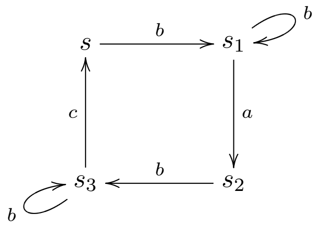
Tutorial 9

Exercise 1*

Consider the following labelled transition system.



Use the fix-point characterization (or optionally the game characterization) for recursive Hennessy-Milner formulae to decide whether the following claims are true or false and discuss what properties the formulae describe:

- $s \overset{?}{\models} X$ where $X \overset{\min}{=} \langle c \rangle tt \vee \langle Act \rangle X$
- $s \overset{?}{\models} X$ where $X \overset{\min}{=} \langle c \rangle tt \vee [Act] X$
- $s \overset{?}{\models} X$ where $X \overset{\max}{=} \langle b \rangle X$
- $s \overset{?}{\models} X$ where $X \overset{\max}{=} \langle b \rangle tt \wedge [a] X \wedge [b] X$

Exercise 2 *

Consider the following four alternative definitions of TCCS agent M:

- $M_1 \overset{\text{def}}{=} \epsilon(3).(\epsilon(2).a.M_1 + b.M_1)$
- $M_2 \overset{\text{def}}{=} \epsilon(5).a.M_2 + \epsilon(3).b.M_2$
- $M_3 \overset{\text{def}}{=} \epsilon(3).(\epsilon(2).a.M_3 + \tau.M_3)$
- $M_4 \overset{\text{def}}{=} \epsilon(5).a.M_4 + \epsilon(3).\tau.M_4$

For which of the above four definitions do we have $M_i \overset{\epsilon(4)}{\longrightarrow}$. In the affirmative case(s) use the SOS rules for TCCS to prove the delay-transition as well as identify the target process P_i such that $M_i \overset{\epsilon(4)}{\longrightarrow} P_i$. Discuss the general relationship between process terms $\epsilon(d).(P + Q)$ and $\epsilon(d).P + \epsilon(d).Q$.

Exercise 3*

Consider the agent M and the three variants of agent N:

- $M \overset{\text{def}}{=} \epsilon(3).(\epsilon(2).\overline{a}.M + \overline{b}.M)$
- $N_1 \overset{\text{def}}{=} \epsilon(5).b.N_1 + \epsilon(3).a.N_1$
- $N_2 \overset{\text{def}}{=} \epsilon(3).(\epsilon(2).a.N_2 + \tau.N_2)$
- $N_3 \overset{\text{def}}{=} \epsilon(5).\tau.N_3 + \epsilon(3).b.N_3$

Indicate the values of i for which *a)* $M|N_i \overset{3}{\longrightarrow}$, *b)* $M|N_i \overset{5}{\longrightarrow}$ and *c)* $M|N_i \overset{8}{\longrightarrow}$. In the affirmative case give proper proofs using the SOS rules for TCCS.

Exercise 4

An *alarm timer* is a process which can be set to time-out after a prescribed time period has elapsed. Here we want to model an alarm timer T, which can be set to time-out after 5, 10 and 30 minutes by discrete actions $set5$ $set10$ and $set30$. After the prescribed time period T signals the time out by the action $\overline{to}$. It is required that the alarm timer can be reset with a new time-out period at any given moment, in particular before the previously set time period has elapsed. You are requested to model T both as a TCCS agent and as a timed automata.

Exercise 5

Consider an autonomous elevator which operates between two floors. The requested behaviour of the elevator is as follows:

- The elevator can stop either at the ground floor or the first floor.
- When the elevator arrives at a certain floor, its door automatically opens. It takes at least 2 seconds from its arrival before the door opens but the door must definitely open within 5 seconds.
- Whenever the elevator's door is open, passengers can enter. They enter one by one and we (optimistically) assume that the elevator has a sufficient capacity to accommodate any number of passengers waiting outside.
- The door can close only 4 seconds after the last passenger entered.
- After the door closes, the elevator waits at least 2 seconds and then travels up or down to the other floor.

Your tasks are:

- Suggest a timed automaton model of the elevator. Use the actions *up* and *down* to model the movement of the elevator, *open* and *close* to describe the door operation and the action *enter* which means that a passenger is entering the elevator.
- Provide two different timed traces of the system starting at the ground floor with the door open.

Exercise 6 (Optional)

(Extension of exercise 8.3.3) Prove that TCCS enjoys the following properties of *time determinism* and *persistency*:

Time Determinism: For any processes P, P', P'' and delay d, if $P \xrightarrow{d} P'$ and $P \xrightarrow{d} P''$ then $P' = P''$.

Persistency: For any processes P, Q, action a and delay d, if $P \xrightarrow{a}$ and $P \xrightarrow{d} Q$, then $Q \xrightarrow{a}$.

For simplicity, you may restrict attention to process terms without occurrences of contants.