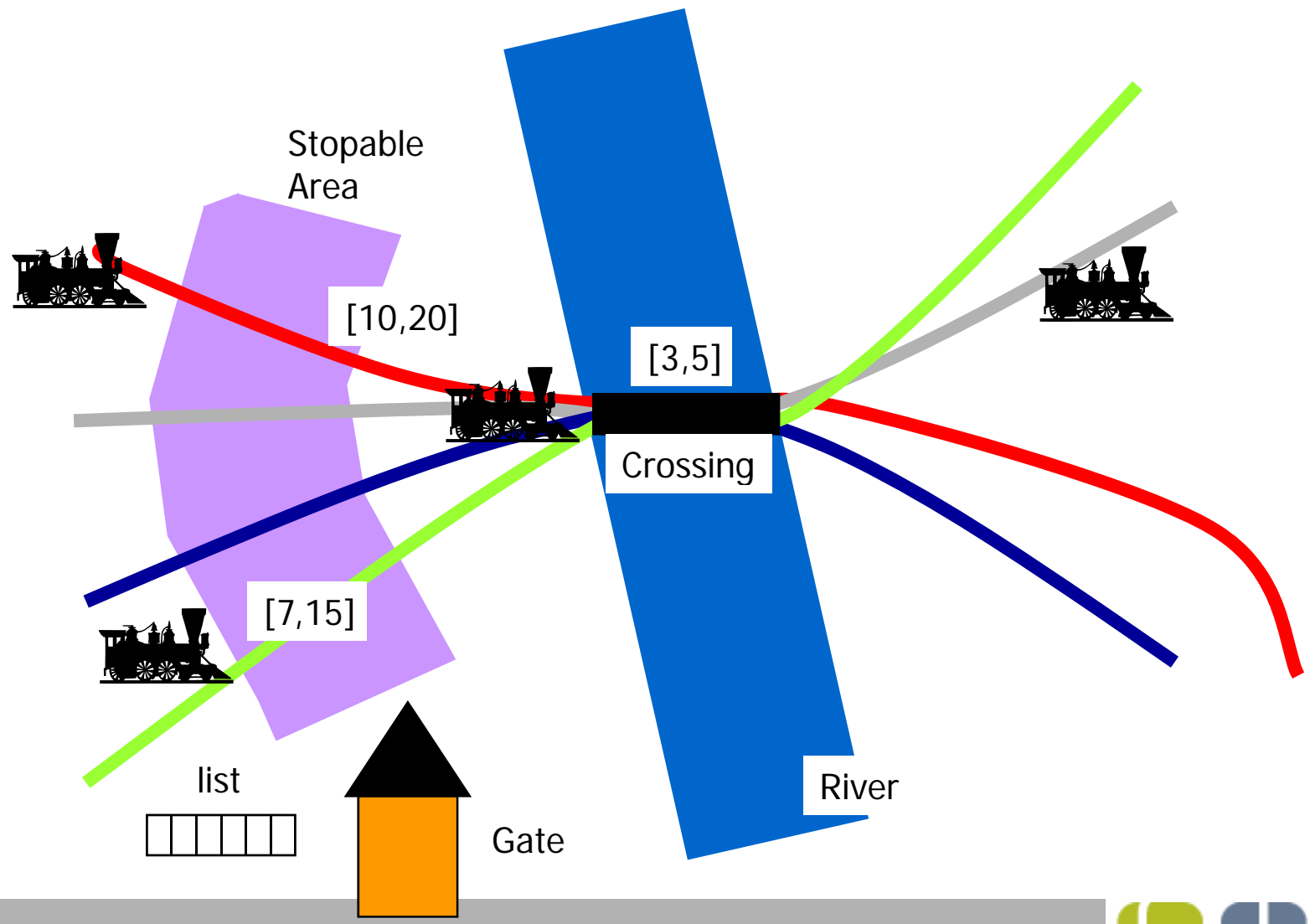


UPPAAL

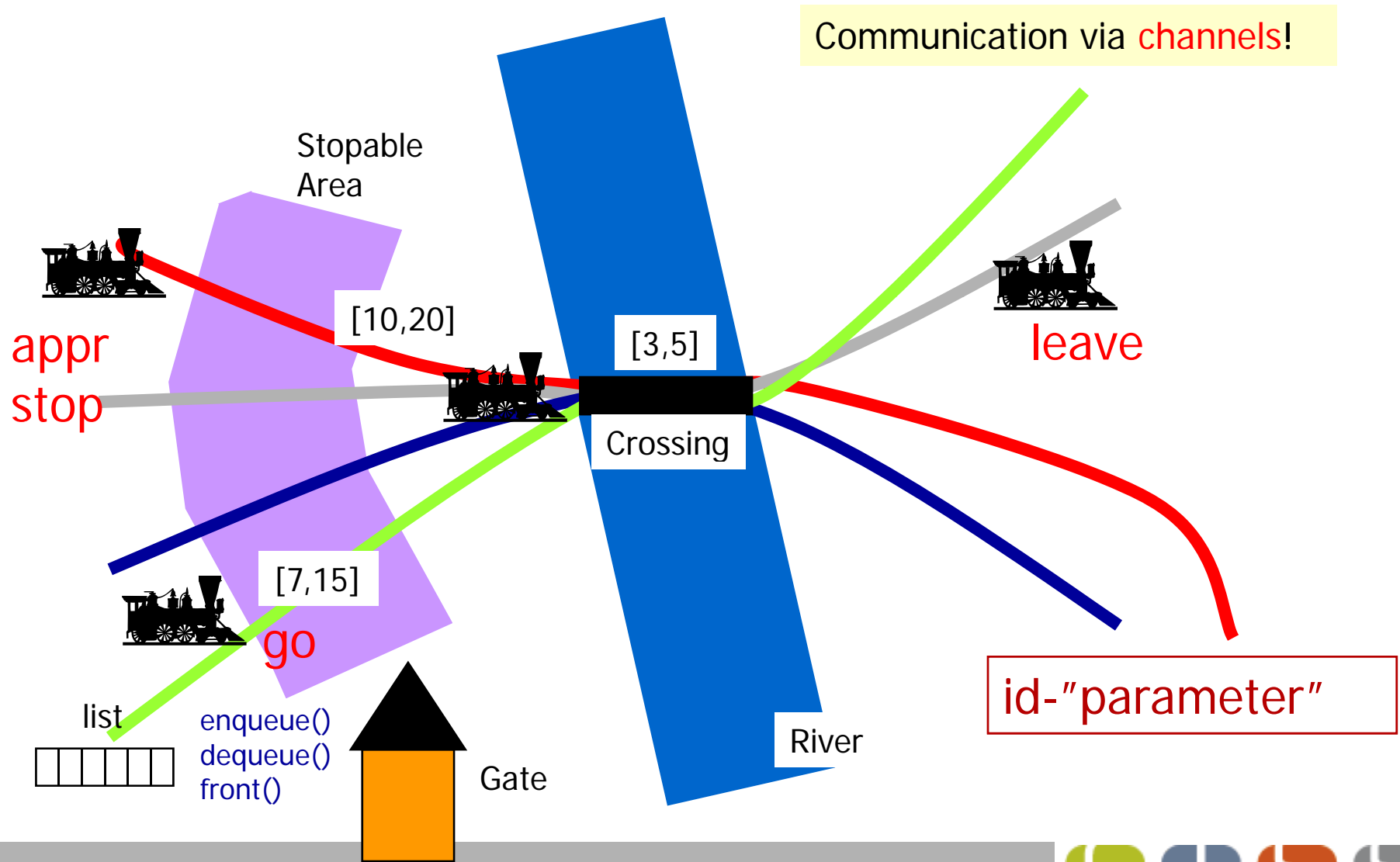
Modeling & Specification



Train Crossing



Train Crossing



Declarations

C:/Documents and Settings/Kim/Desktop/uppaal-3.4.7/demo/train-gate.xml - UPPAAL

File Templates View Queries Options Help

System Editor Simulator Verifier

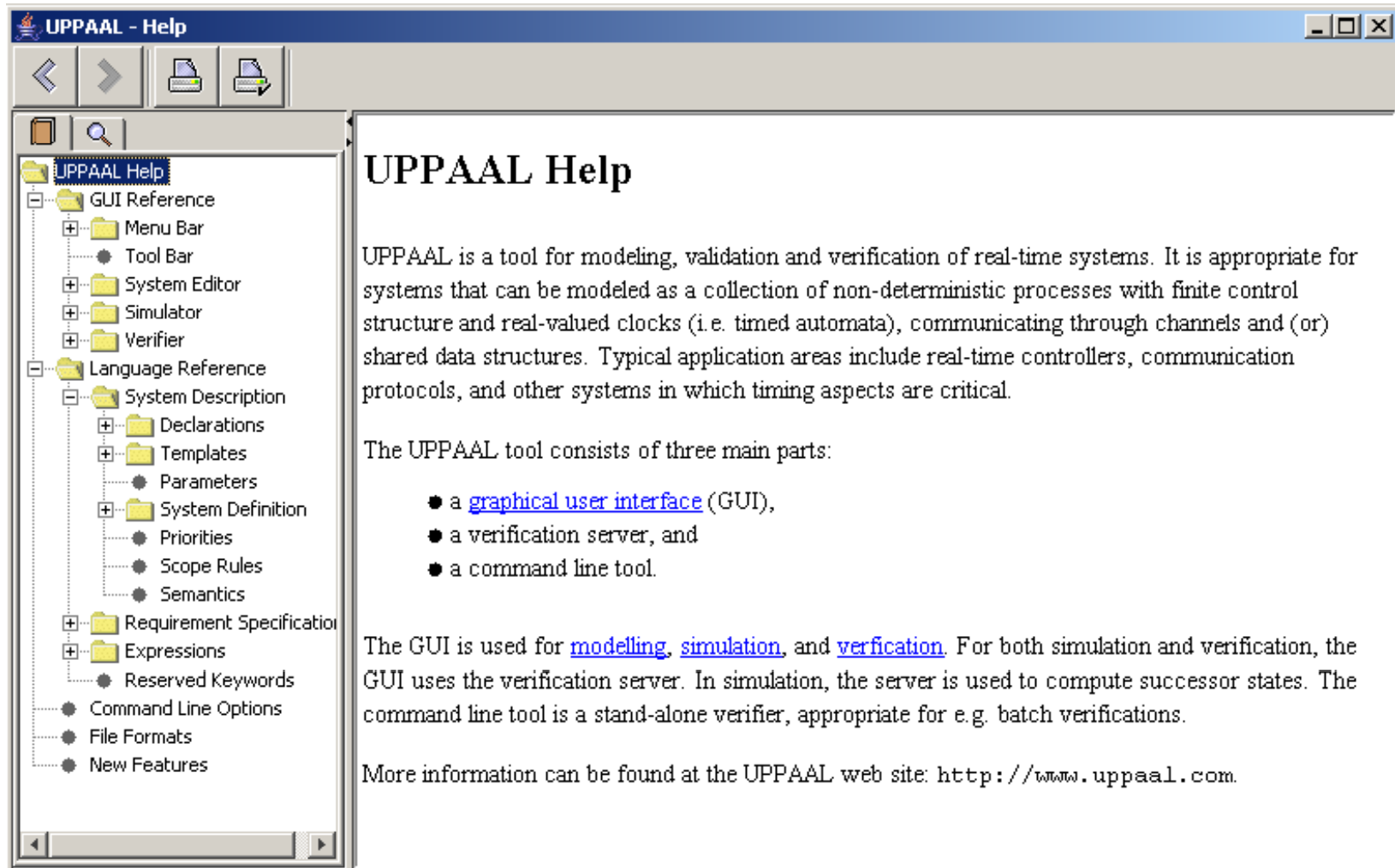
Drag out

- train-gate
 - Global declarations
 - Train
 - Gate
 - IntQueue
 - Process assignments
 - System definition
- train-gate
 - Global declarations
 - Train
 - Declarations
- train-gate
 - Global declarations
 - Train
 - Declarations
 - Gate
 - IntQueue
 - Declarations
- Process assignments
- System definition
- IntQueue
 - Declarations
 - Process assignments
 - System definition

```
/*  
 * For more details about this example, see  
 * "Automatic Verification of Real-Time Communicating Systems by Constraint Solving",  
 * by Wang Yi, Paul Pettersson and Mats Daniels. In Proceedings of the 7th International  
 * Conference on Formal Description Techniques, pages 223-238, North-Holland. 1994.  
 */  
  
const N      5;          // # trains + 1  
int[0,N]    e1;  
chan        appr, stop, go, leave;  
chan        empty, notempty, hd, add, rem;  
  
clock x;  
  
int[0,N] list[N], len, i;  
  
Train1:=Train(e1, 1);  
Train2:=Train(e1, 2);  
Train3:=Train(e1, 3);  
Train4:=Train(e1, 4);  
  
system  
    Train1, Train2, Train3, Train4,  
    appr, stop, go, leave, empty, notempty, hd, add, rem;
```

Constants
Bounded integers
Channels
Clocks
Arrays
Types
Functions
Templates
Processes
Systems

UPPAAL Help



Logical Specifications

- **Validation Properties**
 - Possibly: $E \langle \rangle P$
- **Safety Properties**
 - Invariant: $A[] P$
 - Pos. Inv.: $E[] P$
- **Liveness Properties**
 - Eventually: $A \langle \rangle P$
 - Leadsto: $P \rightarrow Q$
- **Bounded Liveness**
 - Leads to within: $P \rightarrow_{\leq t} Q$

The expressions P and Q must be type safe, side effect free, and evaluate to a boolean.

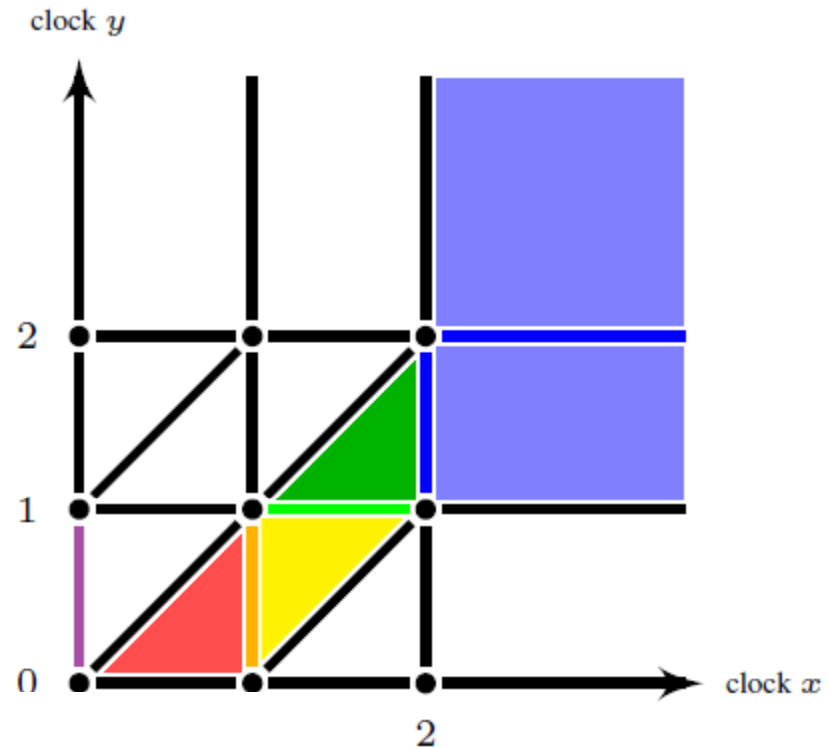
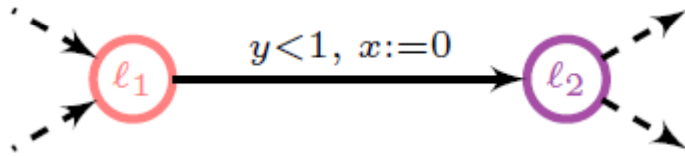
Only references to integer variables, constants, clocks, are allowed (and arrays of these).

Symbolic Verification

The UPPAAL Verification Engine



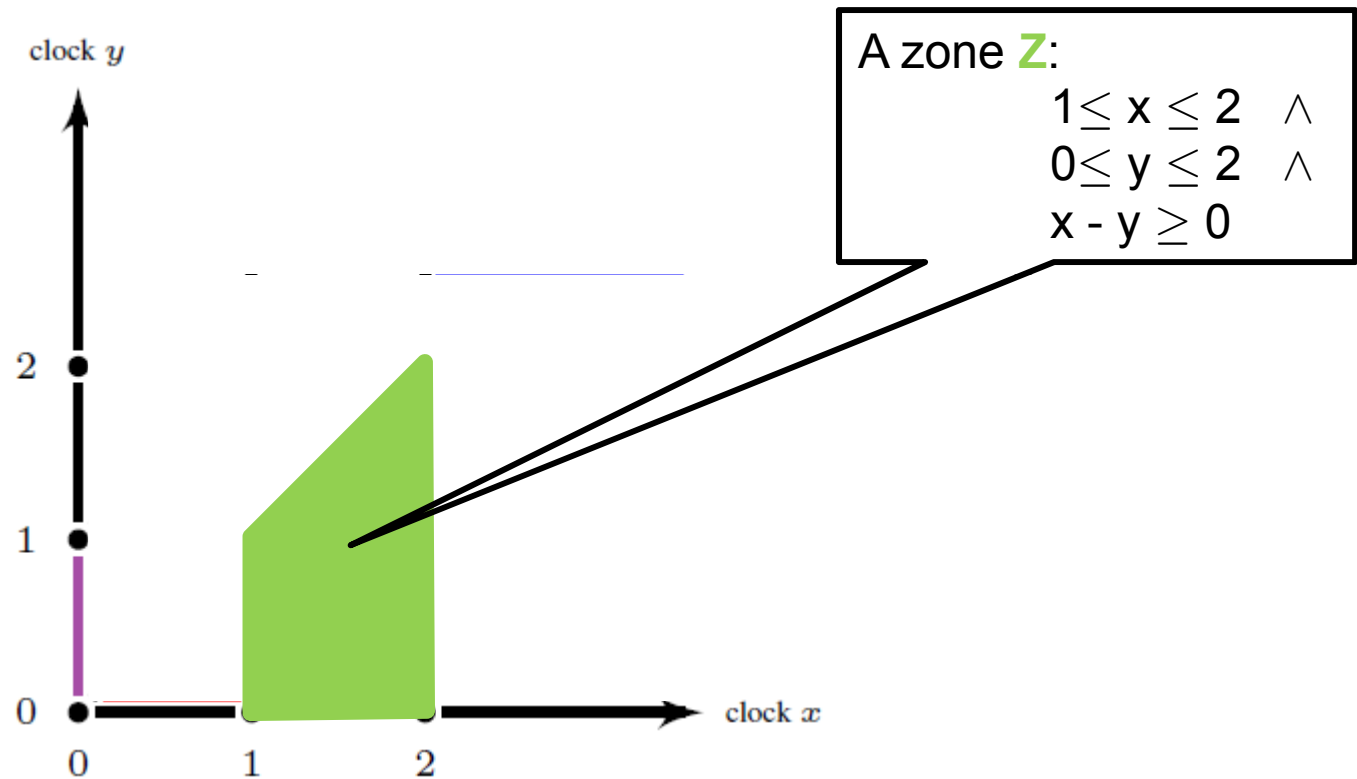
Regions – From Infinite to Finite



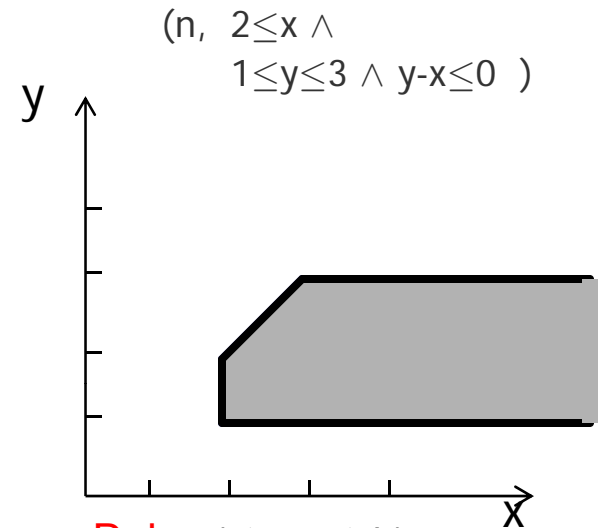
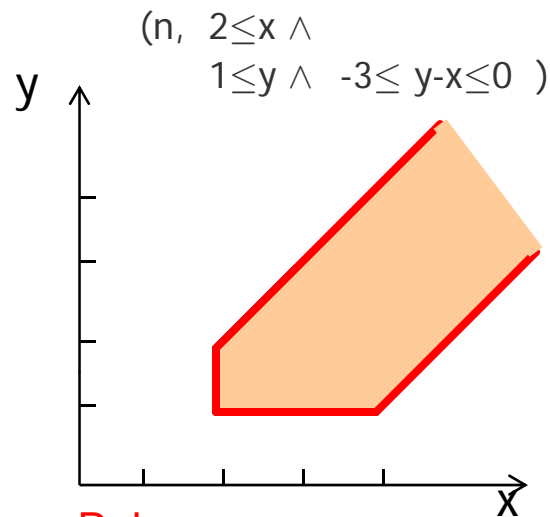
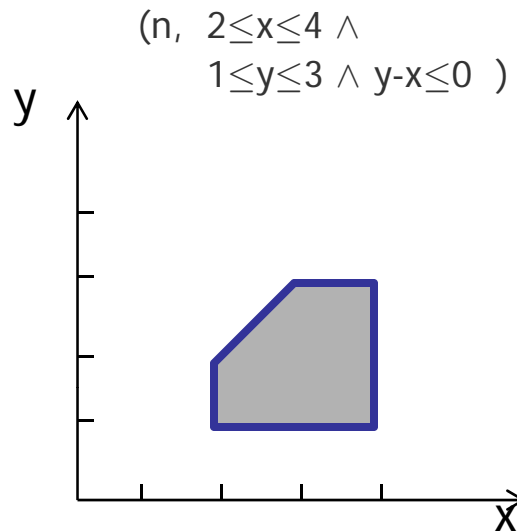
Theorem

The number of regions is $n! \cdot 2^n \cdot \prod_{x \in C} (2c_x + 2)$.

Zones – From Finite to Efficiency

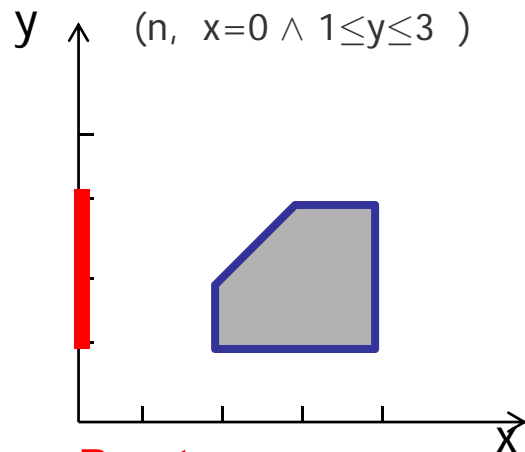


Zones – Operations

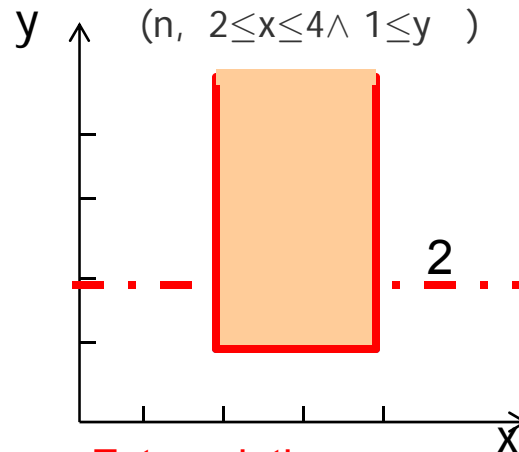


Delay

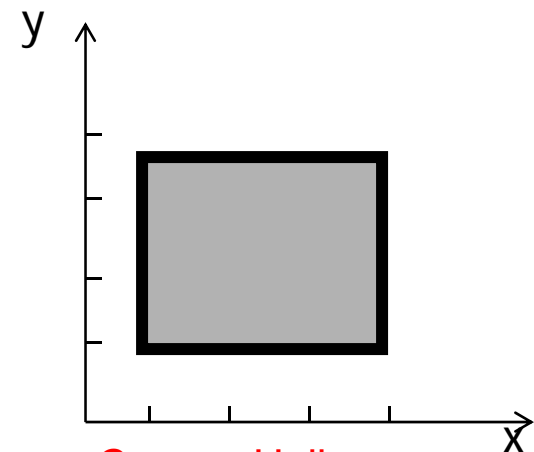
Delay (stopwatch)



Reset



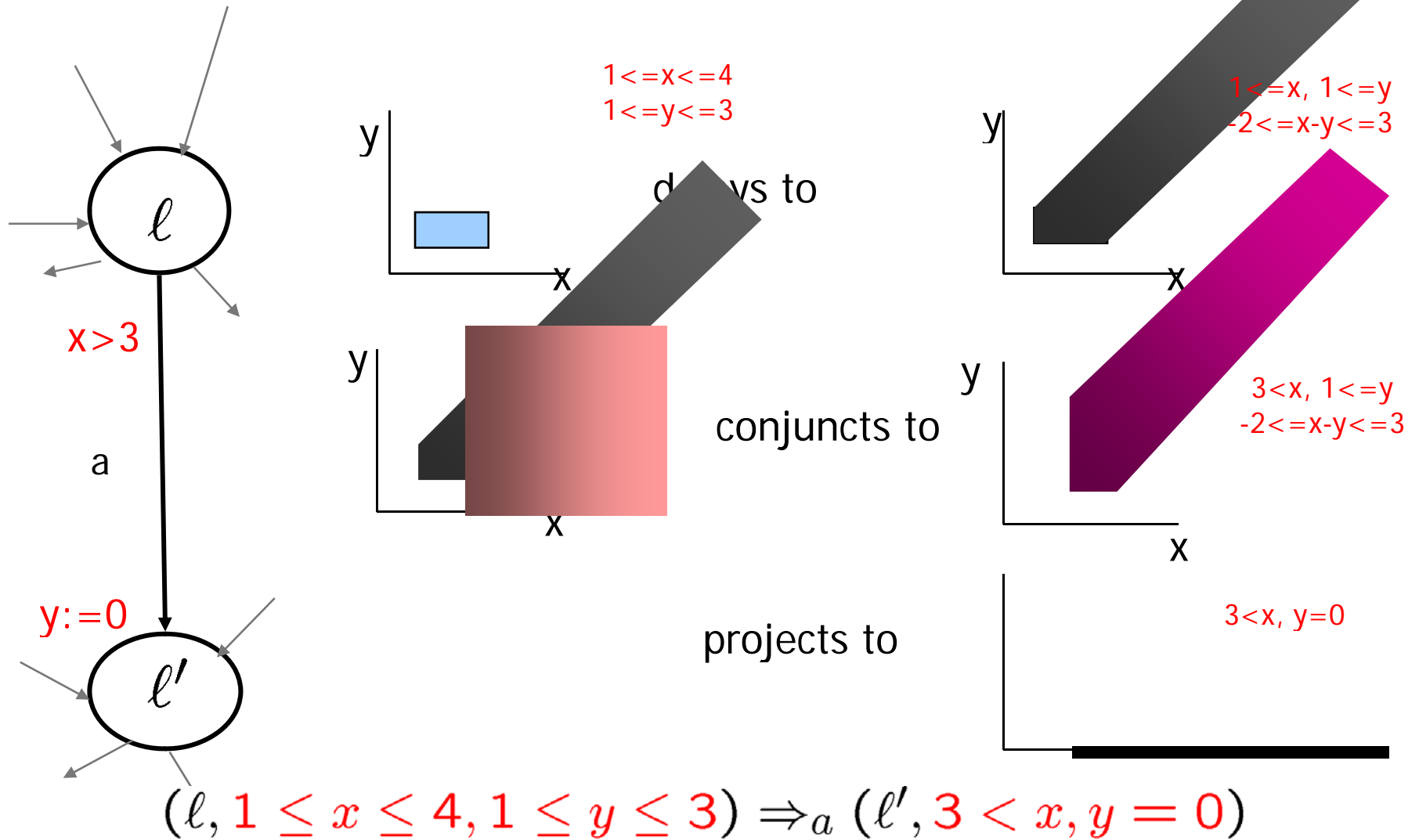
Extrapolation



Convex Hull

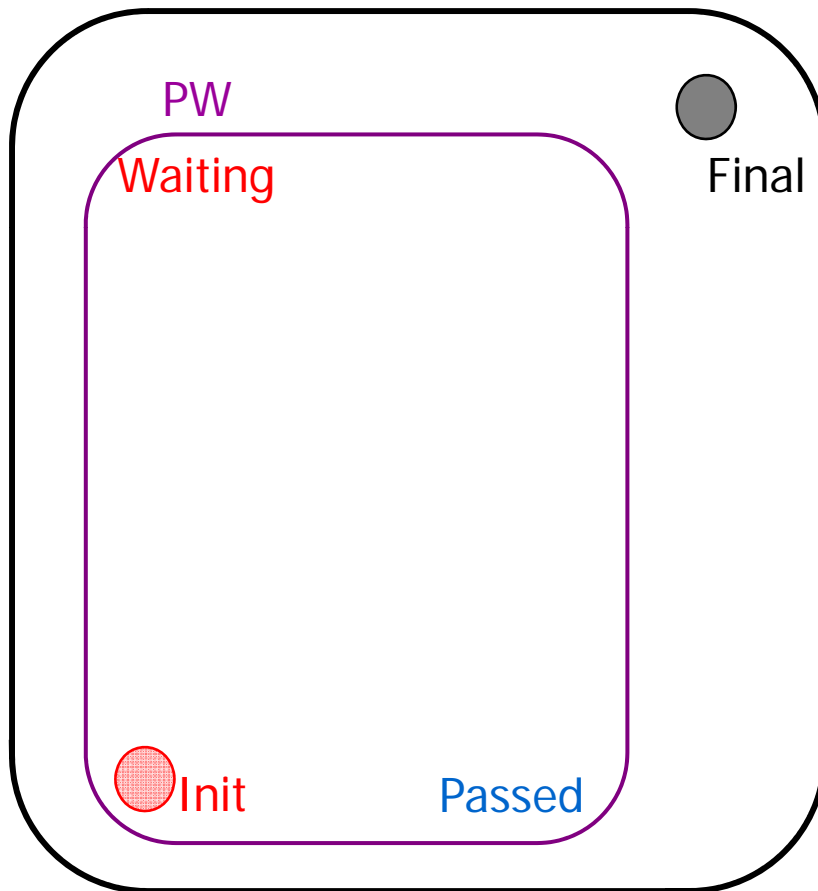


Symbolic Transitions



Forward Reachability

Init -> Final ?



```
INITIAL Passed :=  $\emptyset$ ;  
       Waiting :=  $\{(n_0, Z_0)\}$ 
```

```
REPEAT
```

```
  pick  $(n, Z)$  in Waiting
```

```
  if  $(n, Z) = \text{Final}$  return true
```

```
  for all  $(n, Z) \rightarrow (n', Z')$ :
```

```
    if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue
```

```
    else add  $(n', Z')$  to Waiting
```

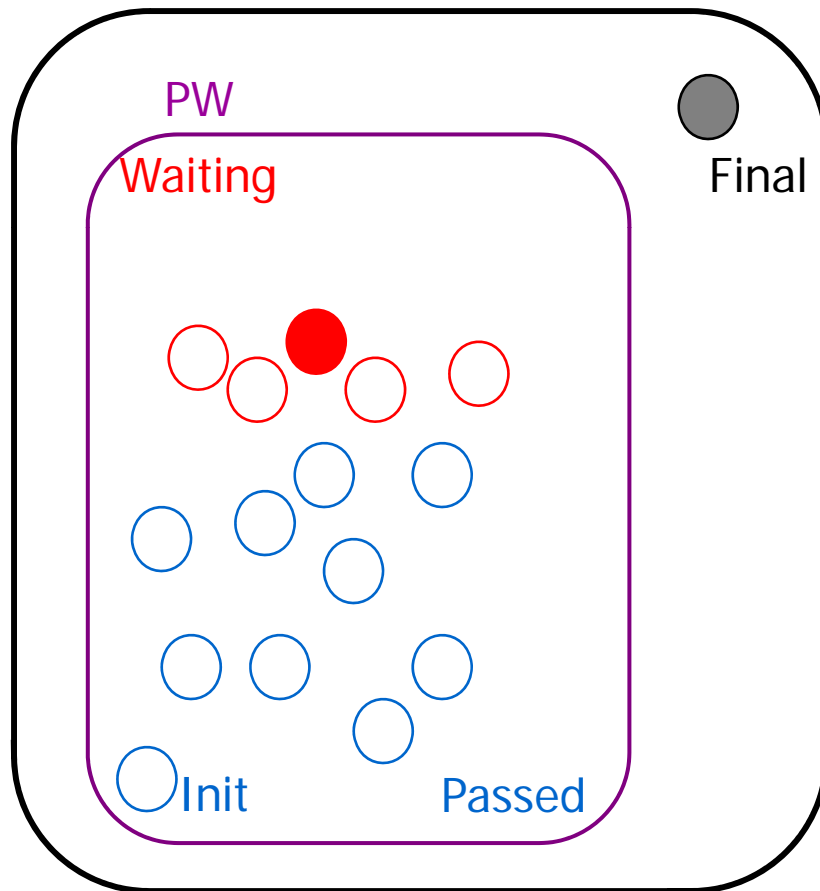
```
    move  $(n, Z)$  to Passed
```

```
UNTIL Waiting =  $\emptyset$ 
```

```
return false
```

Forward Reachability

Init -> Final ?



INITIAL $Passed := \emptyset;$
 $Waiting := \{(n_0, Z_0)\}$

REPEAT

pick (n, Z) in $Waiting$

if $(n, Z) = Final$ return true

for all $(n, Z) \rightarrow (n', Z')$:

if for some (n', Z'') $Z' \subseteq Z''$ continue

else add (n', Z') to $Waiting$

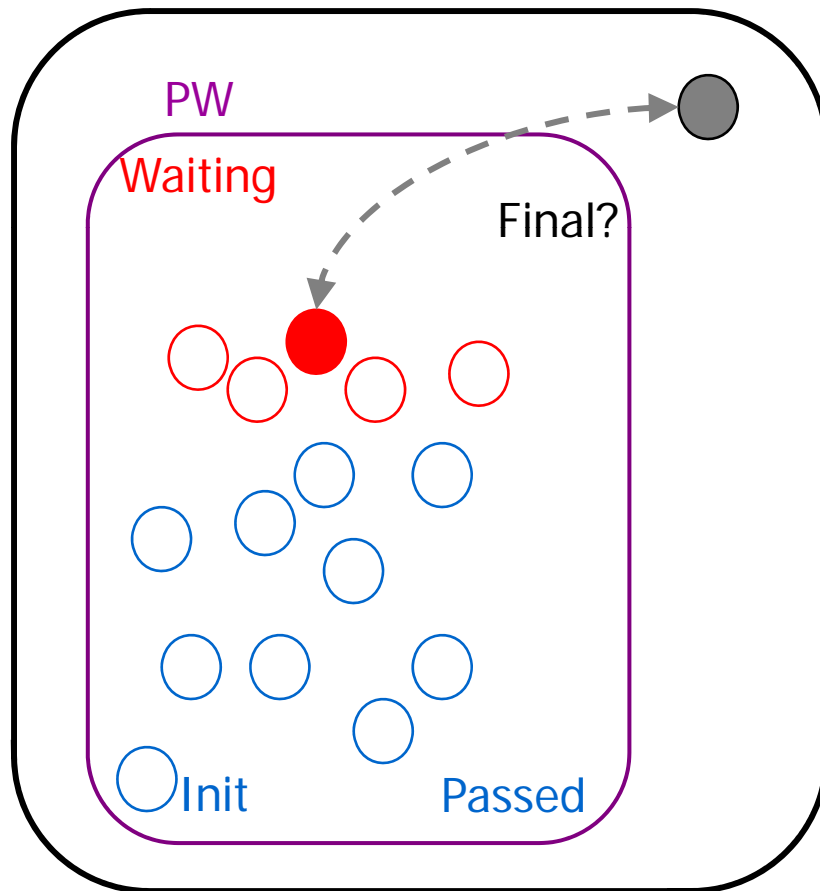
move (n, Z) to $Passed$

UNTIL $Waiting = \emptyset$

return false

Forward Reachability

Init -> Final ?



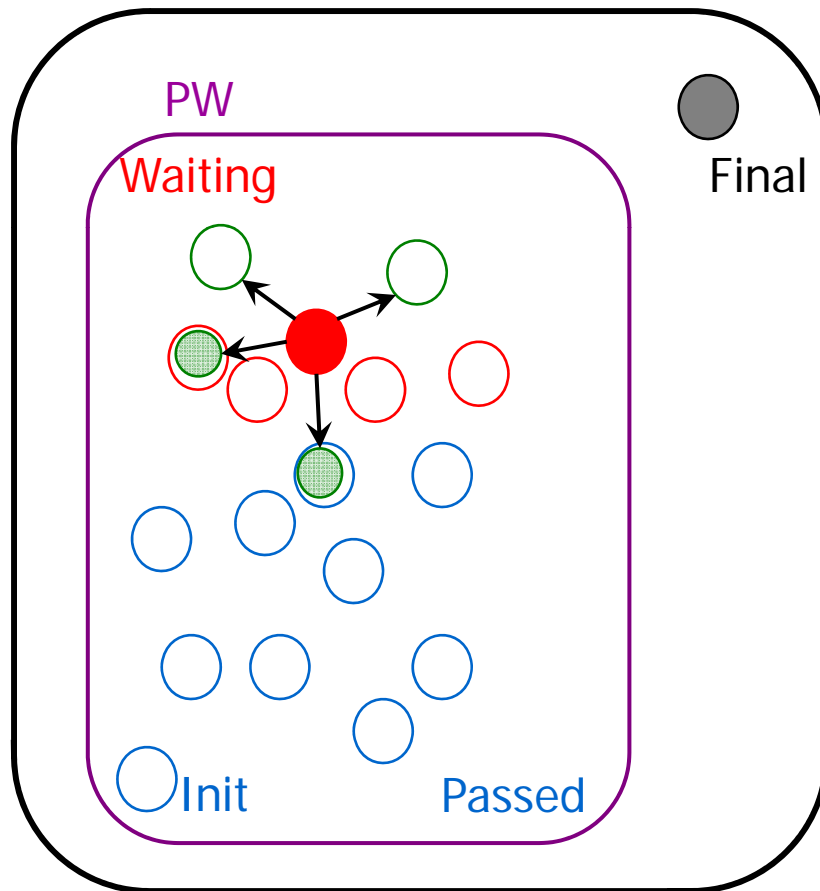
INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT
 pick (n, Z) in **Waiting**
 if $(n, Z) = \text{Final}$ return true
 for all $(n, Z) \rightarrow (n', Z')$:
 if for some (n', Z'') $Z' \subseteq Z''$ continue
 else add (n', Z') to **Waiting**
 move (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
return false

Forward Reachability

Init -> Final ?



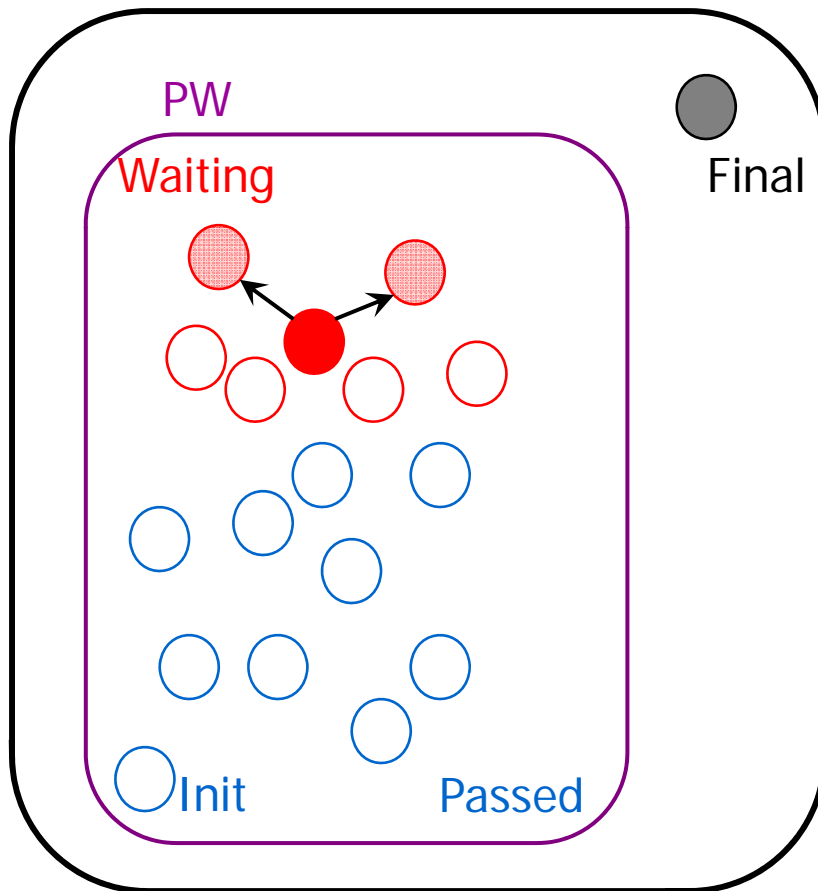
INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT
pick (n, Z) in **Waiting**
if $(n, Z) = \text{Final}$ return true
for all $(n, Z) \rightarrow (n', Z')$:
if for some (n', Z'') $Z' \subseteq Z''$ continue
else add (n', Z') to **Waiting**
move (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
return false

Forward Reachability

Init -> Final ?



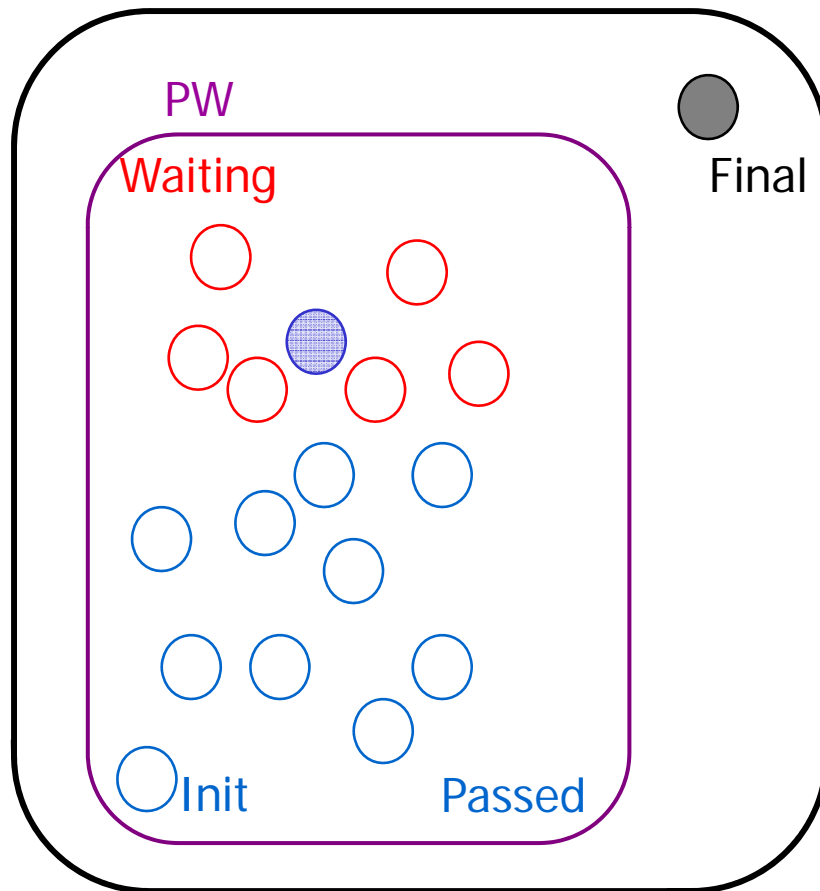
INITIAL $Passed := \emptyset;$
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REPEAT
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if $(n, Z) = Final$ return true
for all $(n, Z) \rightarrow (n', Z')$:
if for some (n', Z'') $Z' \subseteq Z''$ continue
else add (n', Z') to $Waiting$
move (n, Z) to $Passed$

UNTIL $Waiting = \emptyset$
return false

Forward Reachability

Init -> Final ?



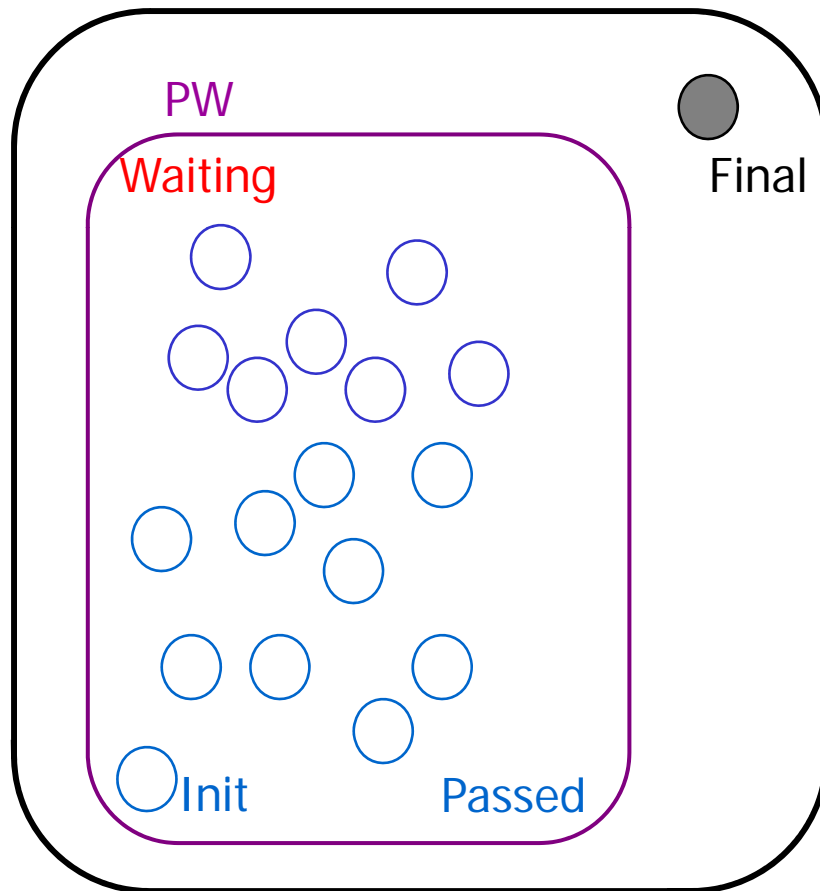
INITIAL $Passed := \emptyset;$
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Forward Reachability

Init -> Final ?

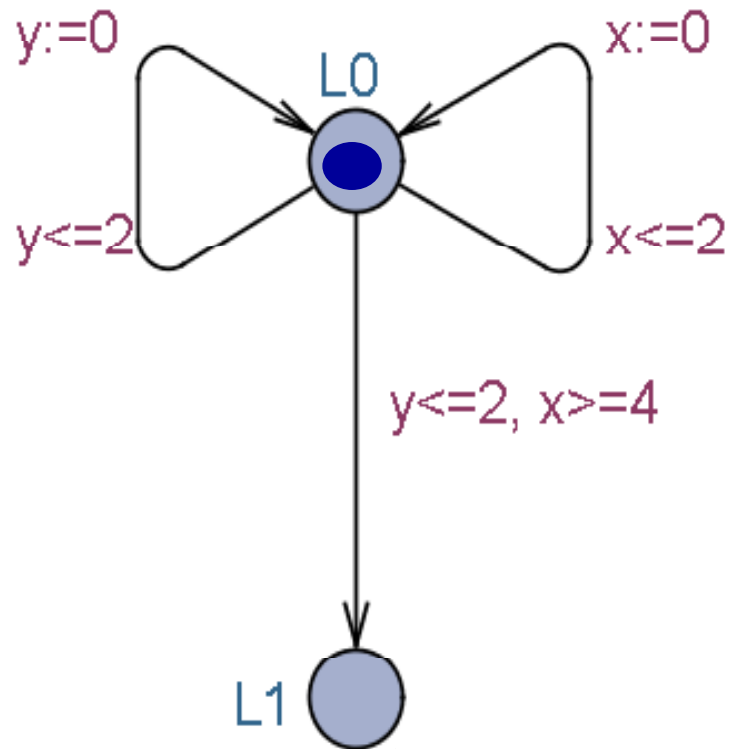


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       Waiting :=  $\{(n_0, Z_0)\}$ 
```

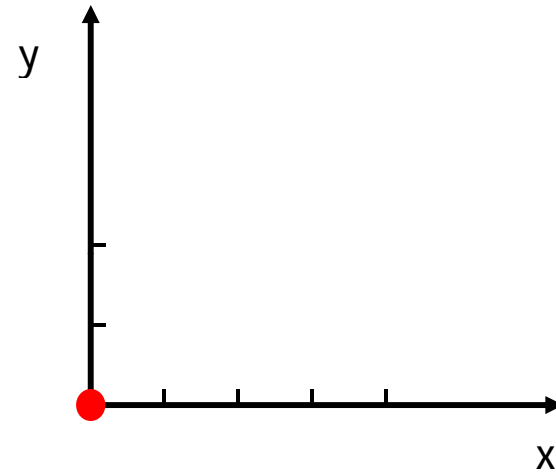
```
REPEAT  
  pick  $(n, Z)$  in Waiting  
  if  $(n, Z) = \text{Final}$  return true  
  for all  $(n, Z) \rightarrow (n', Z')$ :  
    if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue  
    else add  $(n', Z')$  to Waiting  
    move  $(n, Z)$  to Passed
```

```
UNTIL Waiting =  $\emptyset$   
return false
```

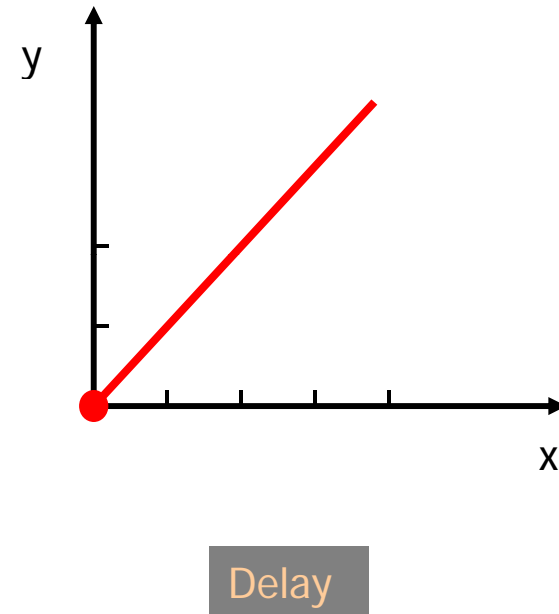
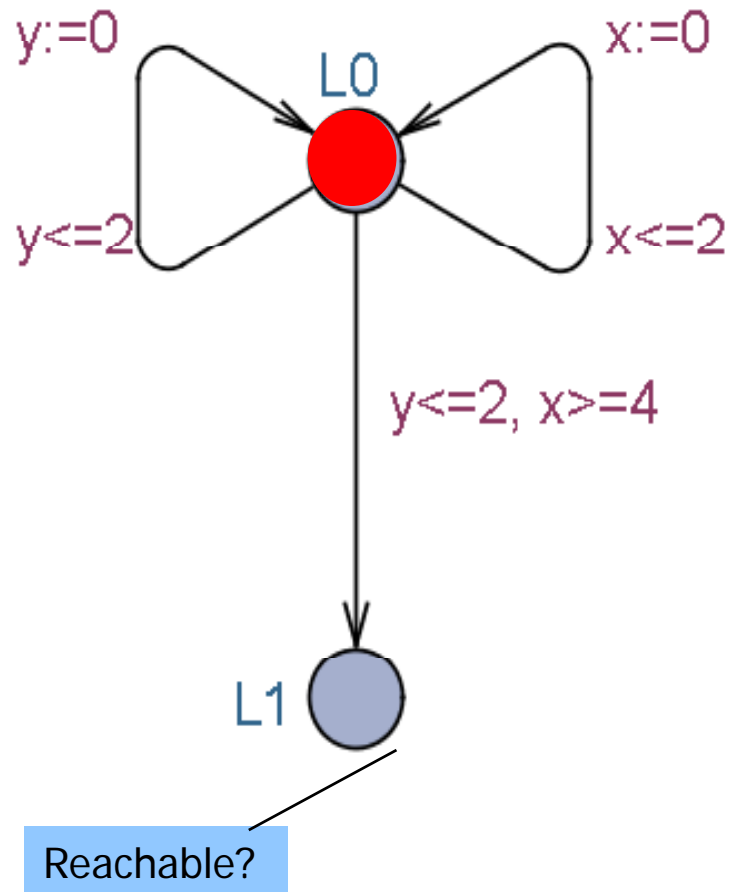
Symbolic Exploration



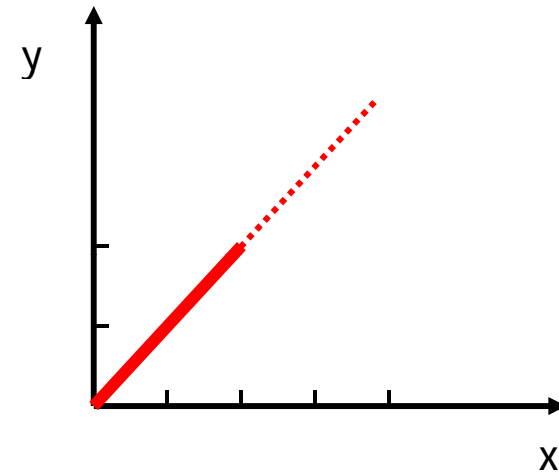
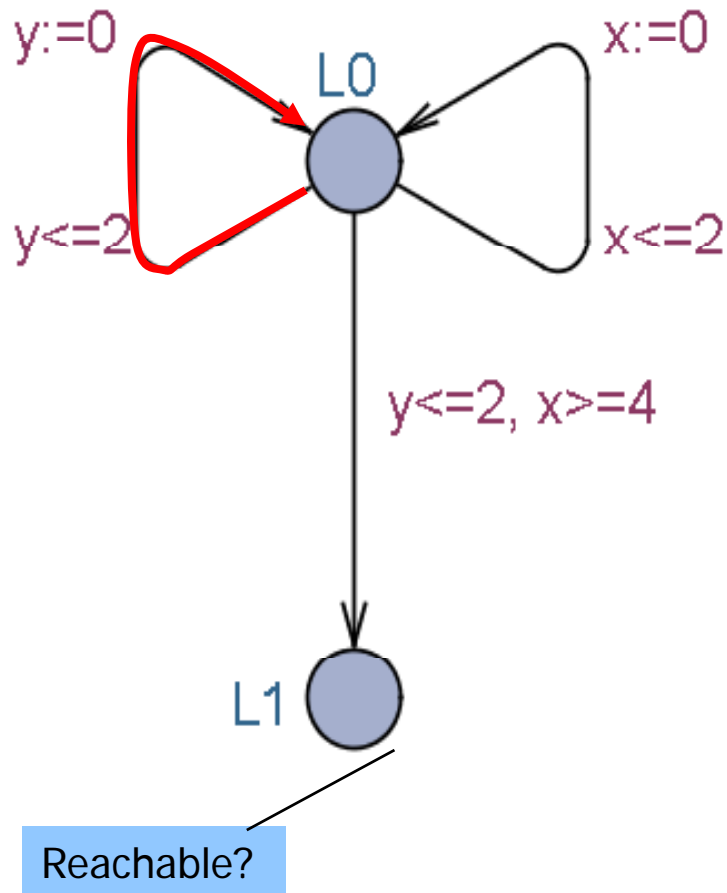
Reachable?



Symbolic Exploration



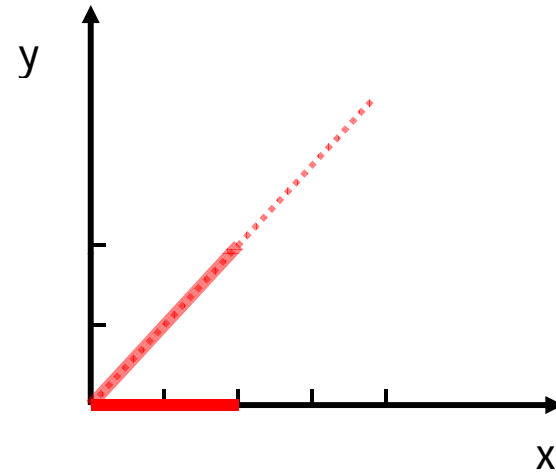
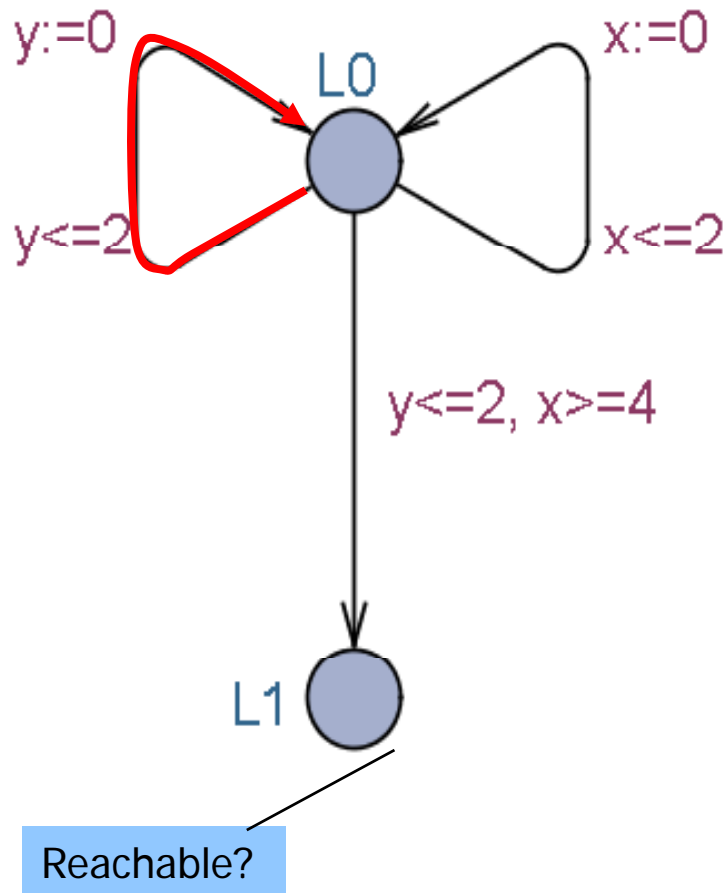
Symbolic Exploration



Left



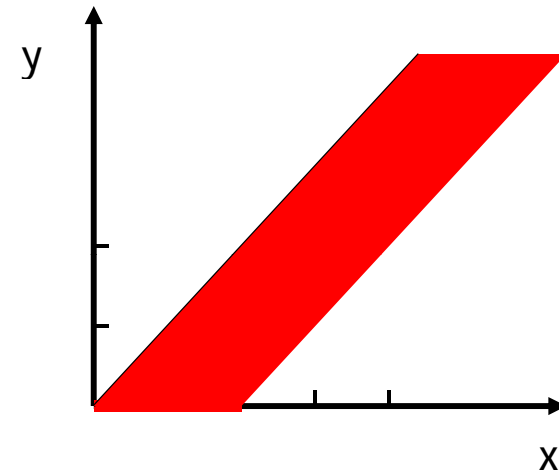
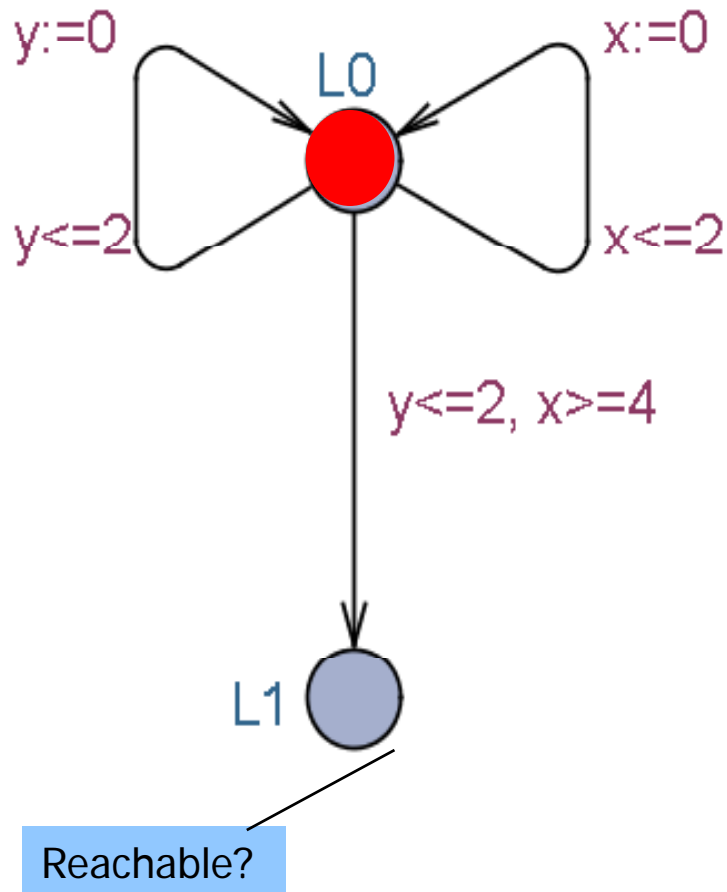
Symbolic Exploration



Left



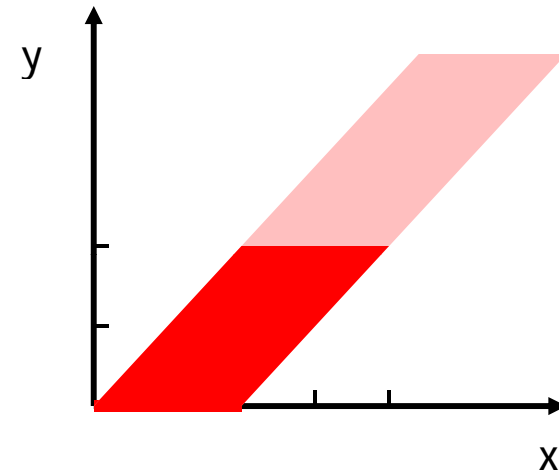
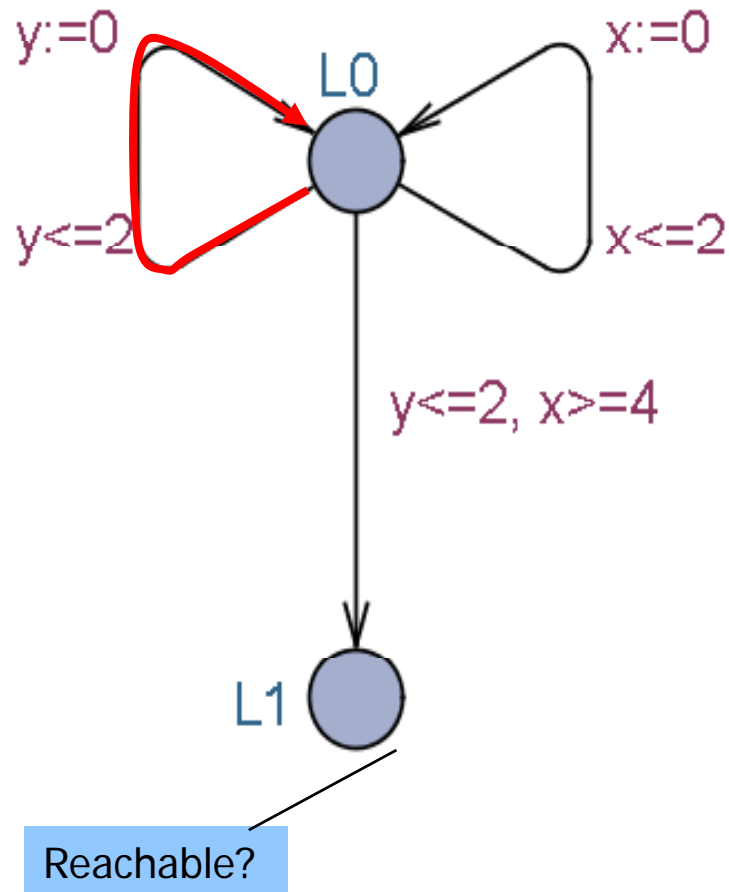
Symbolic Exploration



Delay



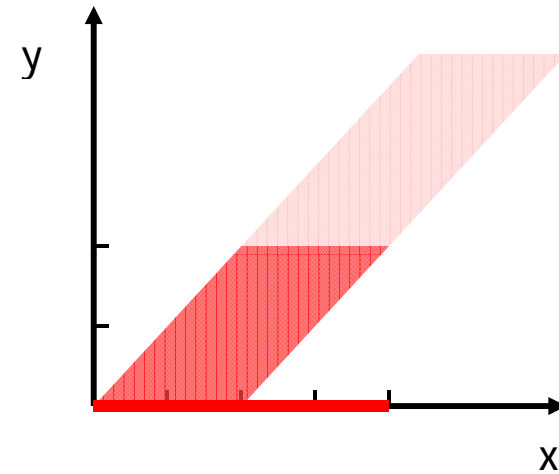
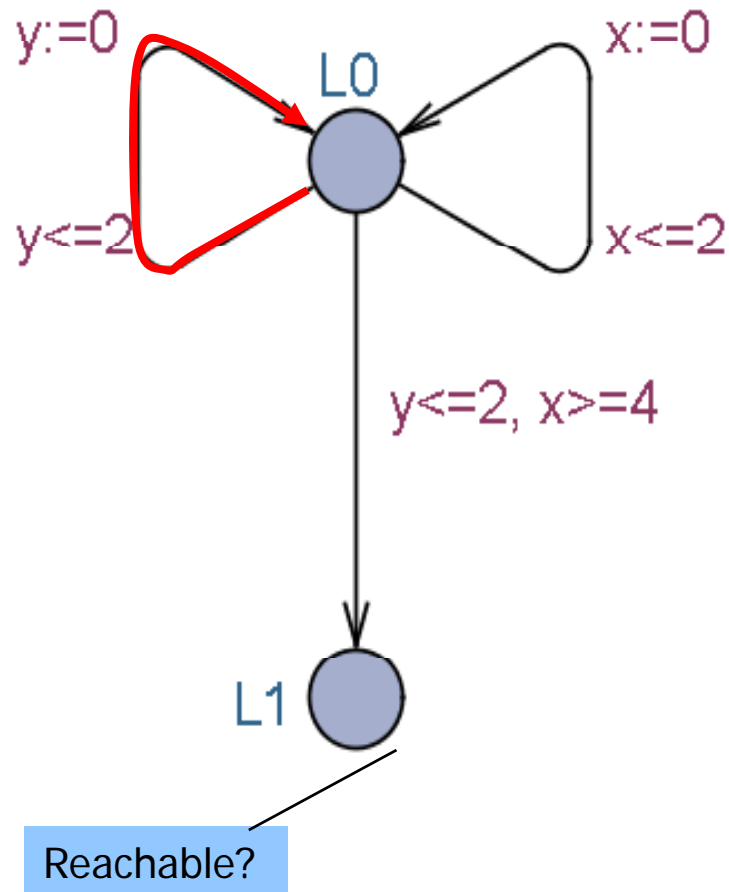
Symbolic Exploration



Left



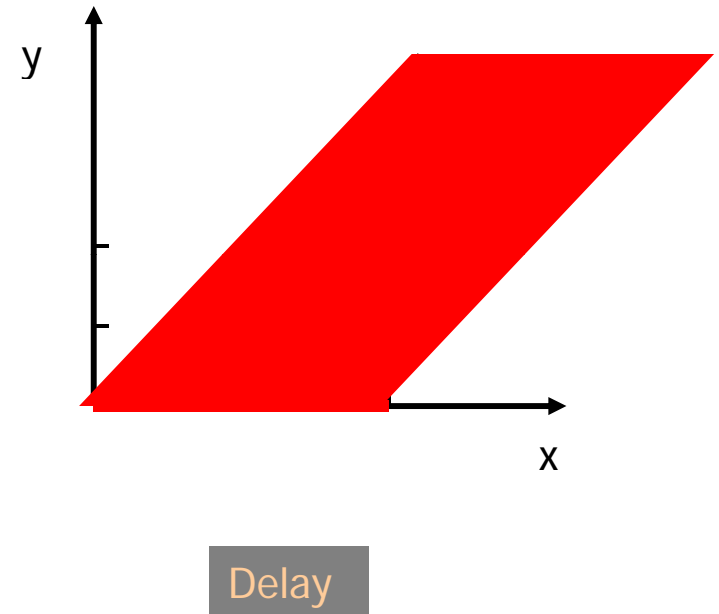
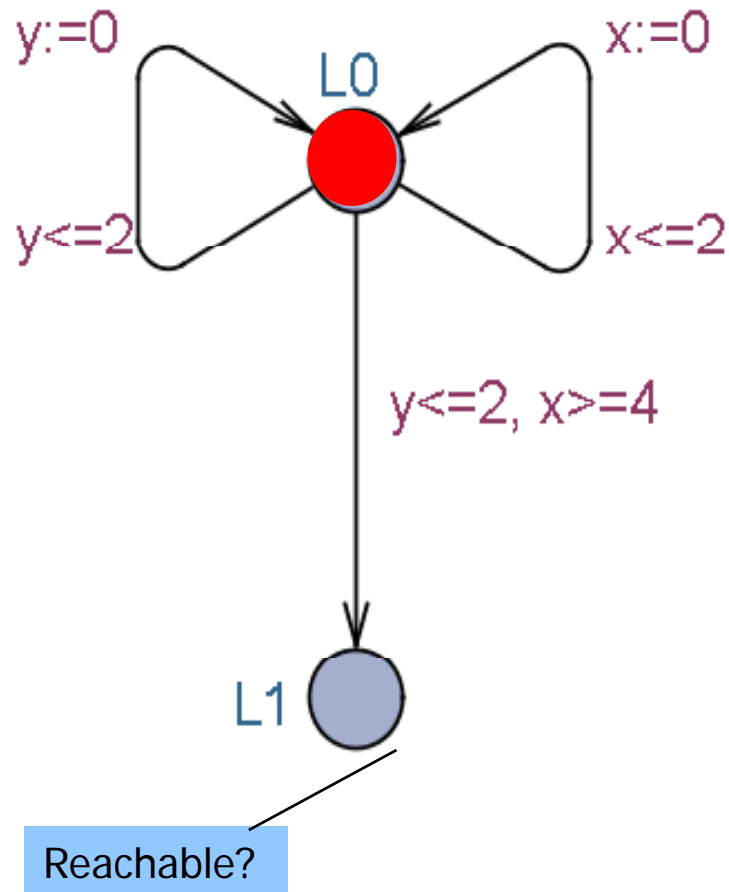
Symbolic Exploration



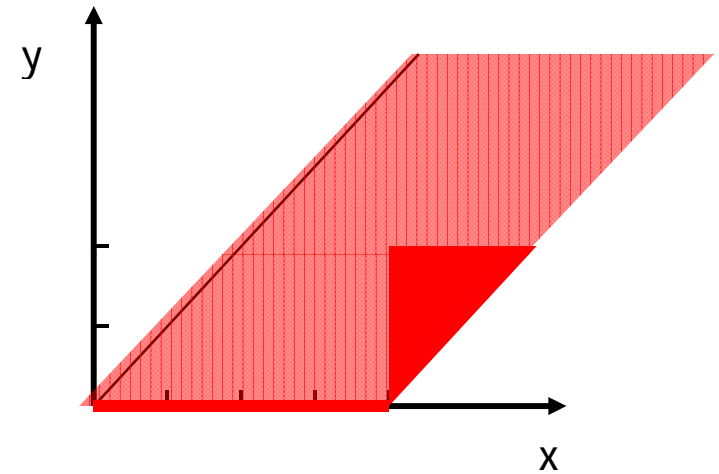
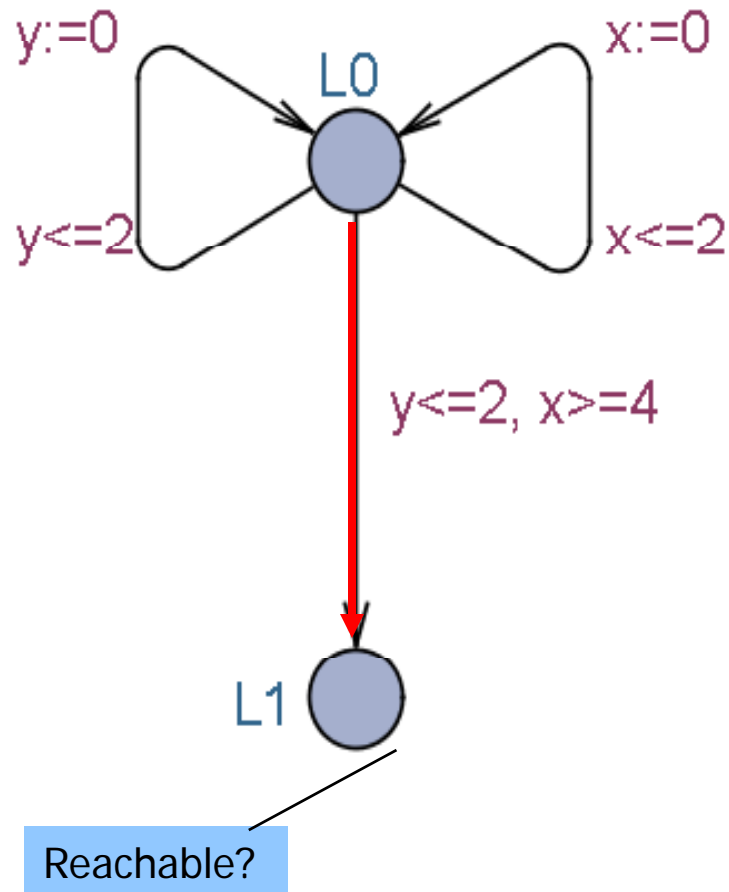
Left



Symbolic Exploration



Symbolic Exploration



Down



Datastructures for Zones

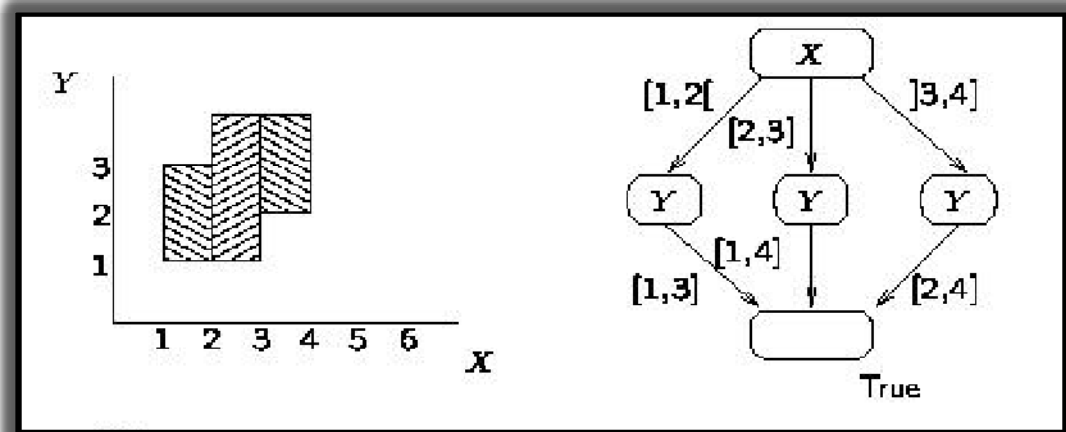
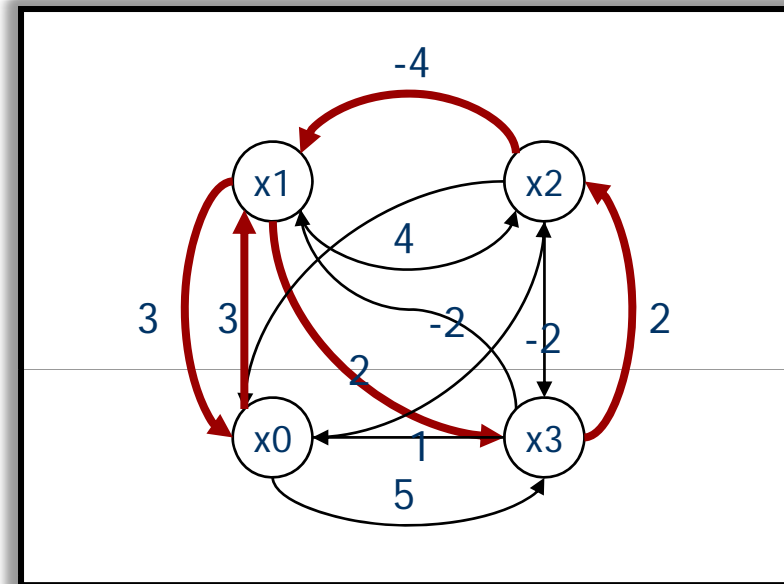
- Difference Bounded Matrices (DBMs)

- Minimal Constraint Form

[RTSS97]

- Clock Difference Diagrams

[CAV99]



Inclusion Checking (DBMs)

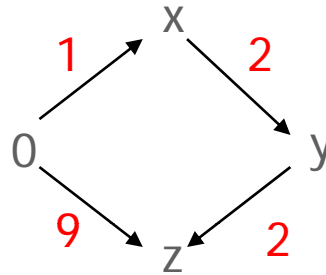
Bellman 1958, Dill 1989

Inclusion

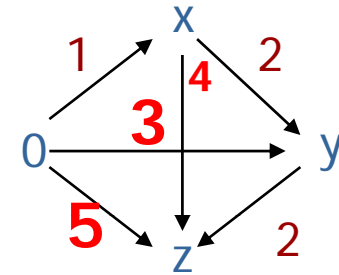
D1

$x \leq 1$
$y - x \leq 2$
$z - y \leq 2$
$z \leq 9$

Graph



Shortest
Path
Closure

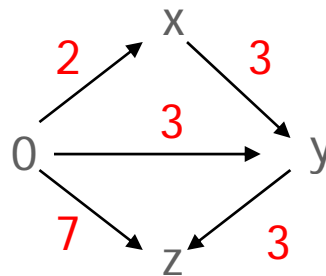


$? \subseteq ?$

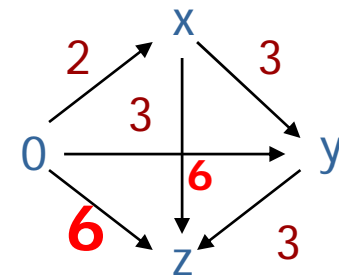
D2

$x \leq 2$
$y - x \leq 3$
$y \leq 3$
$z - y \leq 3$
$z \leq 7$

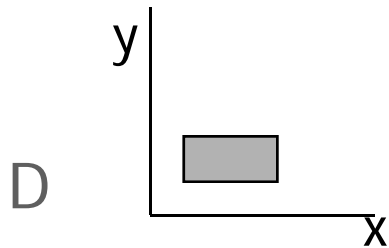
Graph



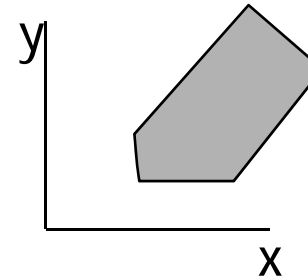
Shortest
Path
Closure



Future (DBMs)

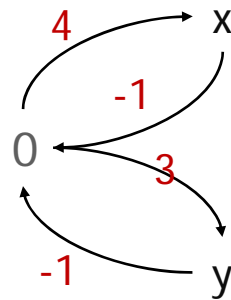


$$\begin{aligned} 1 &\leq x \leq 4 \\ 1 &\leq y \leq 3 \end{aligned}$$

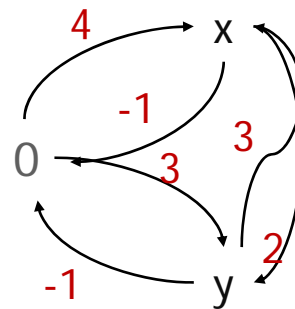


Future D

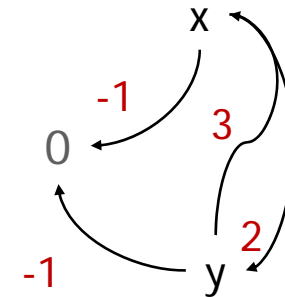
$$\begin{aligned} 1 &\leq x, 1 \leq y \\ -2 &\leq x - y \leq 3 \end{aligned}$$



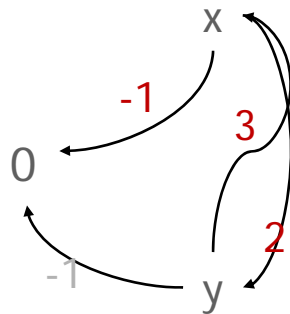
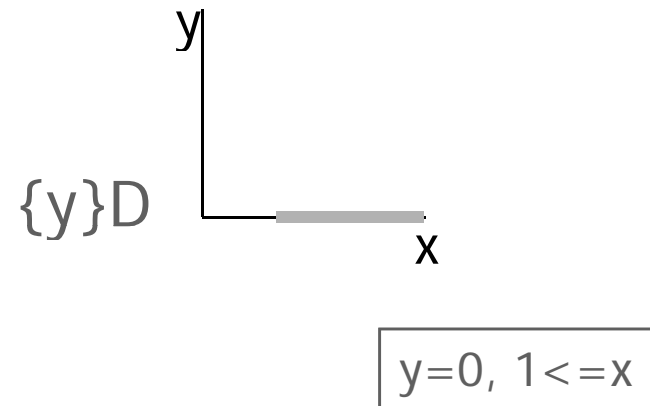
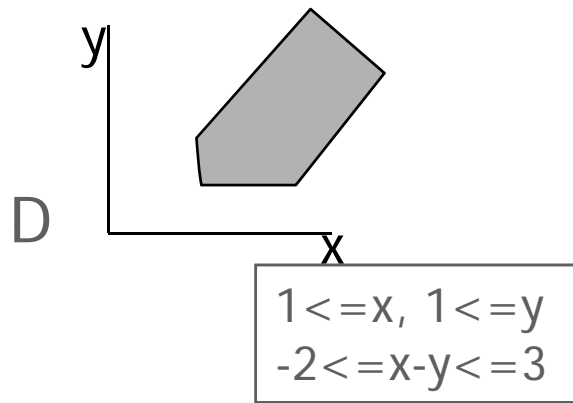
Shortest
Path
Closure



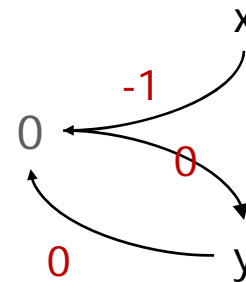
Remove
upper
bounds
on clocks



Reset (DBMs)



Remove all
bounds
involving y
and set y to 0

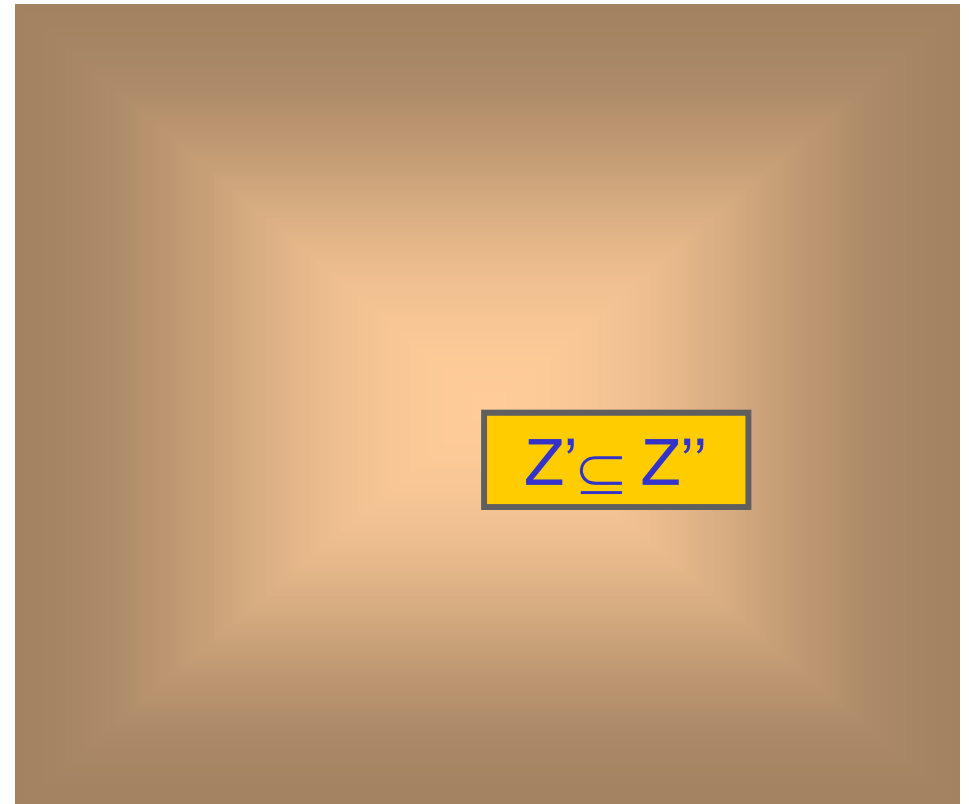
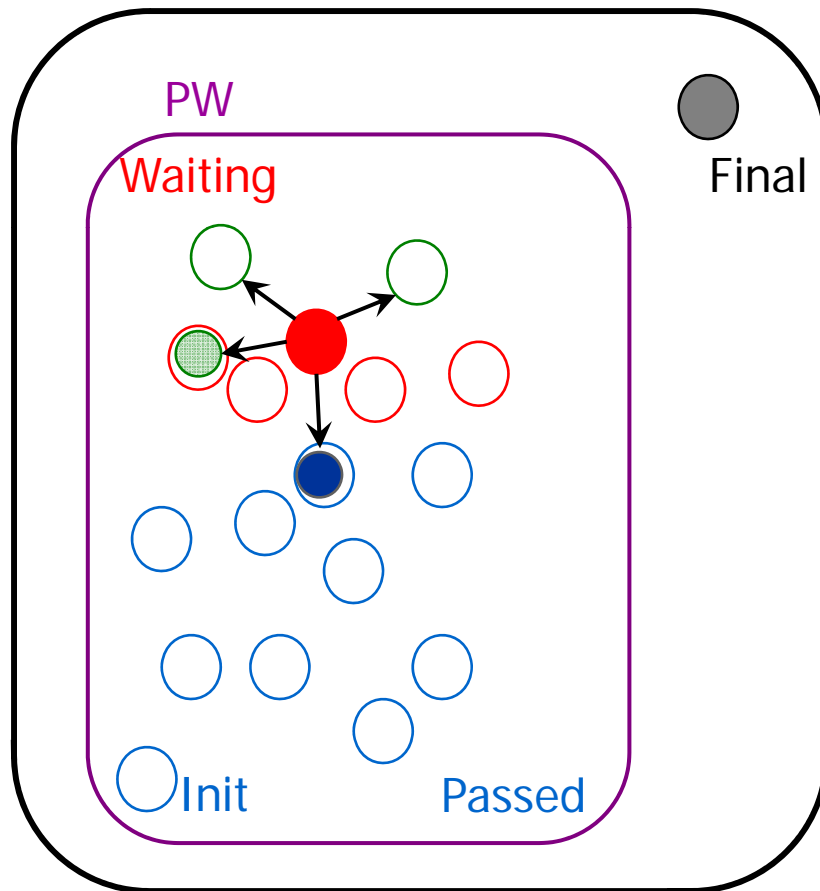


Clock Difference Diagrams



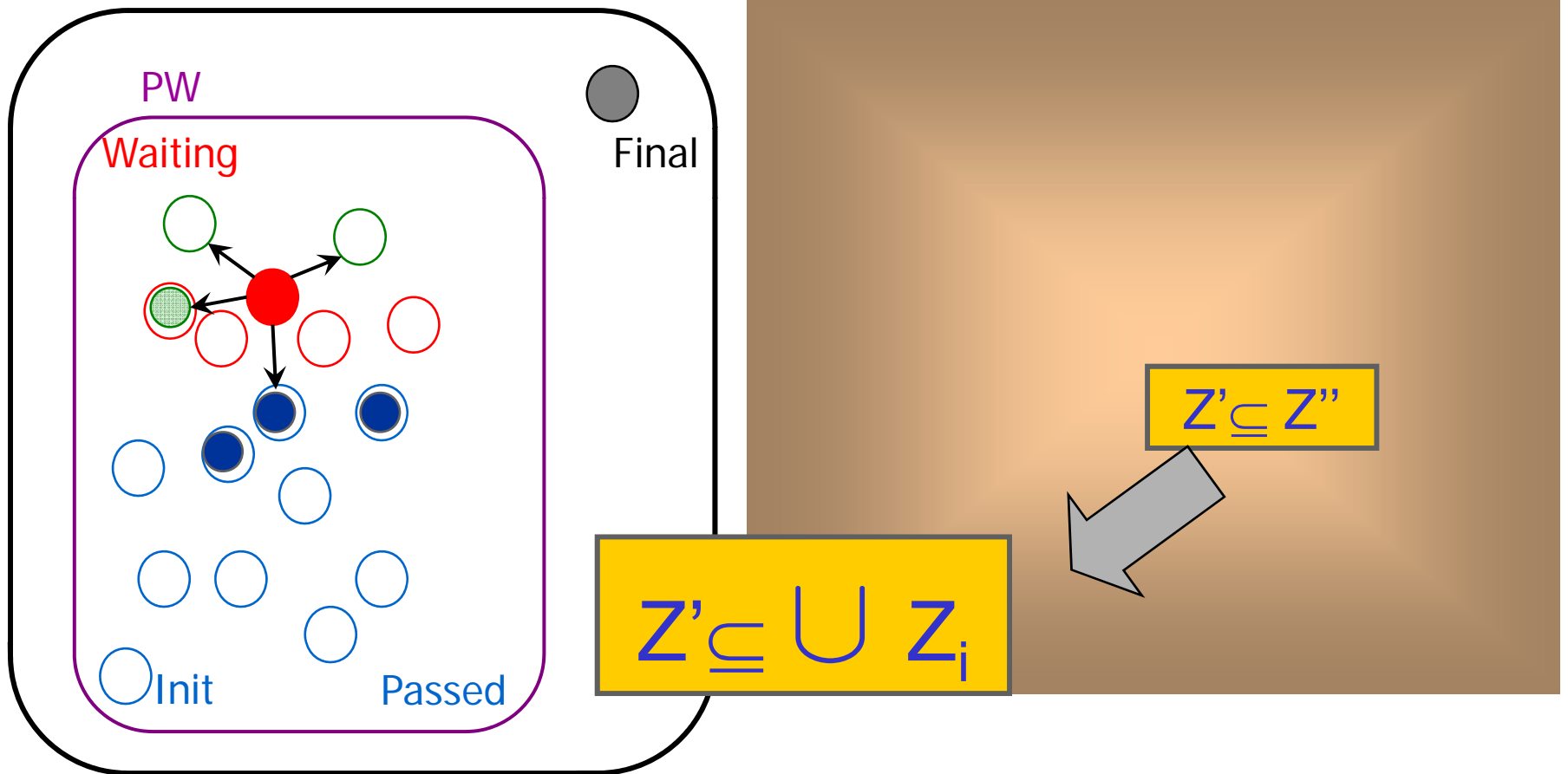
Earlier Termination

Init -> Final ?



Earlier Termination

Init -> Final ?

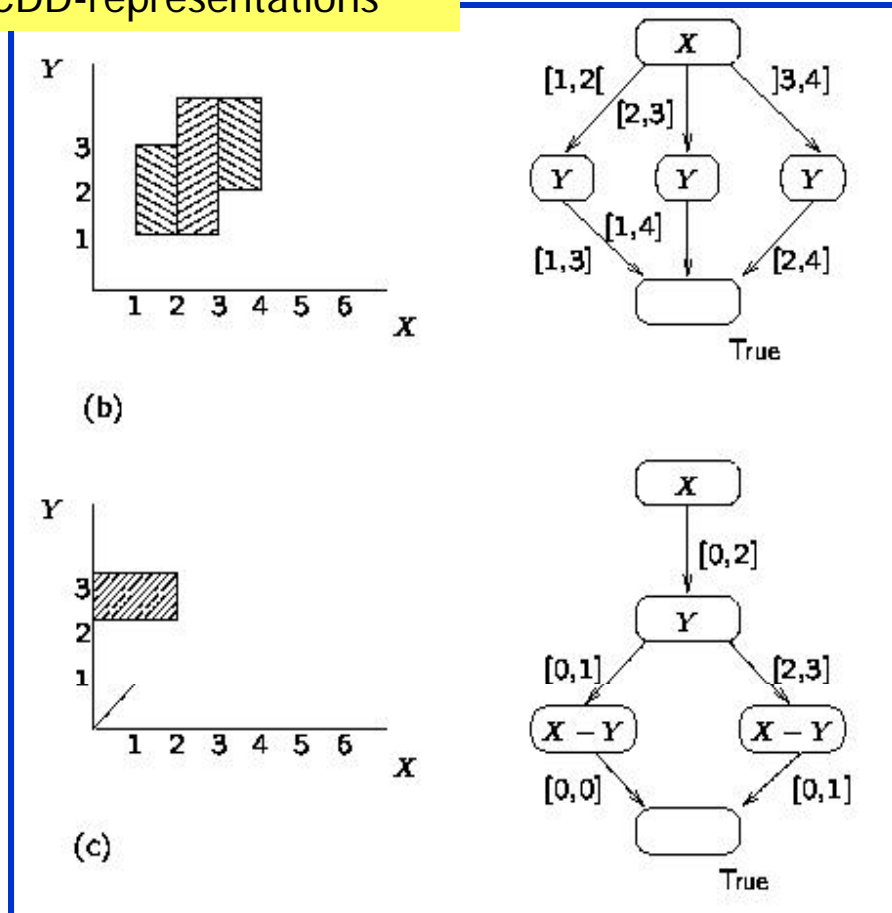


Clock Difference Diagrams

= Binary Decision Diagrams + Difference Bounded Matrices

CAV99

CDD-representations



- Nodes labeled with differences
- Maximal sharing of substructures (also across different CDDs)
- Maximal intervals
- Linear-time algorithms for set-theoretic operations.
- NDD's Maler et. al
- DDD's Møller, Lichtenberg



Clock Difference Diagrams I

A *Clock Difference Diagram* (CDD) is a directed acyclic graph consisting of a set of nodes V and two functions $\text{type} : V \rightarrow \mathcal{T}$ and $\text{succ} : V \rightarrow 2^{\mathcal{I} \times V}$ such that:

- V has exactly two *terminal nodes* called True and False, where $\text{type}(\text{True}) = \text{type}(\text{False}) = (0, 0)$ and $\text{succ}(\text{True}) = \text{succ}(\text{False}) = \emptyset$.
- all other nodes $n \in V$ are *inner nodes*, which have attributed a **type** $\text{type}(n) \in \mathcal{T}$ and a finite set of **successors** $\text{succ}(n) = \{(I_1, n_1), \dots, (I_k, n_k)\}$, where $(I_i, n_i) \in \mathcal{I} \times V$.



Disjoint & Ordered

For each inner node n , the following must hold:

- the successors are *disjoint*: for $(I, m), (I', m') \in \text{succ}(n)$ either $(I, m) = (I', m')$ or $I \cap I' = \emptyset$,
- the successor set is an \mathbb{R} -*cover*: $\bigcup \{I \mid \exists m. n \xrightarrow{I} m\} = \mathbb{R}$,
- the CDD is *ordered*: for all m , whenever $n \xrightarrow{I} m$ then $\text{type}(m) \sqsubseteq \text{type}(n)$



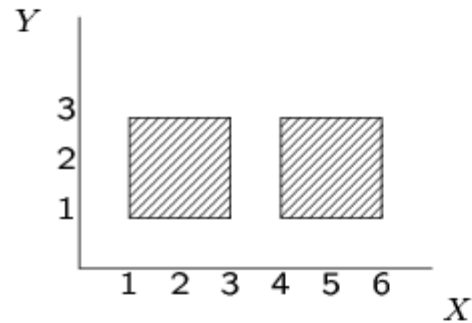
Reduced

Further, the CDD is assumed to be *reduced*, i.e.

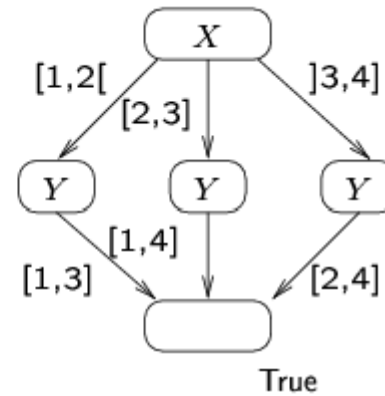
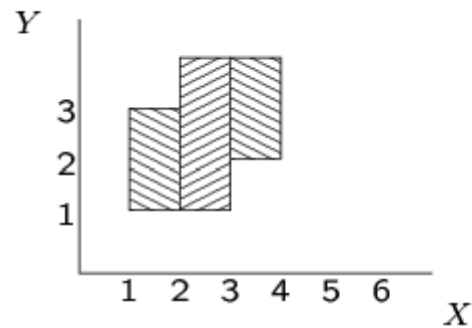
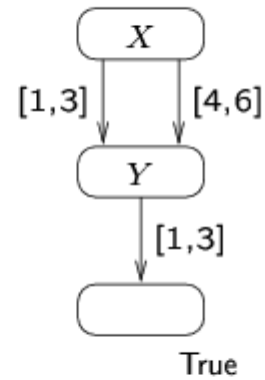
- it has *maximal sharing*: for all $n, m \in V$, whenever $\text{succ}(n) = \text{succ}(m)$ then $n = m$,
- it has *no trivial edges*: whenever $n \xrightarrow{I} m$ then $I \neq \mathbb{R}$,
- all intervals are *maximal*: whenever $n \xrightarrow{I_1} m, n \xrightarrow{I_2} m$ then $I_1 = I_2$ or $I_1 \cup I_2 \notin \mathcal{I}$



Clock Difference Diagrams



(a)



...



Makenode

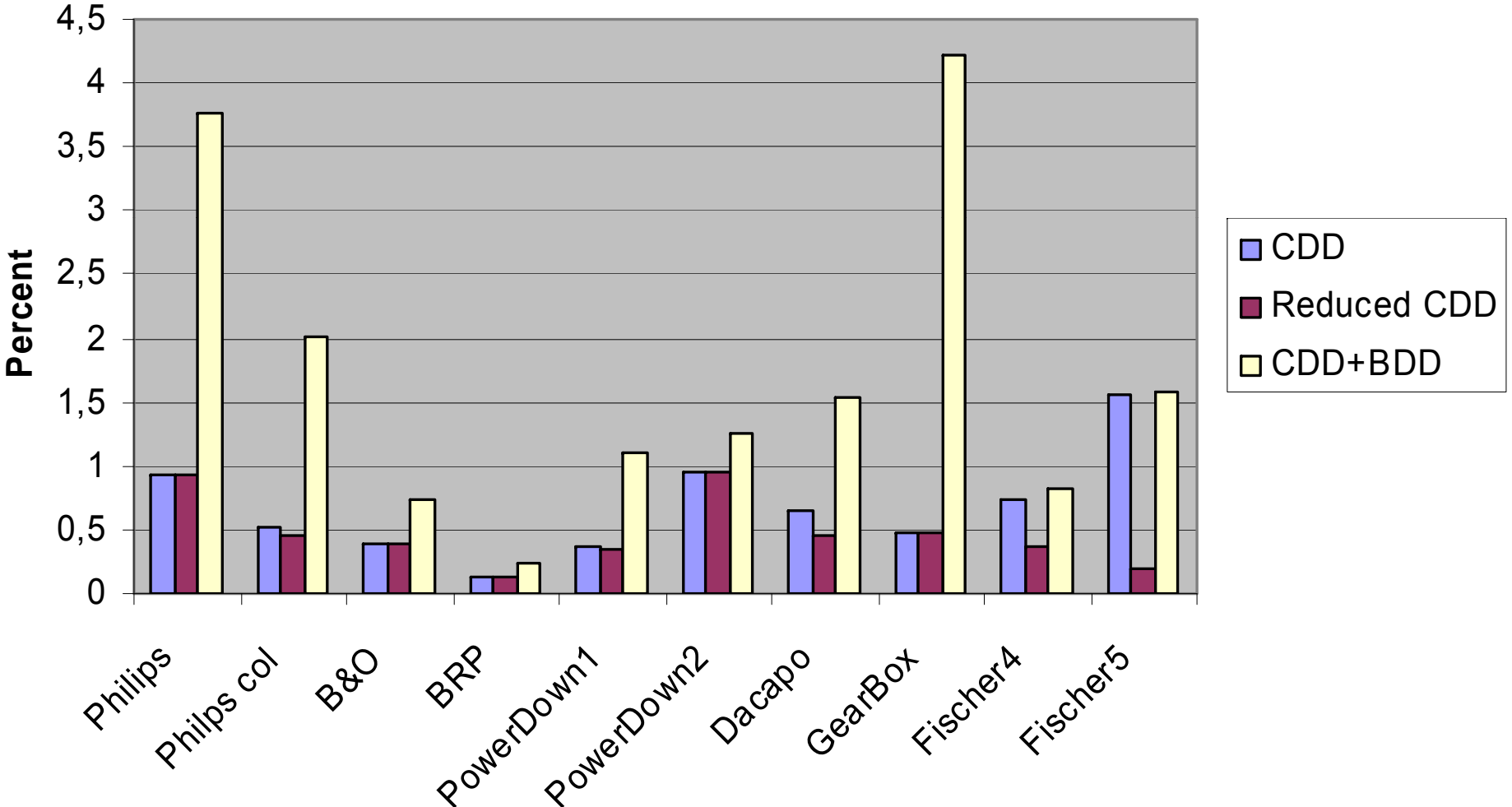
Let t be a **type** and $S = \{(I_1, n_1), \dots, (I_k, n_K)\}$ a **successor set**. We want to extend a given CDD $C = (V, \text{type}, \text{succ})$ with a node n with these attributes.

```
MN( $t, S$ ):  
  if ( $\exists n \in V. \text{type}(n) = t \wedge \text{succ}(n) = S$ )  
  then return  $n$   
  else  
     $V := V \cup \{n\}$  //  $n$  fresh  
     $\text{type} := \text{type} \cup \{n \mapsto t\}$   
     $\text{succ} := \text{succ} \cup \{n \mapsto S\}$   
    return  $n$   
  endif
```

MN in “constant” time



SPACE PERFORMANCE



Union

```
union( $n_1, n_2$ )
  if  $n_1 = \text{True}$  or  $n_2 = \text{True}$  then return True
  elseif  $n_1 = \text{False}$  then return  $n_2$ 
  elseif  $n_2 = \text{False}$  then return  $n_1$ 
  else
    if type( $n_1$ ) = type( $n_2$ ) then
      return MN(type( $n_1$ ),  $\{(I_1 \cap I_2,$ 
        union( $n'_1, n'_2$ )) |  $n_1 \xrightarrow{I_1} n'_1, n_2 \xrightarrow{I_2} n'_2, I_1 \cap I_2 \neq \emptyset\}$ )
    elseif type( $n_1$ )  $\sqsubseteq$  type( $n_2$ ) then
      return MN(type( $n_1$ ),  $\{(I_1,$  union( $n'_1, n_2$ )) |  $n_1 \xrightarrow{I_1} n'_1\}$ )
    elseif type( $n_2$ )  $\sqsubseteq$  type( $n_1$ ) then
      return MN(type( $n_2$ ),  $\{(I_2,$  union( $n_1, n'_2$ )) |  $n_2 \xrightarrow{I_2} n'_2\}$ )
    endif
  endif
endif
```

11



Complement

```
complement(n)  
  if n = True return False  
  elseif n = False return True  
  elseif return MN(type(n), {(I, complement(m)) | n  $\xrightarrow{I}$  m})  
  endif
```



Zones, CDD, Subset

Single zones may be represented as single path CDD's in the obvious manner.

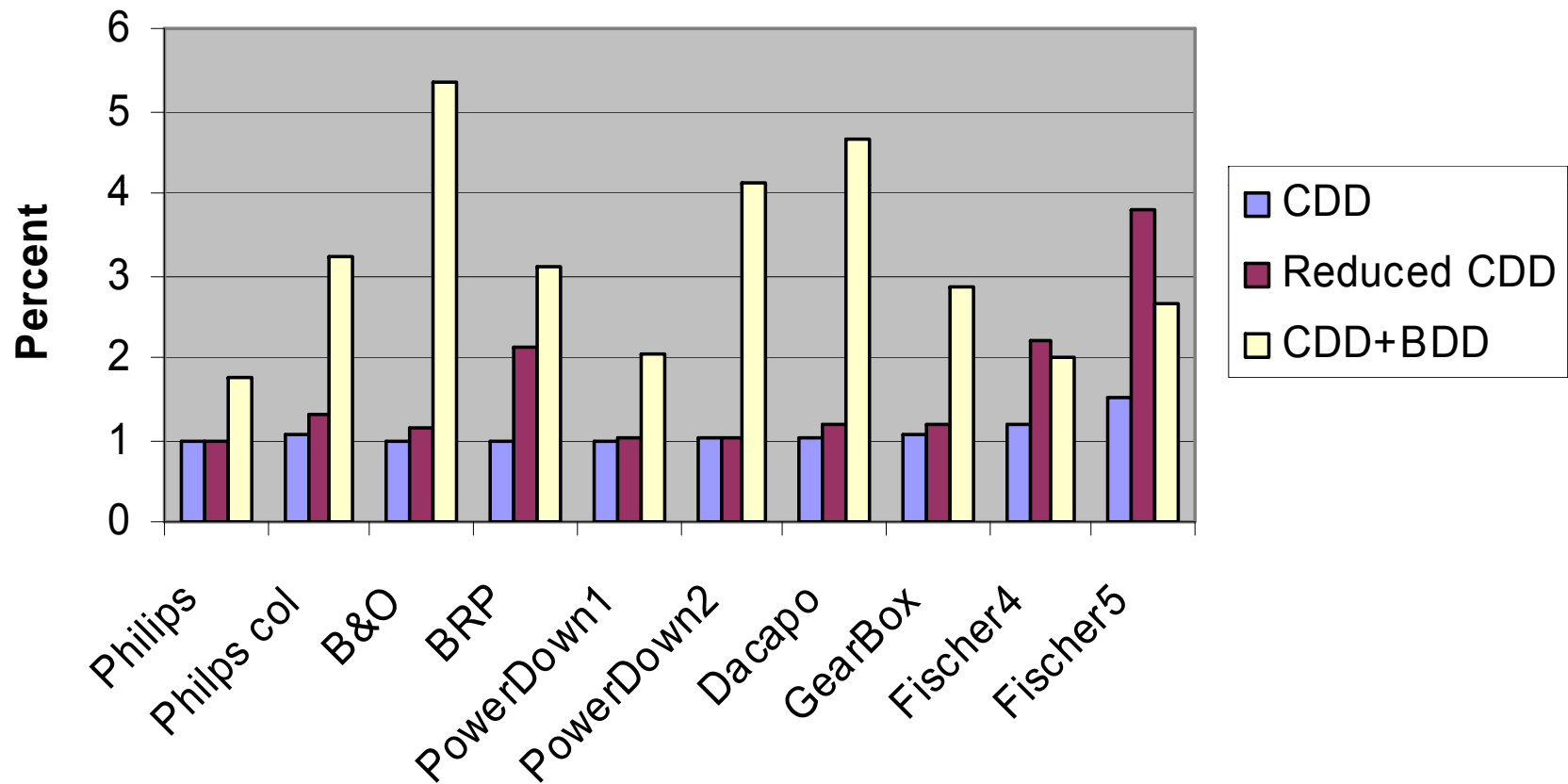
It may be advantageous to represent zones using a *minimal* set of constraints (see [RTSS97]).

```
subset( $D, n$ ) //  $D$  const. sys.,  $n$  CDD-node  
if  $D = \text{false}$  or  $n = \text{True}$  then return true  
elseif  $n = \text{False}$  then return false  
else return  $\bigwedge_{n \rightarrow m} \text{subset}(D \wedge I_n), m$   
endif
```

where I_n is the constraint $X_i - X_j \in I$ if $\text{type}(n) = (i, j)$.



TIME PERFORMANCE



Related & Recent Work

- DDD: Andersen et al.
- NDD: Asarin, Bozga, Kerbrat, Maler, Pnueli, Rasse.
- IDD: Strehl, Thiele.

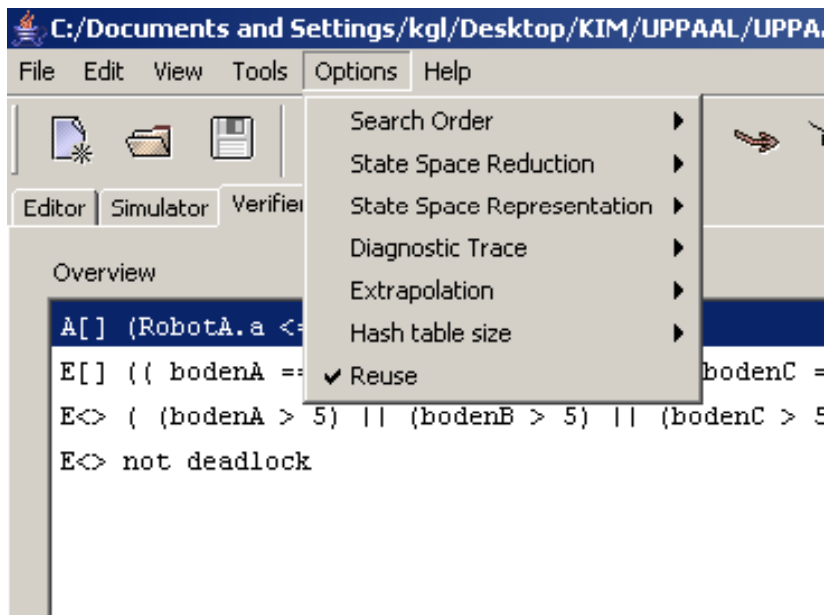
- Recent work on fully symbolic engine for TA:
 - Georges Morbe, Florian Pigorsch and Christoph Scholl:
Fully Symbolic Model Checking for Timed Automata.
CAV 2011.



Verification Options



Verification Options



Search Order

- Depth First
- Breadth First

State Space Reduction

- None
- Conservative
- Aggressive

State Space Representation

- DBM
- Compact Form
- Under Approximation
- Over Approximation

Diagnostic Trace

- Some
- Shortest
- Fastest

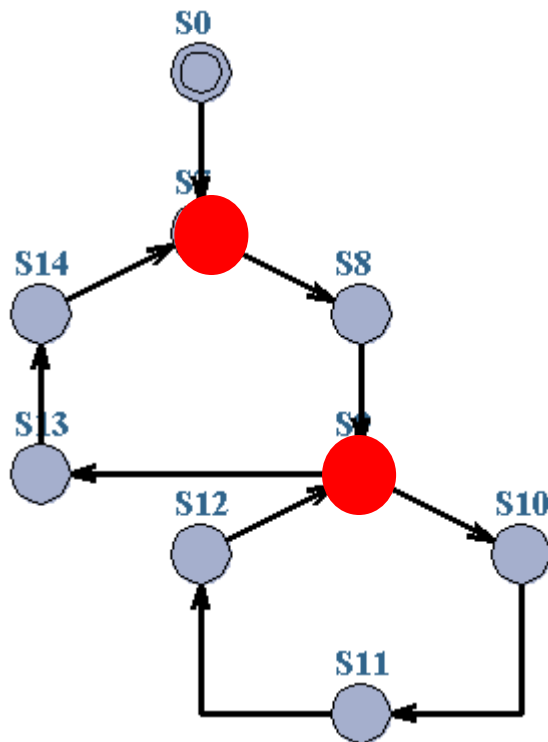
Extrapolation

Hash Table size

- Reuse



State Space Reduction



Cycles:

Only symbolic states involving loop-entry points need to be saved on **Passed** list

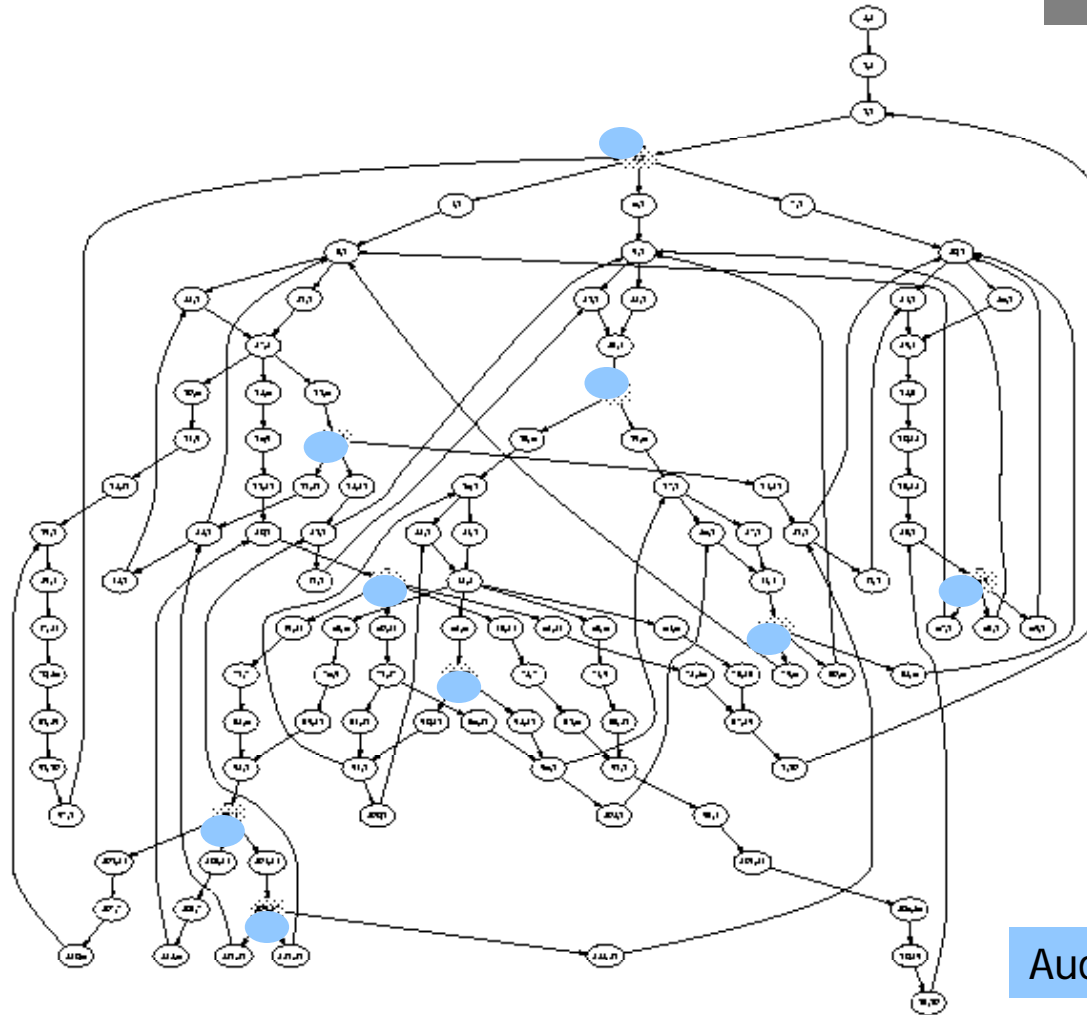


To Store or Not To Store

Behrmann, Larsen,
Pelanek 2003

117 states_{total}
→
81 states_{entrypoint}
→
9 states

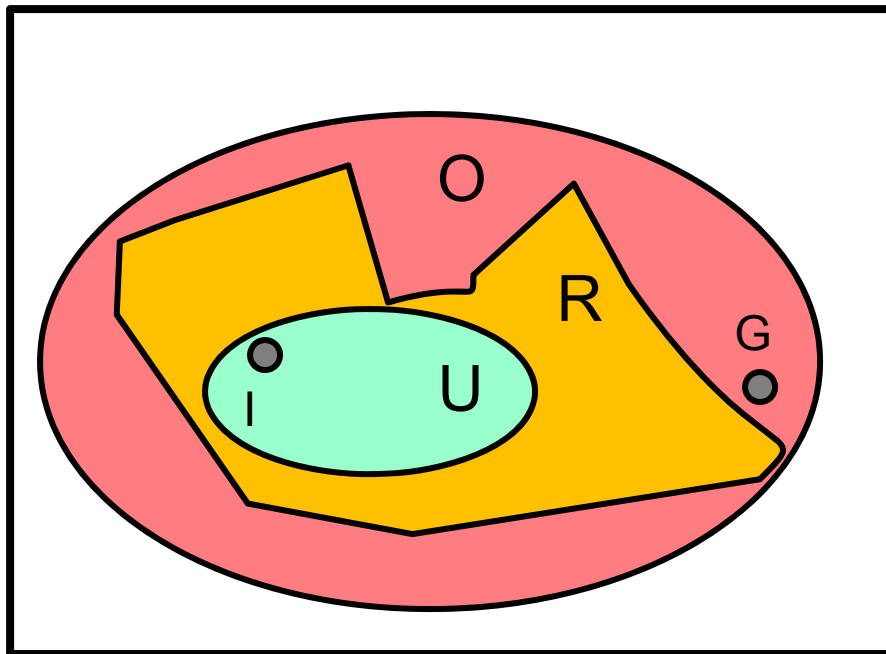
Time OH
less than 10%



Audio Protocol



Over/Under Approximation



Declared State Space

Question:

$G \in R ?$

How to use:

$G \in O ?$

$G \in U ?$

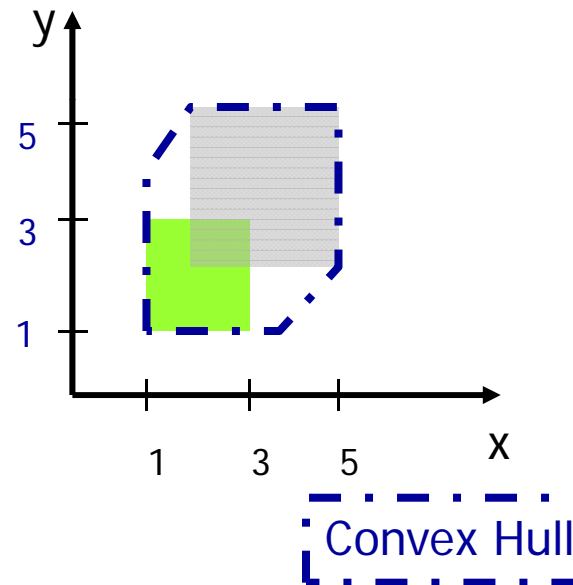
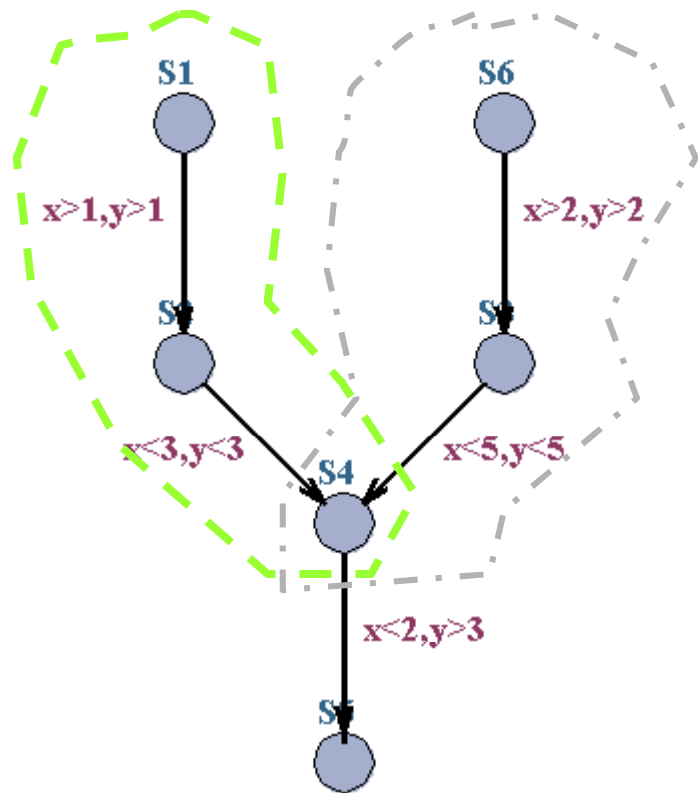
$G \in U \Rightarrow G \in R$

$\neg(G \in O) \Rightarrow \neg(G \in R)$



Over-approximation

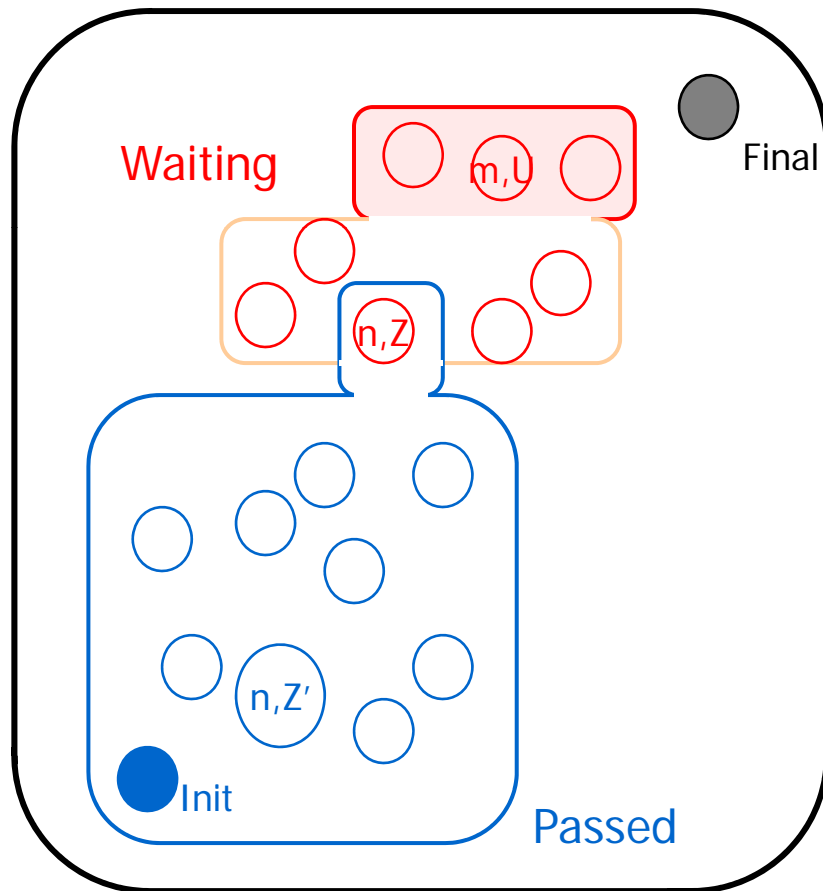
Convex Hull



TACAS04: An **EXACT** method performing as well as Convex Hull has been developed based on abstractions taking max constants into account distinguishing between clocks, locations and \leq & \geq

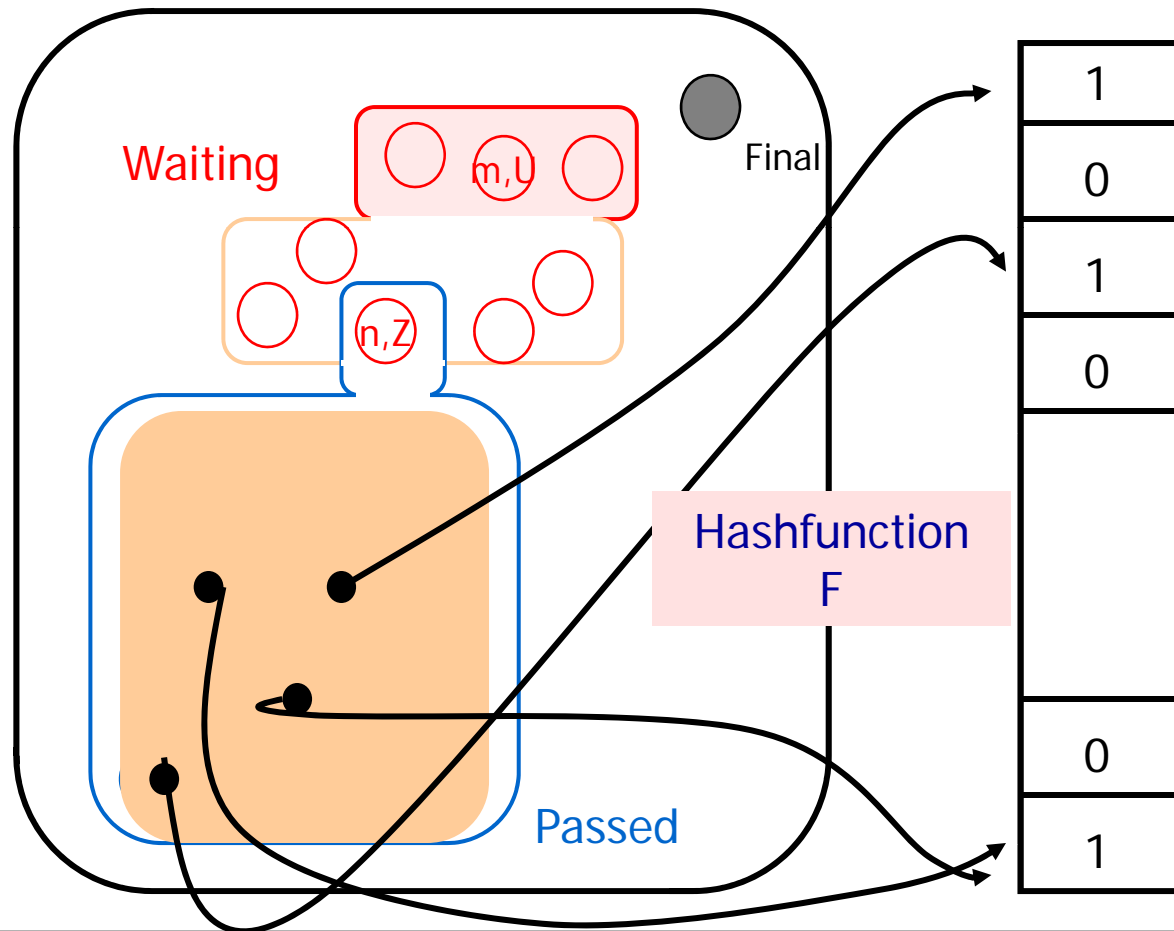
Under-approximation

Bitstate Hashing



Under-approximation

Bitstate Hashing

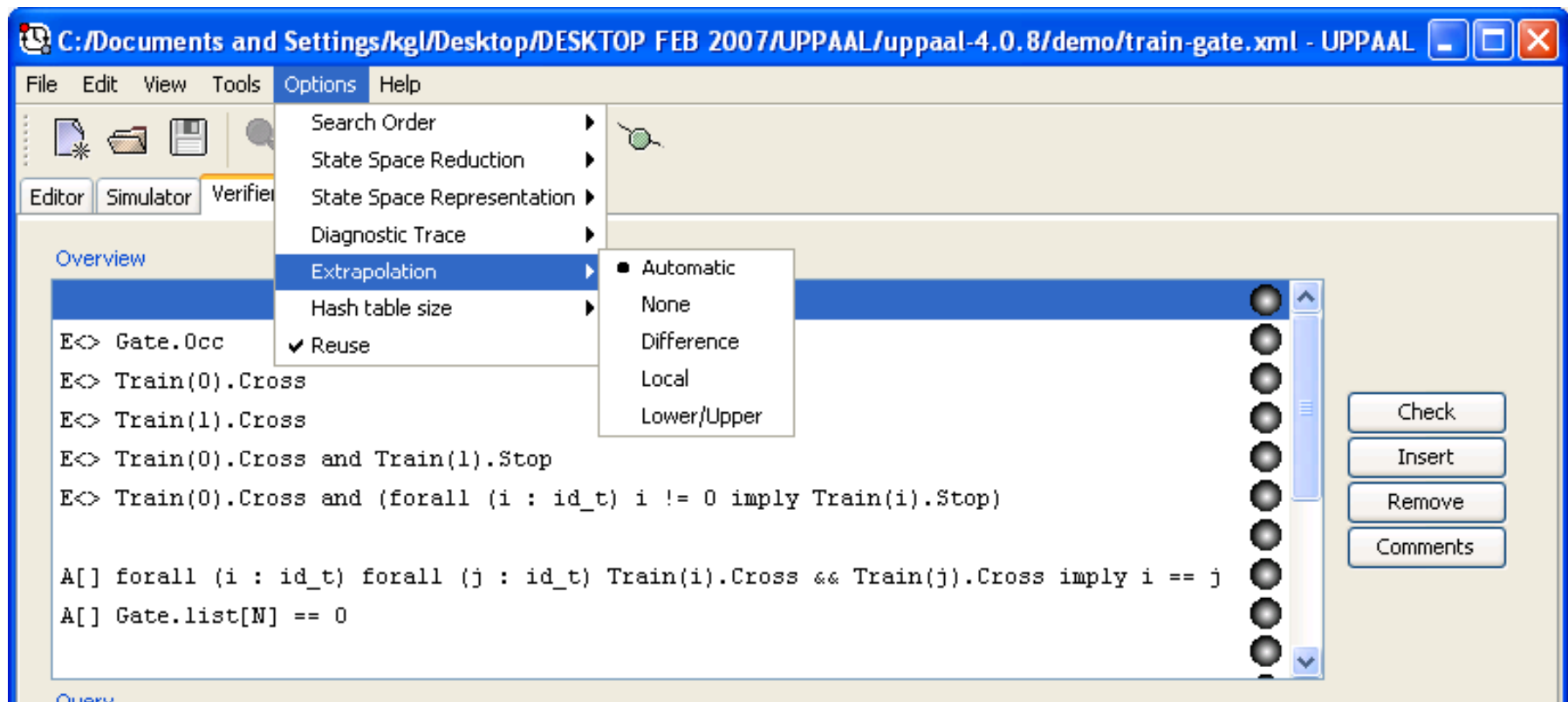


Passed=
Bitarray

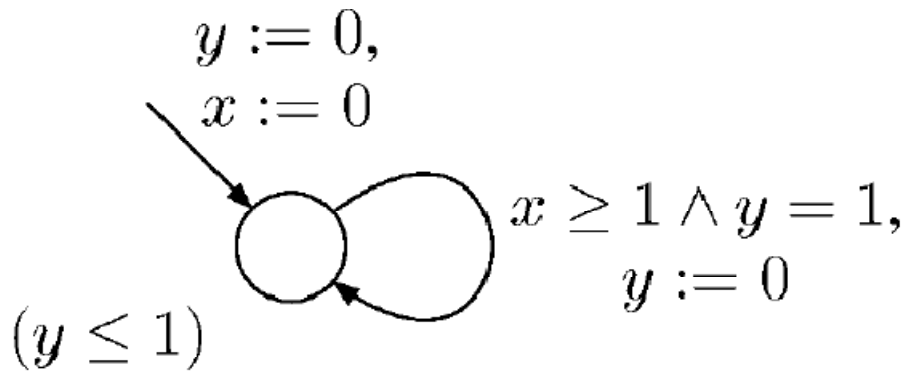
UPPAAL
4 - 512 Mbits



Extrapolation

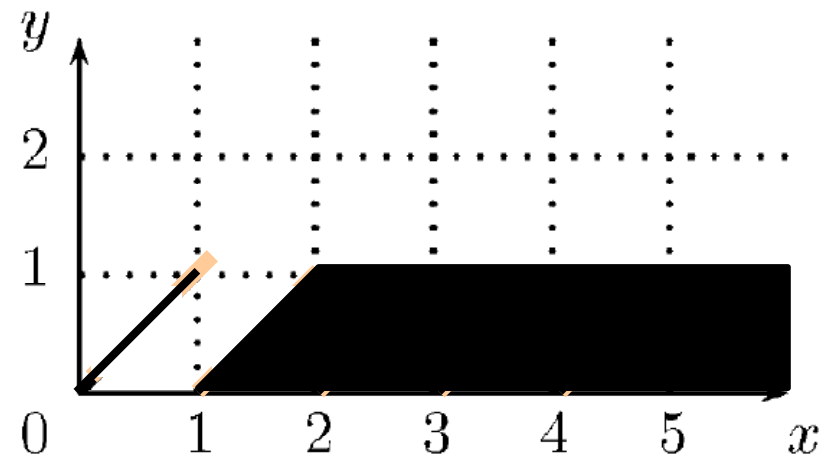


Forward Symbolic Exploration



TERMINATION
not
garanteed

Need for
Finite
Abstractions



Abstractions

$a : \mathcal{P}(R_{\geq 0}^X) \hookrightarrow \mathcal{P}(R_{\geq 0}^X)$ such that $W \subseteq a(W)$

$$\frac{(\ell, W) \Rightarrow (\ell', W')}{(\ell, W) \Rightarrow_a (\ell', a(W'))} \quad \text{if } W = a(W)$$

We want \Rightarrow_a to be:

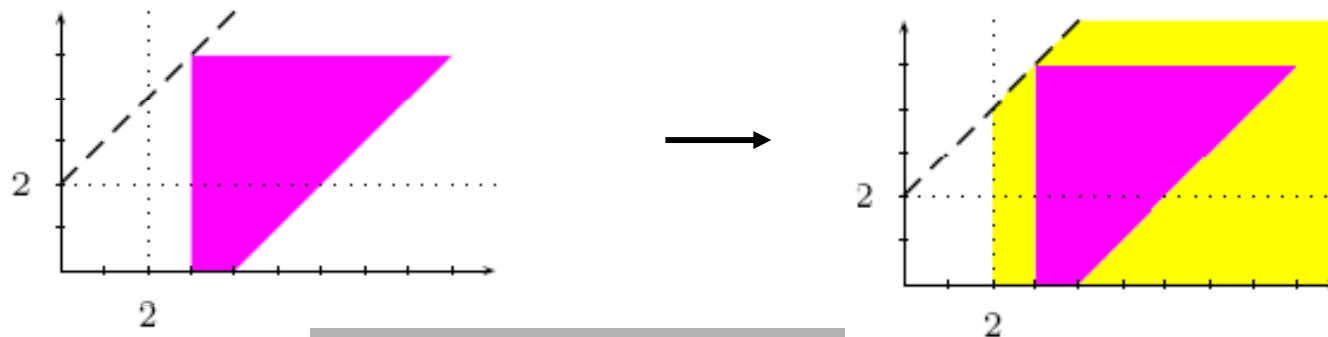
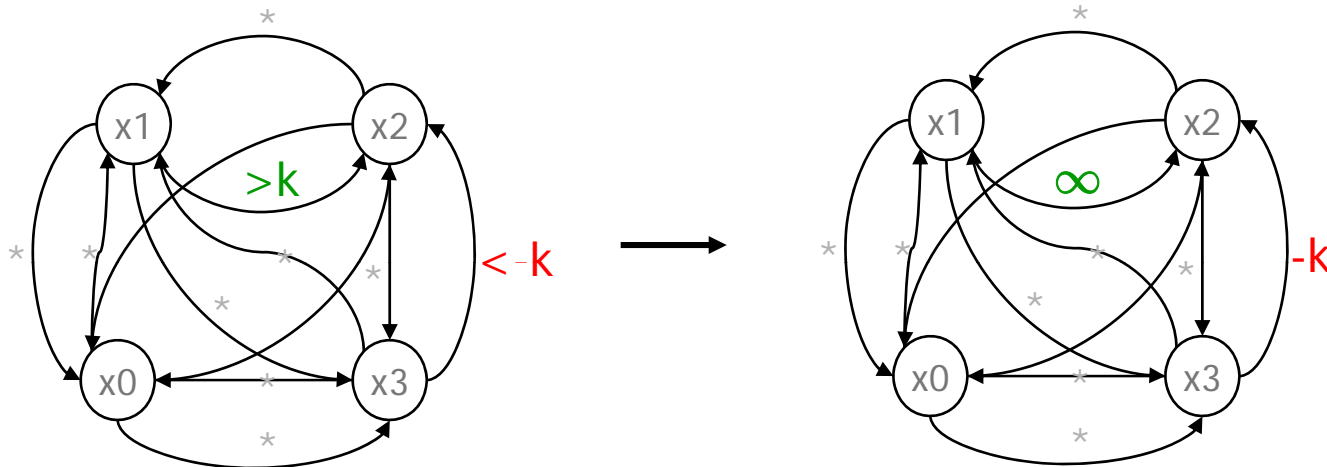
- **sound** & complete wrt reachability
- **finite**
- **easy** to compute
- as **coarse** as possible



Abstraction by Extrapolation

[Daws, Tripakis 98]

Let k be the largest constant appearing in the TA

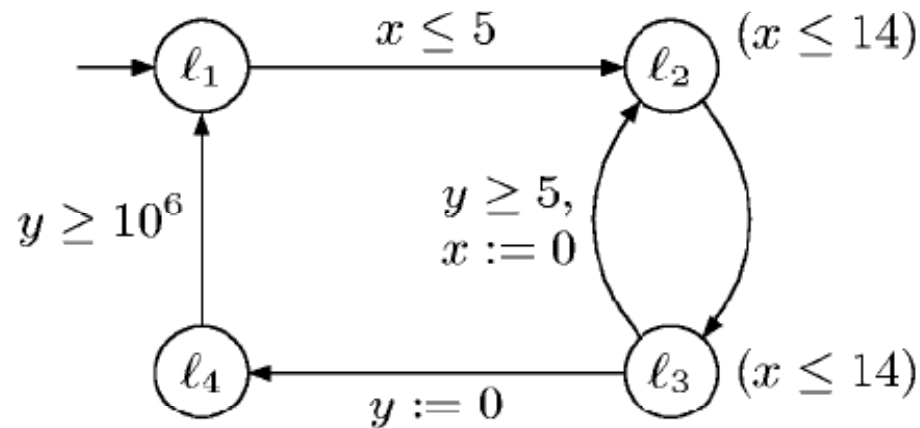


Sound & Complete
Ensures Termination



Location Dependency

[Behrmann, Bouyer, Fleury, Larsen 03]



$$k_x = 5 \quad k_y = 10^6$$

Will generate all symbolic states of the form

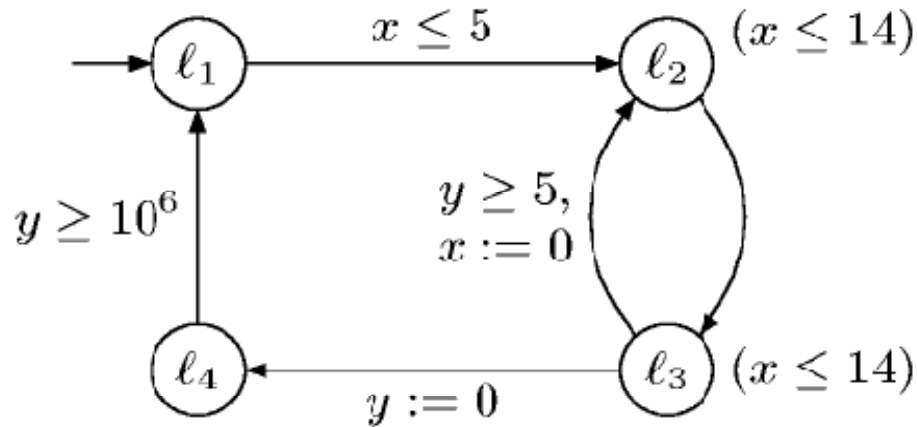
$$(l_2, x \in [0, 14], y \in [5, 14n], y-x \in [5, 14n-14])$$

for $n \leq 10^6/14$!!

But $y \geq 10^6$ is not RELEVANT in l_2



Location Dependent Constants



$$k_x = 5 \quad k_y = 10^6$$

$$\begin{array}{ll}
 k_x^i & = 14 \quad \text{for } i \in \{1, 2, 3, 4\} \\
 k_y^i & = 5 \quad \text{for } i \in \{1, 2, 3\} \\
 & k_y^4 = 10^6
 \end{array}$$

k_j^i may be found as solution to simple linear constraints!

Active Clock Reduction:

$$k_j^i = -\infty$$



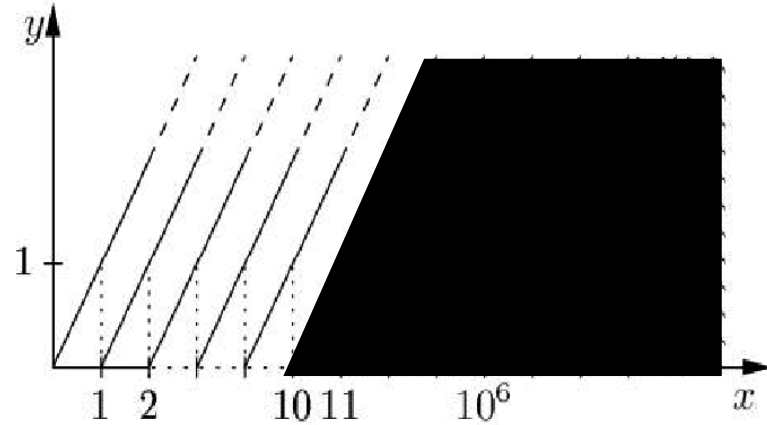
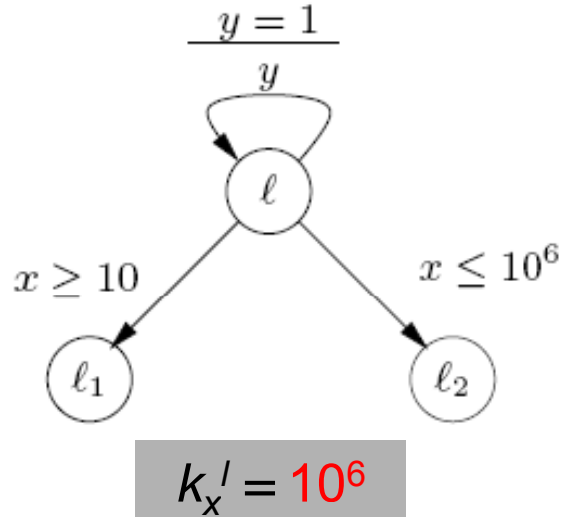
Experiments

	<i>Constant BIG</i>	<i>Global Method</i>	<i>Active-clock Reduction</i>	<i>Local Constants</i>
<i>Naive Example</i>	10^3	0.05s/1MB	0.05s/1MB	0.00s/1MB
	10^4	4.78s/3MB	4.83s/3MB	0.00s/1MB
	10^5	484s/13MB	480s/13MB	0.00s/1MB
	10^6	stopped	stopped	0.00s/1MB
<i>Two Processes</i>	10^3	3.24s/3MB	3.26s/3MB	0.01s/1MB
	10^4	5981s/9MB	5978s/9MB	0.37s/2MB
	10^5	stopped	stopped	72s/5MB
<i>Asymmetric Fischer</i>	10^3	0.01s/1MB	0.01s/1MB	0.01s/1MB
	10^4	2.20s/3MB	2.20s/3MB	0.85s/2MB
	10^5	333s/19MB	333s/19MB	160s/13MB
	10^6	33307s/122MB	33238s/122MB	16330s/65MB
<i>Bang & Olufsen</i>	25000	stopped	159s/243MB	123s/204MB



Lower and Upper Bounds

[Behrmann, Bouyer,
Larsen, Pelanek 04]



Given that $x \leq 10^6$ is an *upper* bound implies that

(l, v_x, v_y) *simulates* (l, v'_x, v_y)

whenever $v'_x \geq v_x \geq 10$.

For reachability downward
closure wrt **simulation**
suffices!

Simulation

\preceq is the largest relation satisfying

1. if $(\ell_1, \nu_1) \preceq (\ell_2, \nu_2)$ then $\ell_1 = \ell_2$
2. if $(\ell_1, \nu_1) \preceq (\ell_2, \nu_2)$ and $(\ell_1, \nu_1) \longrightarrow (\ell'_1, \nu'_1)$, then there exists (ℓ'_2, ν'_2) such that $(\ell_2, \nu_2) \longrightarrow (\ell'_2, \nu'_2)$ and $(\ell'_1, \nu'_1) \preceq (\ell'_2, \nu'_2)$
3. if $(\ell_1, \nu_1) \preceq (\ell_2, \nu_2)$ and $(\ell_1, \nu_1) \xrightarrow{\epsilon(\delta)} (\ell_1, \nu_1 + \delta)$, then there exists δ' such that $(\ell_2, \nu_2) \xrightarrow{\epsilon(\delta')} (\ell_2, \nu_2 + \delta')$ and $(\ell_1, \nu_1 + \delta) \preceq (\ell_2, \nu_2 + \delta')$

Proposition

If $(\ell, \nu_1) \preceq (\ell, \nu_2)$ and if a discrete state ℓ' is reachable from (ℓ, ν_1) , then it is also reachable from (ℓ, ν_2) .



Maximal Bounds

$M(x)$: the maximum constant k with $x \sim k$,

$L(x)$: the maximum constant k with $x \{ \geq, > \} k$,

$U(x)$: the maximum constant k with $x \{ \leq, < \} k$.

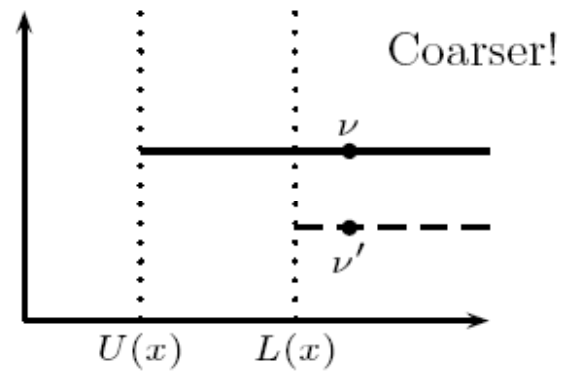
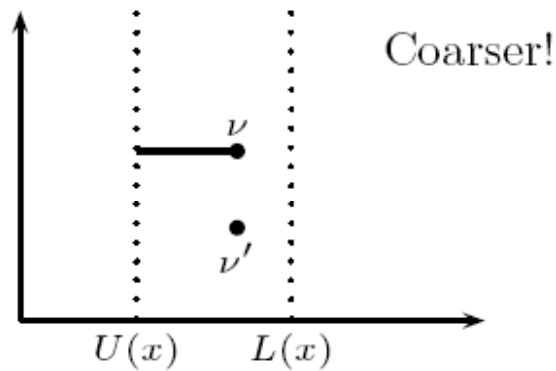
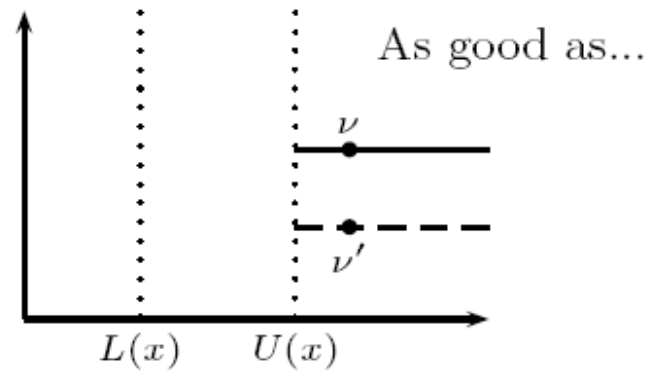
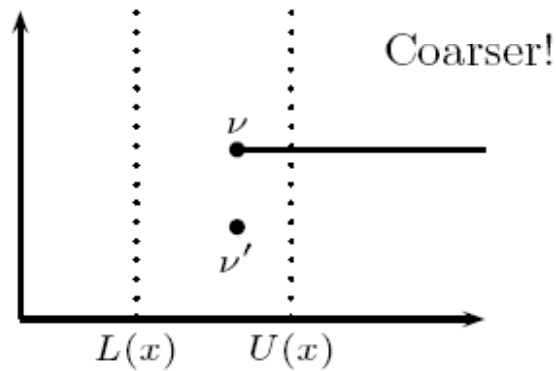
$$\nu \equiv_M \nu' \stackrel{\text{def}}{\iff}$$

$$\forall x \in X : \text{either } \nu(x) = \nu'(x) \text{ or } (\nu(x) > M(x) \text{ and } \nu'(x) > M(x))$$

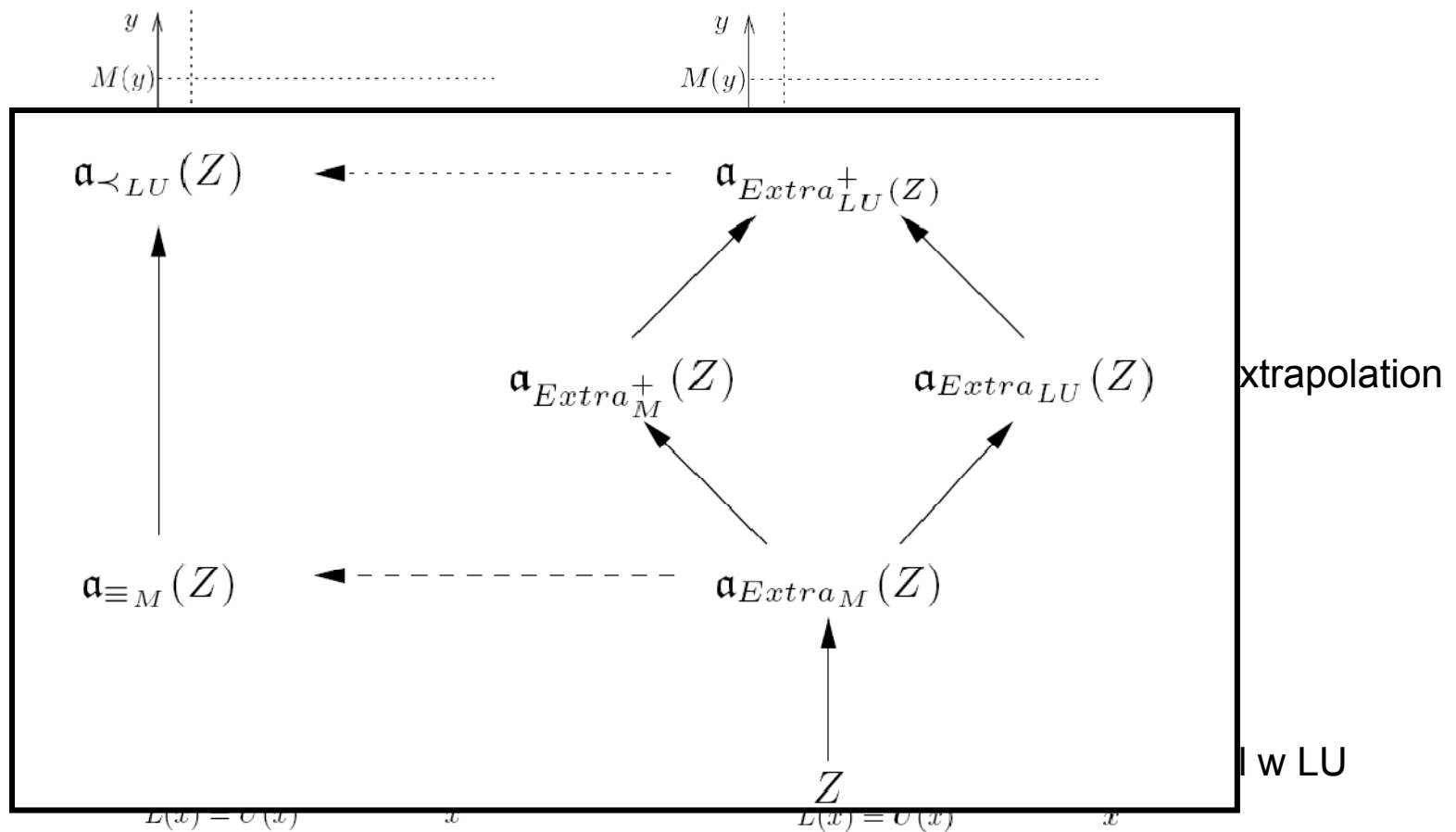
$$\nu' \prec_{LU} \nu \stackrel{\text{def}}{\iff} \text{for each clock } x, \begin{cases} \text{either } \nu'(x) = \nu(x) \\ \text{or } L(x) < \nu'(x) < \nu(x) \\ \text{or } U(x) < \nu(x) < \nu'(x) \end{cases}$$



Maximum Bounds Abstraction



Extrapolation Using Zones



Experiments

		Classical			Loc. dep. Max			Loc. dep. LU			Convex Hull		
		-n1			-n2			-n3			-A		
Model		Time	States	Mem	Time	States	Mem	Time	States	Mem	Time	States	Mem
Fischer	f5	4.02	82,685	5	0.24	16,980	3	0.03	2,870	3	0.03	3,650	3
	f6	597.04	1,489,230	49	6.67	158,220	7	0.11	11,484	3	0.10	14,658	3
	f7				352.67	1,620,542	46	0.47	44,142	3	0.45	56,252	5
	f8							2.11	164,528	6	2.08	208,744	12
	f9							8.76	598,662	19	9.11	754,974	39
	f10							37.26	2,136,980	68	39.13	2,676,150	143
	f11							152.44	7,510,382	268			
CSMA/CD	c5	0.55	27,174	3	0.14	10,569	3	0.02	2,027	3	0.03	1,651	3
	c6	19.39	287,109	11	3.63	87,977	5	0.10	6,296	3	0.06	4,986	3
	c7				195.35	813,924	29	0.28	18,205	3	0.22	14,101	4
	c8							0.98	50,058	5	0.66	38,060	7
	c9							2.90	132,623	12	1.89	99,215	17
	c10							8.42	341,452	29	5.48	251,758	49
	c11							24.13	859,265	76	15.66	625,225	138
	c12							68.20	2,122,286	202	43.10	1,525,536	394
	bus	102.28	6,727,443	303	66.54	4,620,666	254	62.01	4,317,920	246	45.08	3,826,742	324
	philips	0.16	12,823	3	0.09	6,763	3	0.09	6,599	3	0.07	5,992	3
	sched	17.01	929,726	76	15.09	700,917	58	12.85	619,351	52	55.41	3,636,576	427

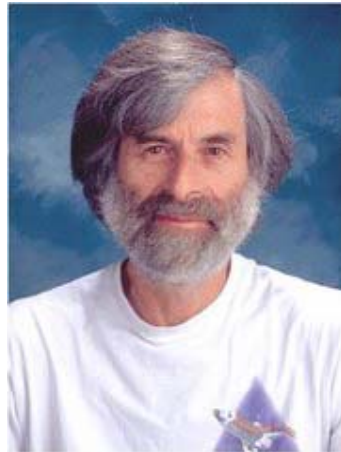


Additional “secrets”

- Sharing among symbolic states
 - location vector / discrete values / zones
- Distributed implementation of UPPAAL
- Symmetry Reduction
- Sweep Line Method
- Guiding wrt Heuristic Value
 - User-supplied / Auto-generated
- Slicing wrt “C” Code



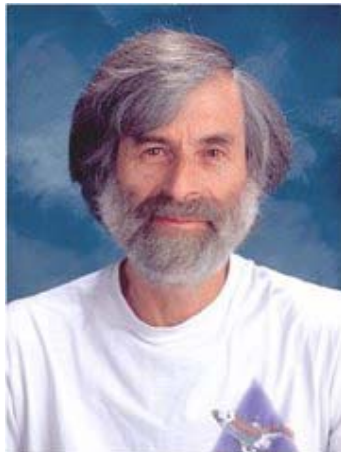
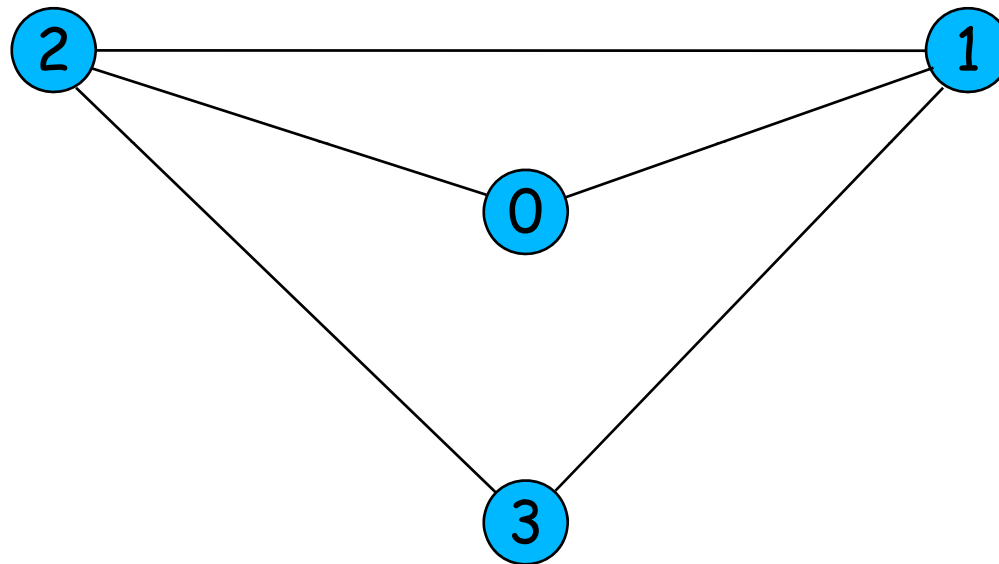
Leader Election Protocol



Protocol analysed in UPPAAL by
Leslie Lamport
CHARME'05



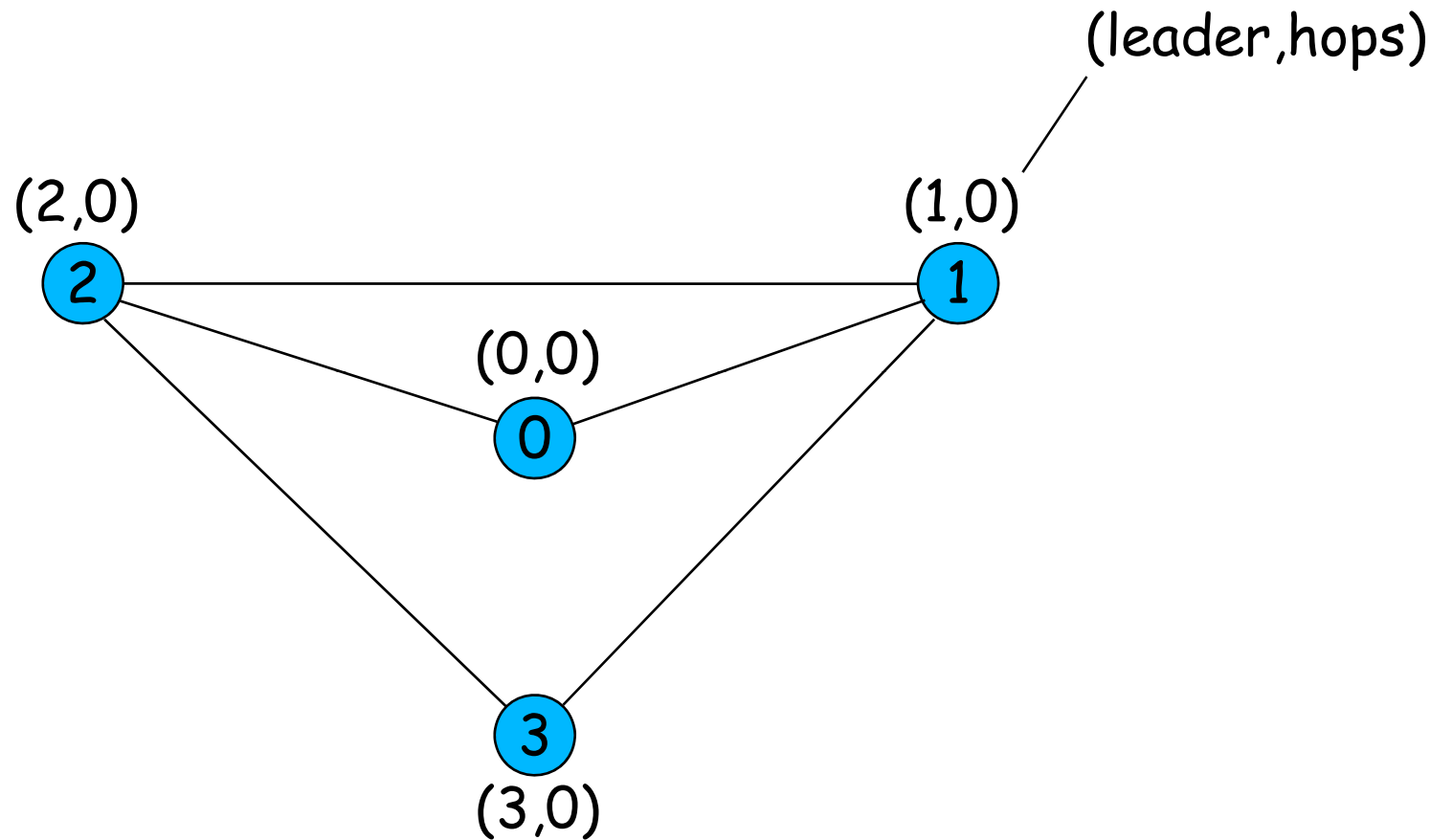
Leader Election



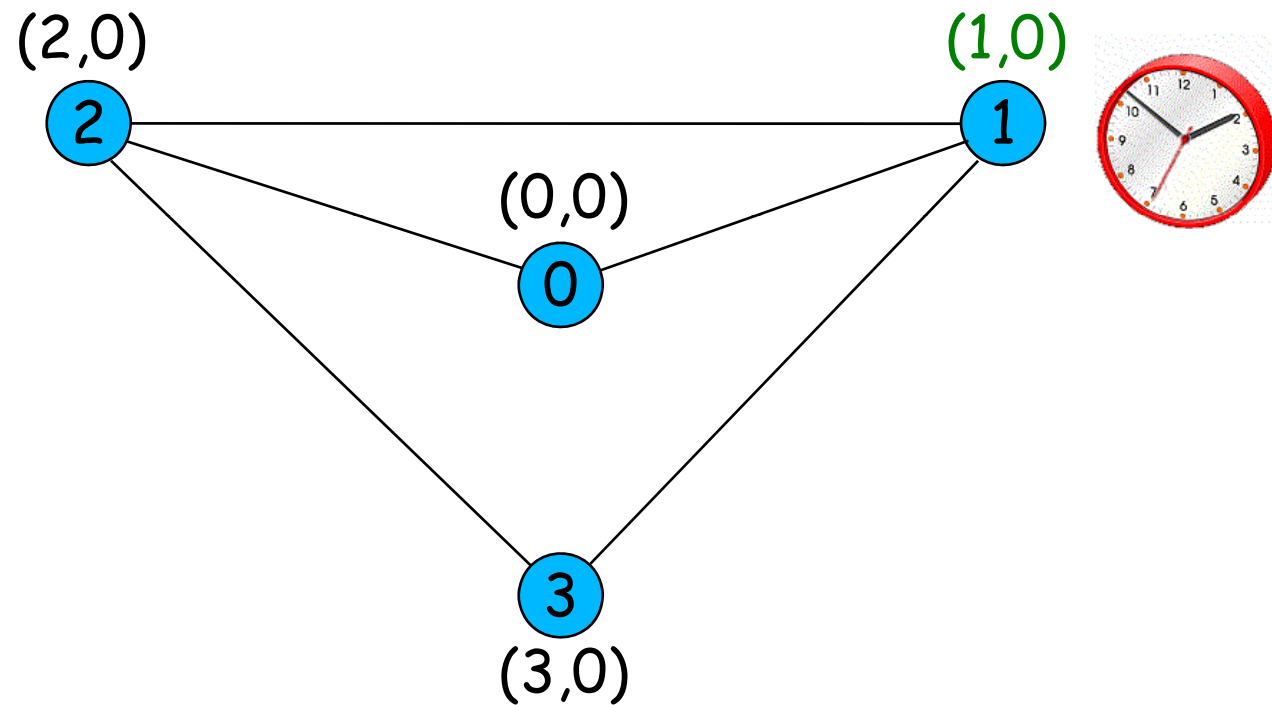
Protocol by
Leslie Lamport



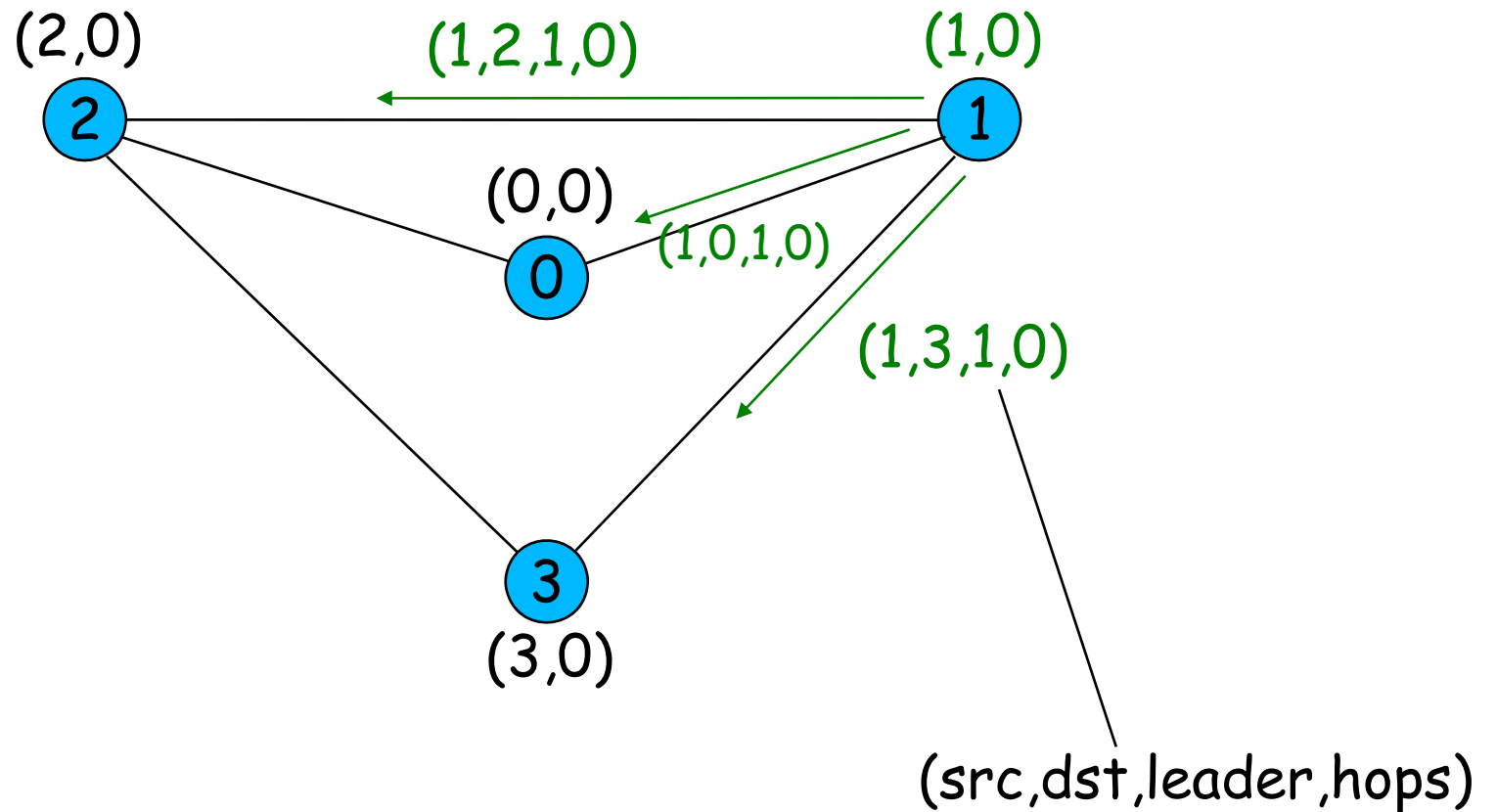
Leader Election



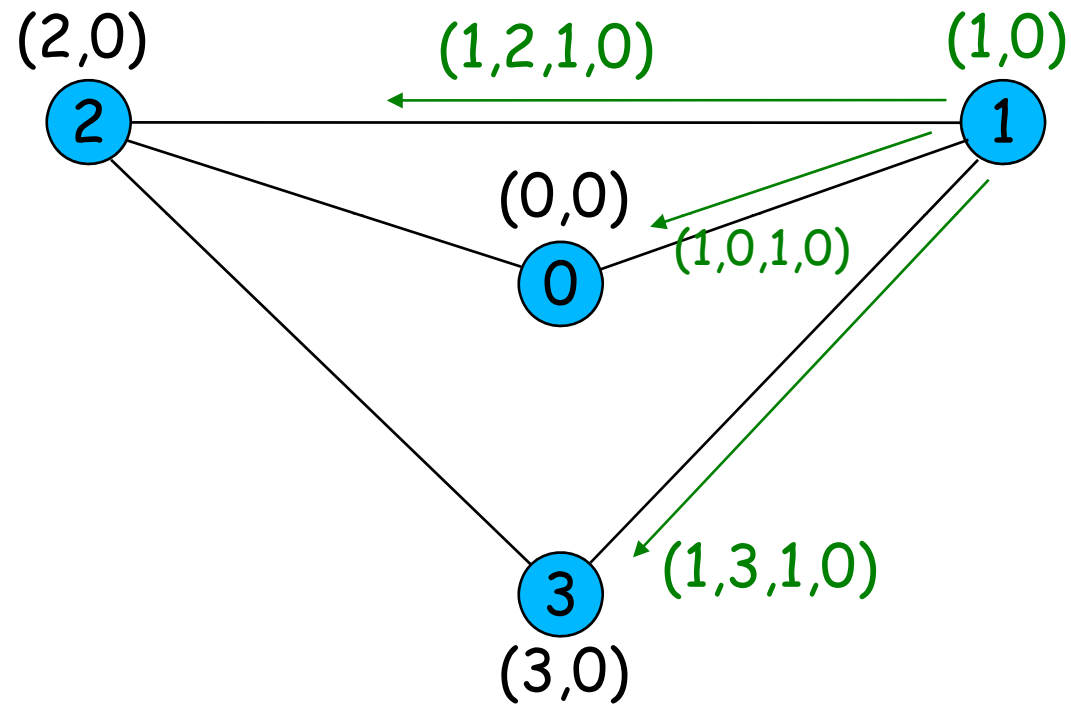
Timeout



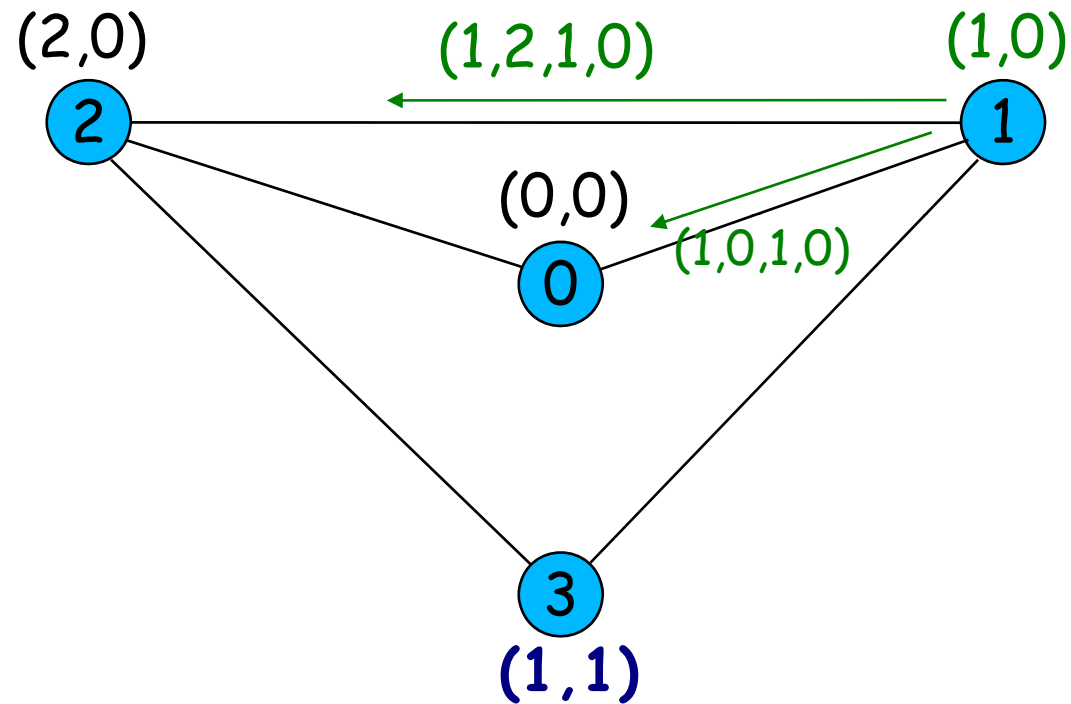
Flooding



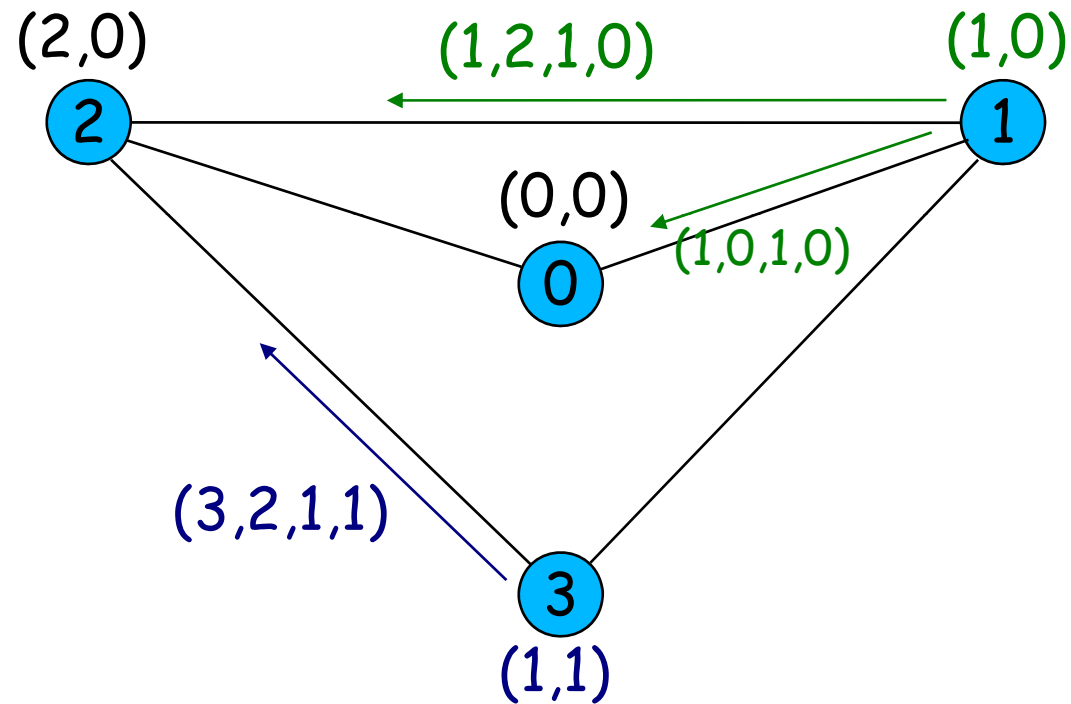
Flooding



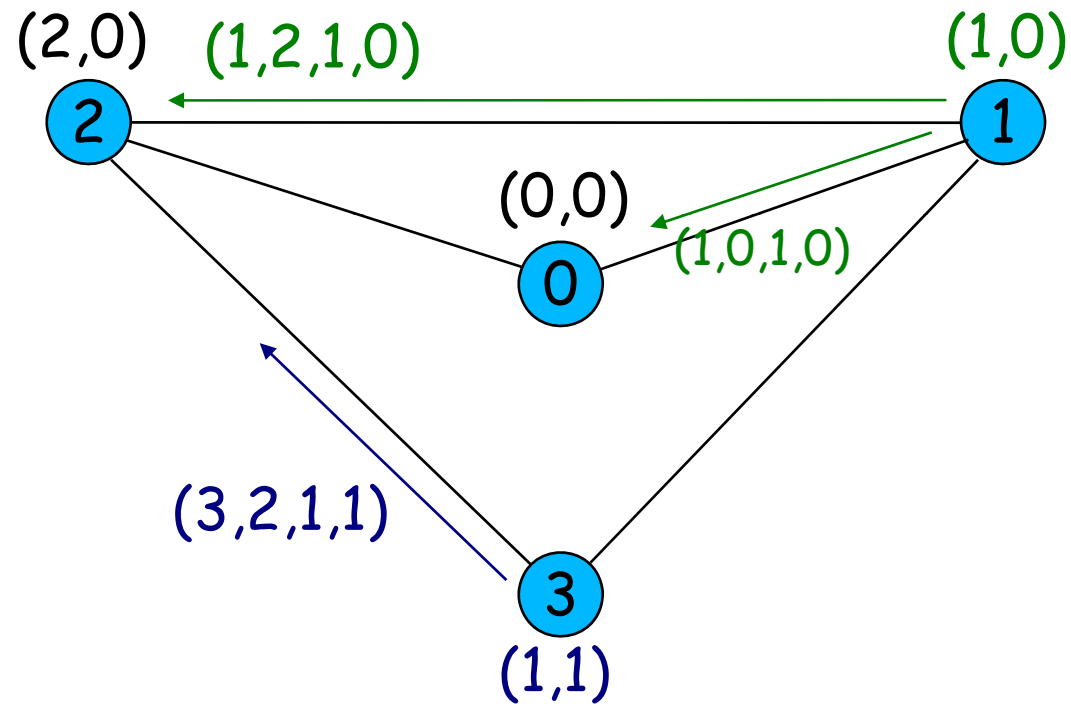
Flooding



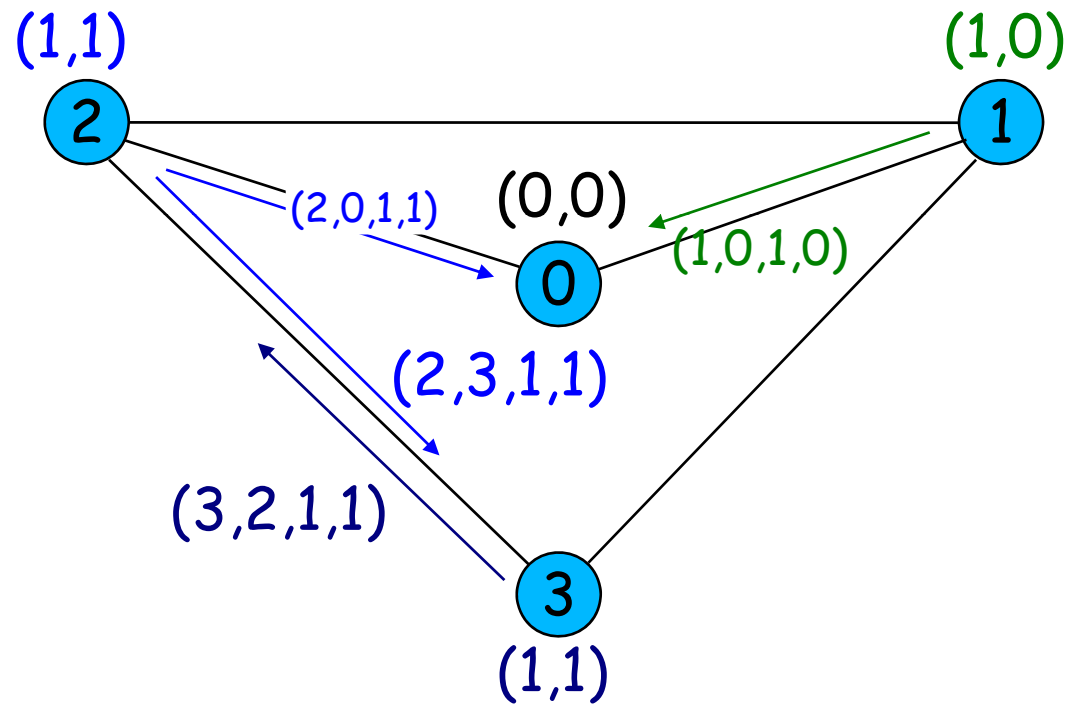
Forwarding



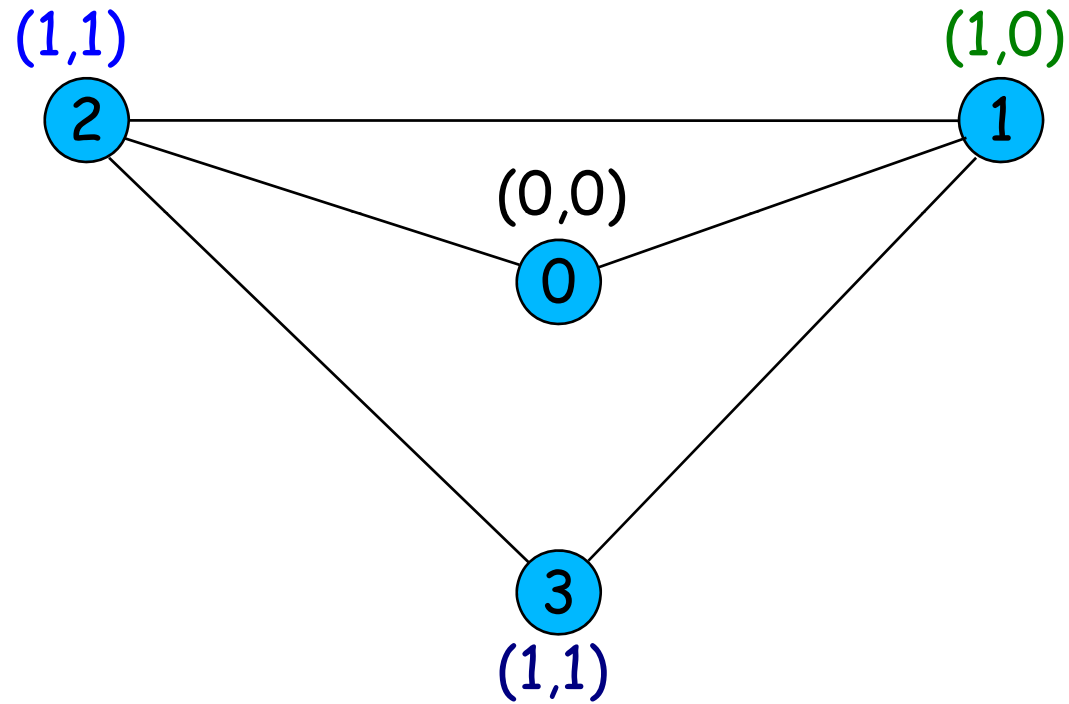
Forwarding



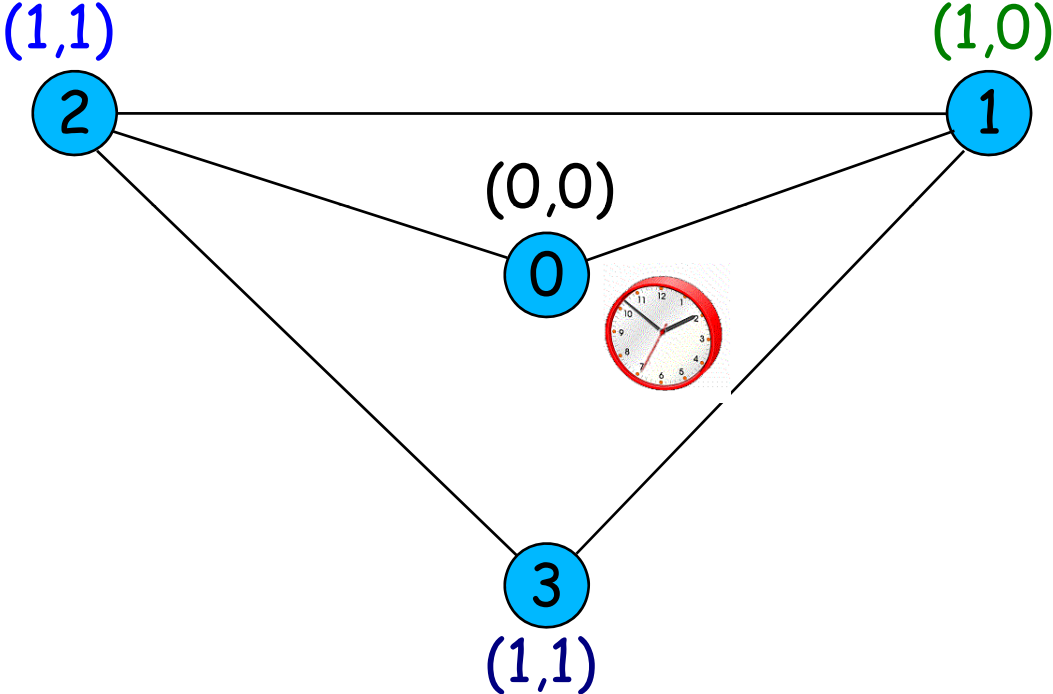
Forwarding



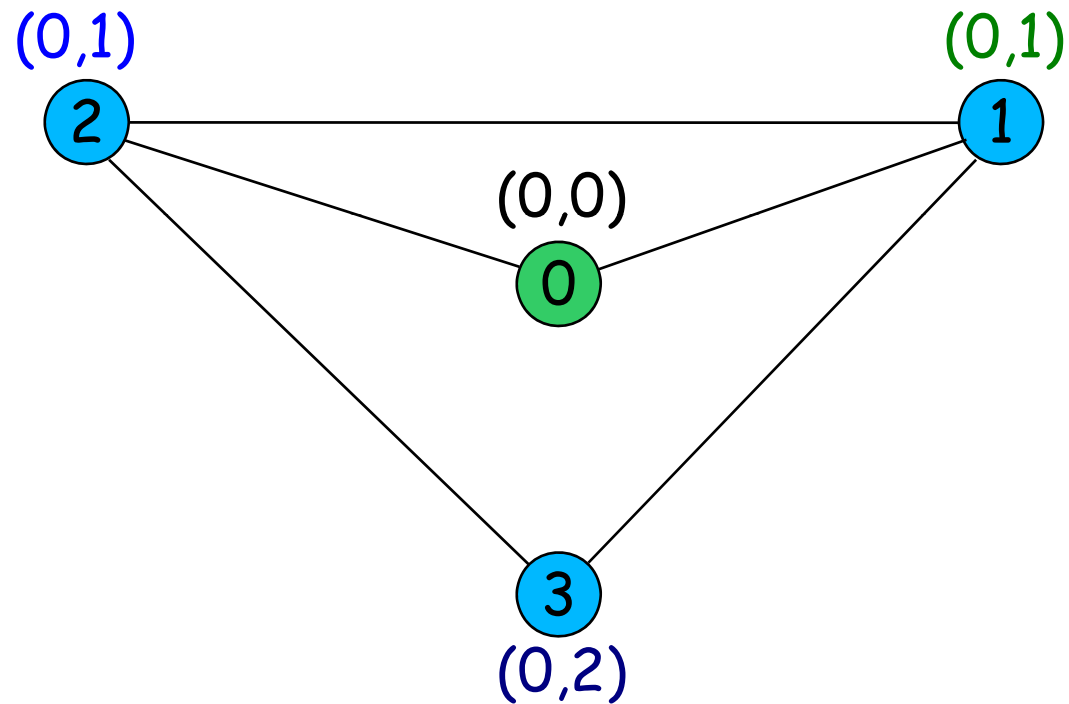
Leader Election



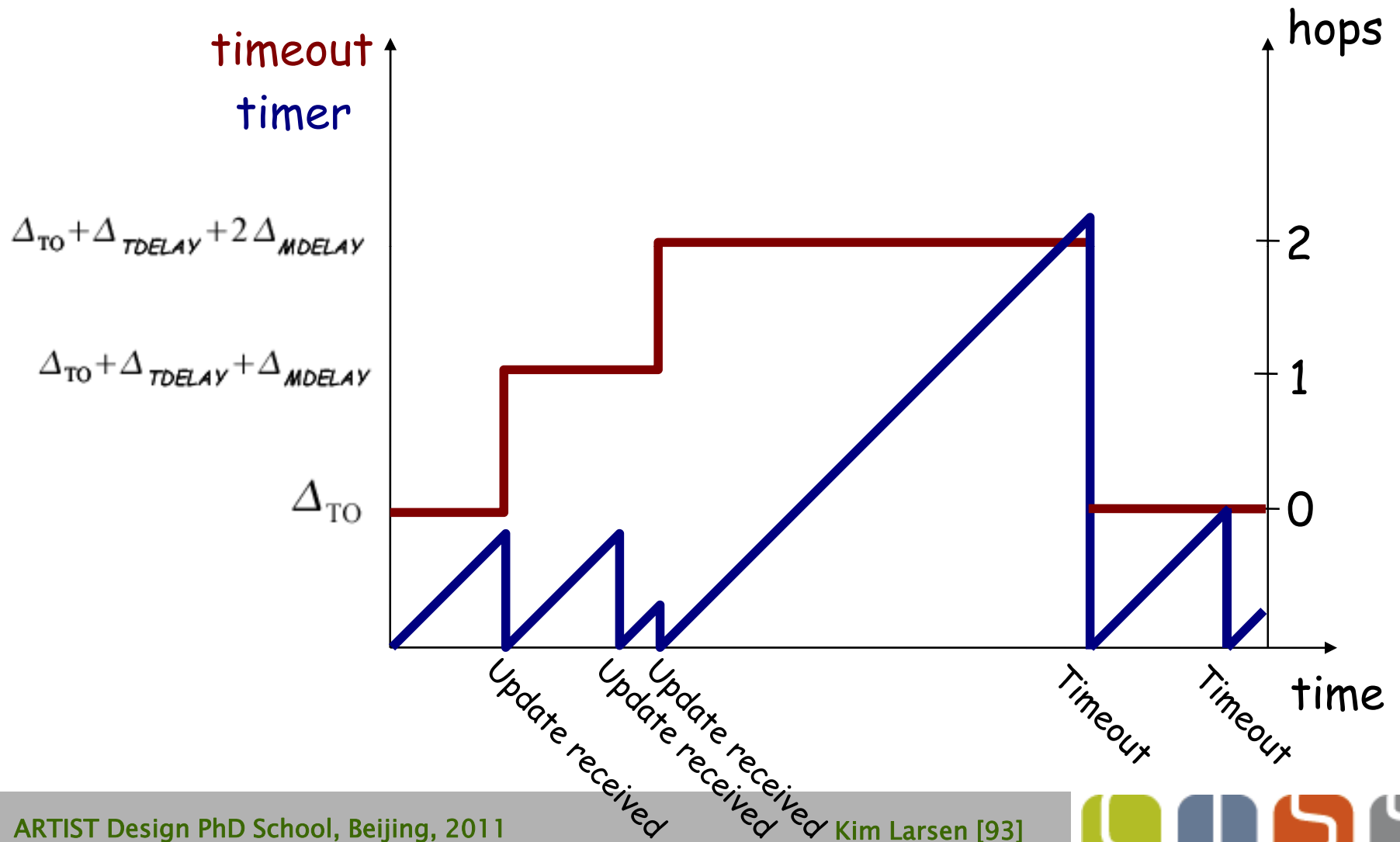
Leader Election



Leader Election



Variable timeout



Leader Election

Claim to be verified

Correct leader is known at a node i after

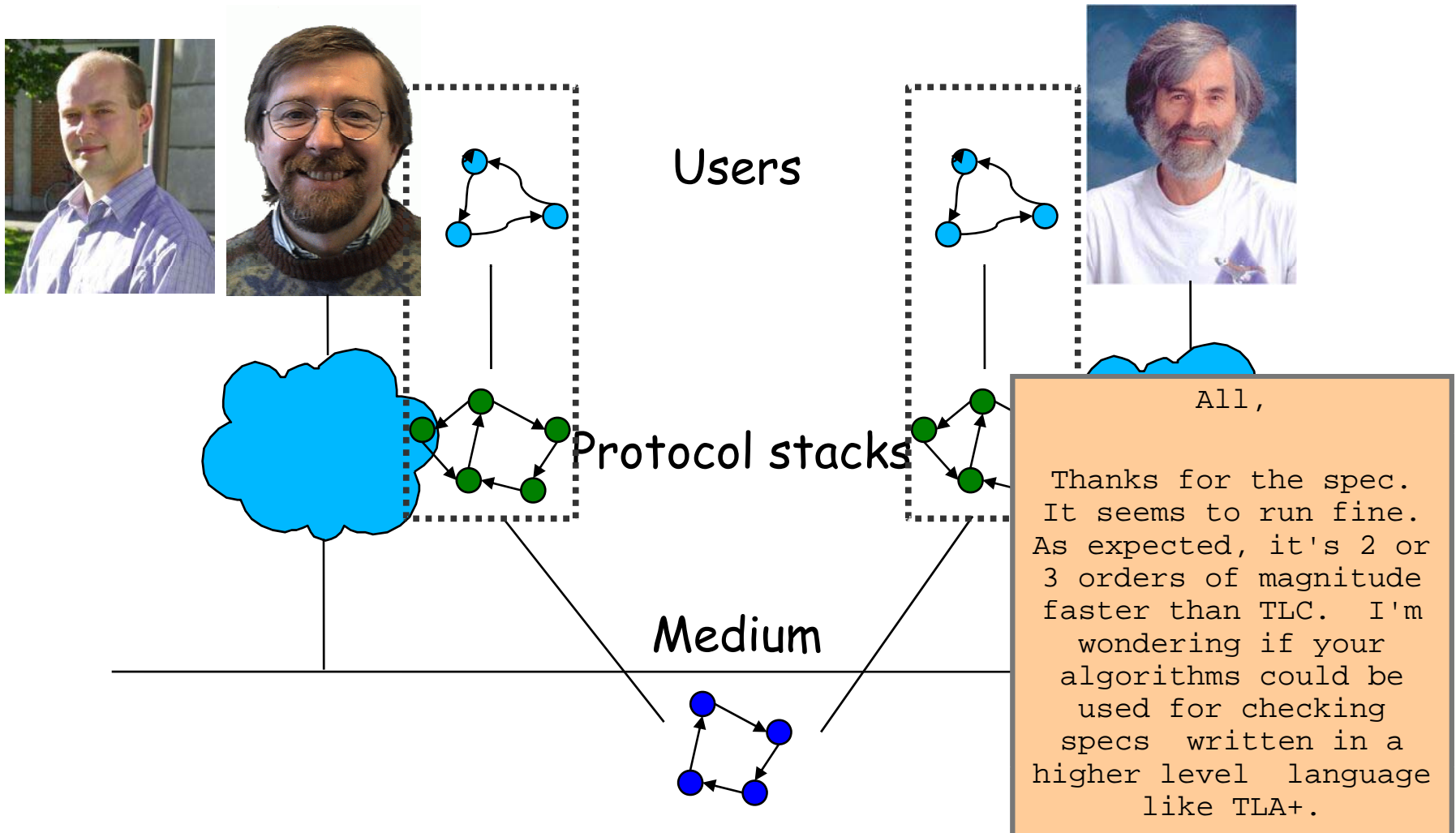
$$t(i) = \Delta_{TO} + \Delta_{TDELAY} + d_i \cdot \Delta_{MDELAY}$$

A model checking problem

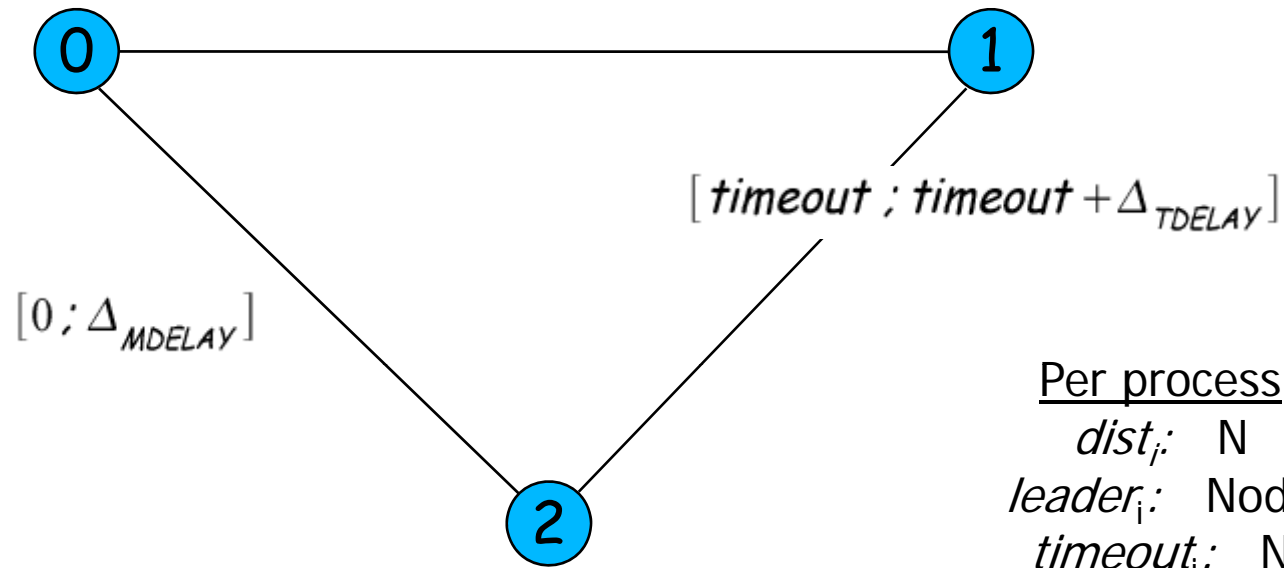
$$\text{IMP} \models \square_{>t(i)} I(i)=L(i)$$

for all i .

Modelling (RT) protocols



Modelling the election protocol



Static

$Topology : Node \times Node \rightarrow B$

Message
src: Node
dst: Node
leader: Node
hopss: N



Global Declaration

```
void setMsg(msg_t &msg, id_t src, id_t dst, id_t leader, int[0,N] hops)
{
    msg.src = src;
    msg.dst = dst;
    msg.leader = leader;
    msg.hops = hops;
}

chan send;
chan receive[N];
msg_t shared;

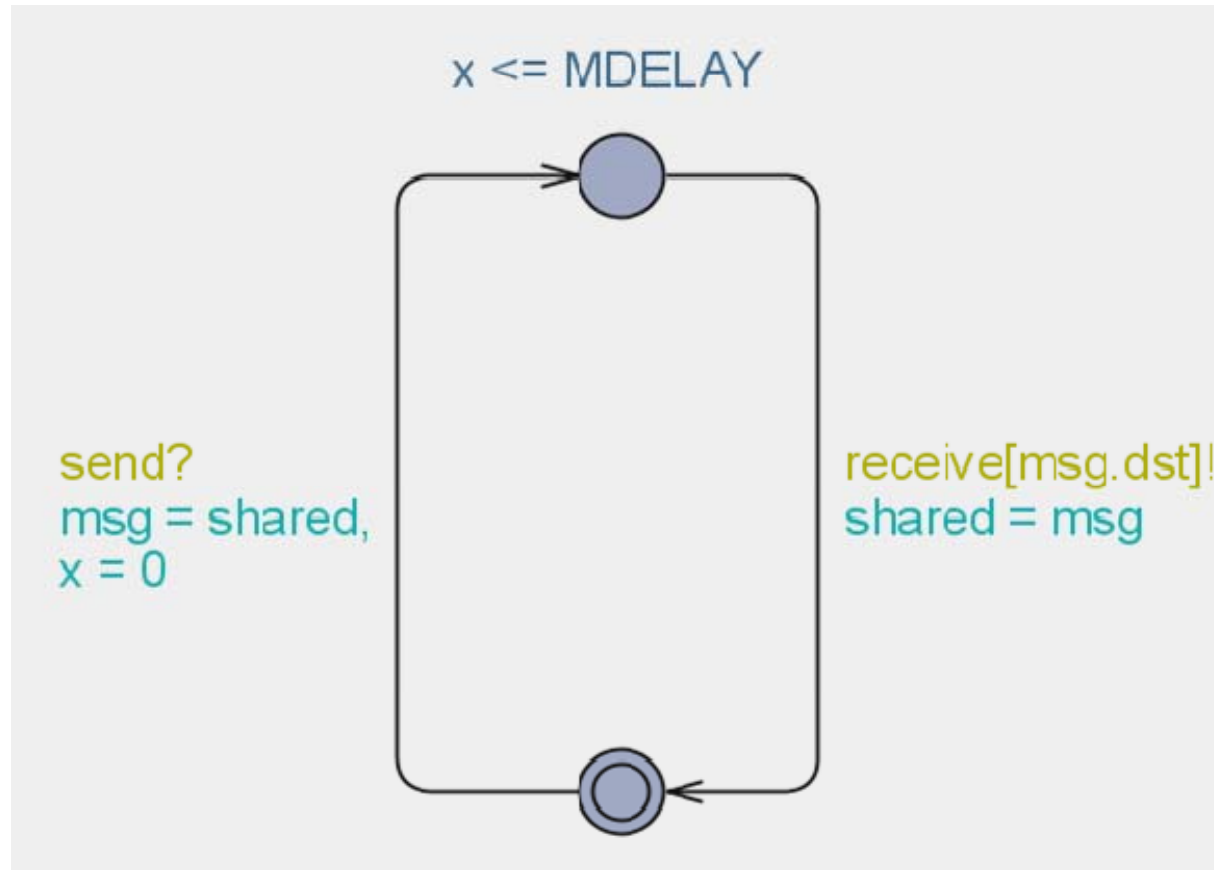
const int link[N][N] = {
    { 0,1,1 },
    { 1,0,1 },
    { 1,1,0 }
};

const int N = 3;
const int MDELAY = 3;
const int TDELAY = 5;
const int TO = 10;

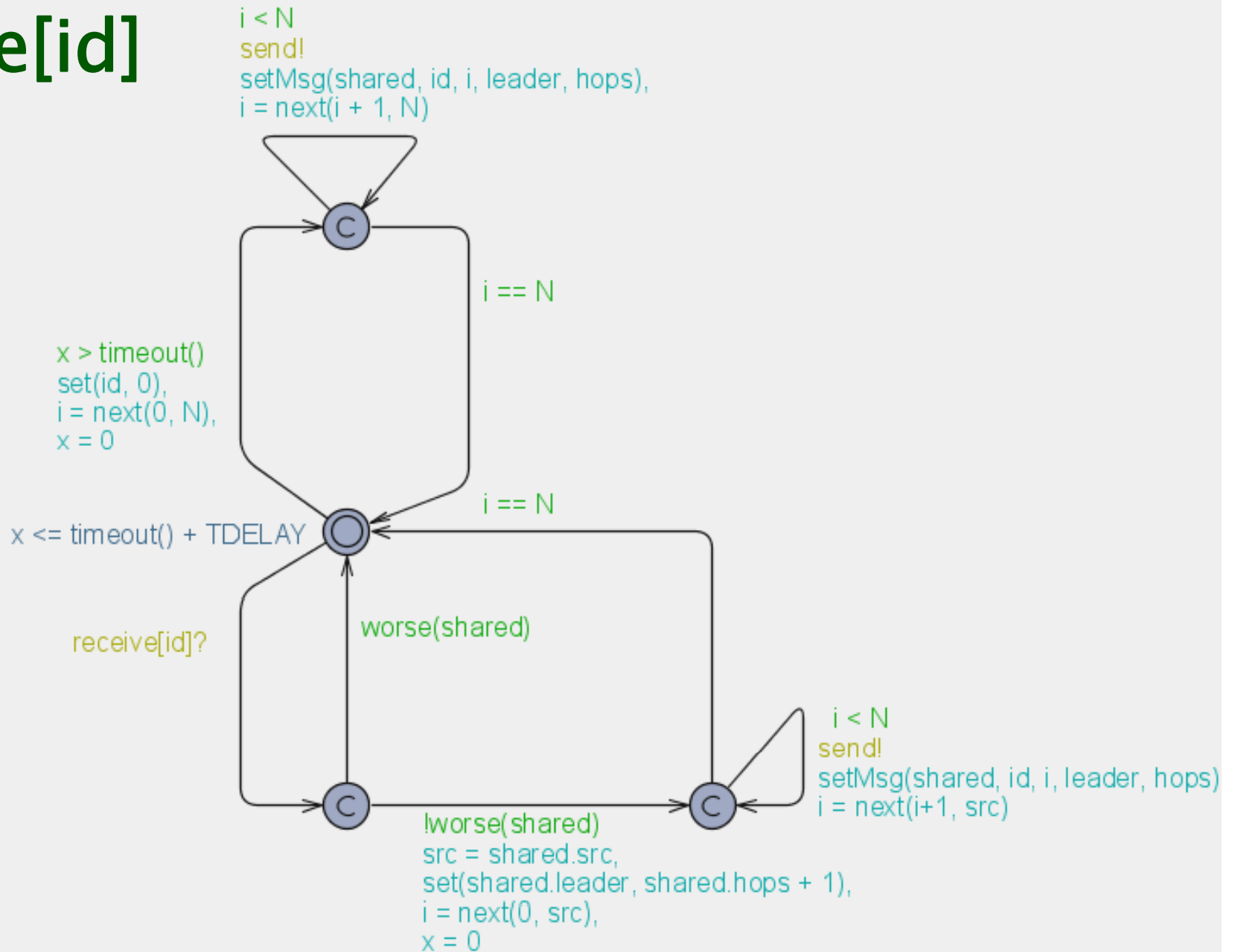
typedef int[0,N-1] id_t;
typedef struct
{
    id_t src;
    id_t dst;
    id_t leader;
    int[0,N] hops;
} msg_t;
```



Message



Node[id]



Local Declarations (Node[id])

```
id_t leader = id;
int[0,N] hops;
clock x;
int[0,N] i;
id_t src;

void set(id_t l, int[0,N] h)
{
    leader = l;
    hops = h;
}

int[0,N] next(int[0,N] i, int[0,N] src)
{
    while (i < N && (!link[id][i] || i == src))
    {
        i++;
    }
    return i;
}
```

```
int[0,1000] timeout()
{
    if (hops > 0)
        return TO + TDELAY + hops * MDELAY;
    return TO;
}

bool worse(const msg_t &msg)
{
    return msg.leader > leader || msg.leader
        == leader && msg.hops > hops;
}
```



Demo

The screenshot displays the UPPAAL simulator interface. The title bar shows the file path: `C:\Documents and Settings\vg\Desktop\DESKTOP_FEB_2007\UPPAAL\UPPAAL_examples\TECS06\leader4.xml - UPPAAL`. The menu bar includes File, Edit, View, Tools, Options, and Help. Below the menu bar are toolbars for navigation and simulation control.

The interface is divided into several panels:

- Drag out (Left):** Contains a list of "Enabled Transitions" (N1) and a "Simulation Trace" showing the sequence of events: `send: N2 --> Message(3)`, `receive[msg.dst]: Message(0) --> N1`, and `receive[msg.dst]: Message(3) --> N1`. It also includes a "Trace File" field and playback controls (Prev, Next, Replay, Open, Save, Random).
- Drag out (Top):** Lists global variables and their values: `shared.src = 2`, `shared.dst = 1`, `shared.leader = 0`, `shared.hops = 1`, `N0.leader = 0`, `N0.hops = 0`, `N0.i = 0`, `N0.src = 0`, `N1.leader = 1`, `N1.hops = 0`, `N1.i = 0`, `N1.src = 0`, `N2.leader = 0`, `N2.hops = 1`, `N2.i = 0`, `N2.src = 0`.
- Main Workspace:** Displays three state transition diagrams for nodes N0, N1, and N2. Below these are eight message diagrams (Message(0) to Message(7)) showing the state of the nodes during each message exchange. A timeline at the bottom shows the sequence of messages and a red arrow indicating a `receive[msg.dst]` event.



Optimisations

- **Reducing** the number of active variables
 - If variable is never used until next reset, then the value does not matter.
- **Symmetry** of message processes
 - The message processes are symmetric: It does not matter which is used to transfer a message.