Specification Theories for Probabilistic Systems

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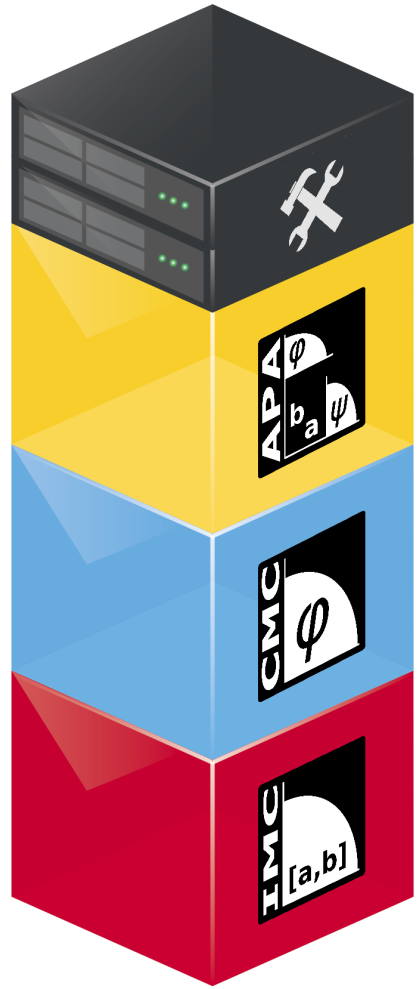

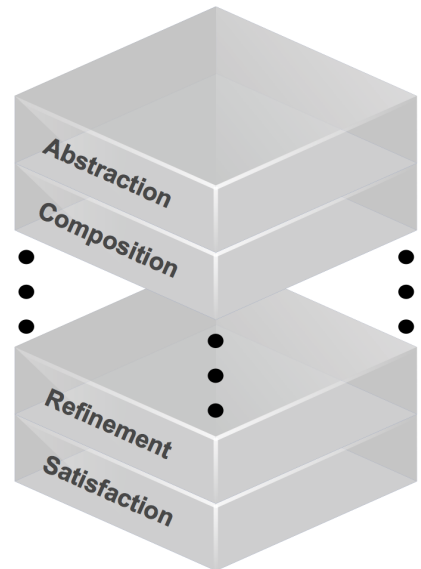

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Motivation

Context

Embedded systems

- "An embedded system is an engineering artifact involving computation that is subject to physical constraints" Henzinger & Sifakis 2006

- Examples: Braking systems in cars, GPS systems in planes etc.

Motivation

Initiating problem

How do we eliminate errors in embedded systems?

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- Errors are expensive (Ariane 5 501, $ 500 million)
- Errors can be life-threatening (Therac-25, 3 persons deceased)

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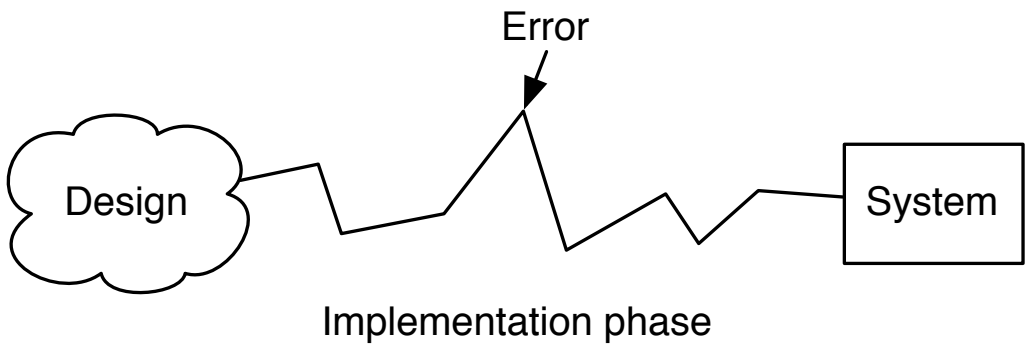


Motivation

Initiating problem

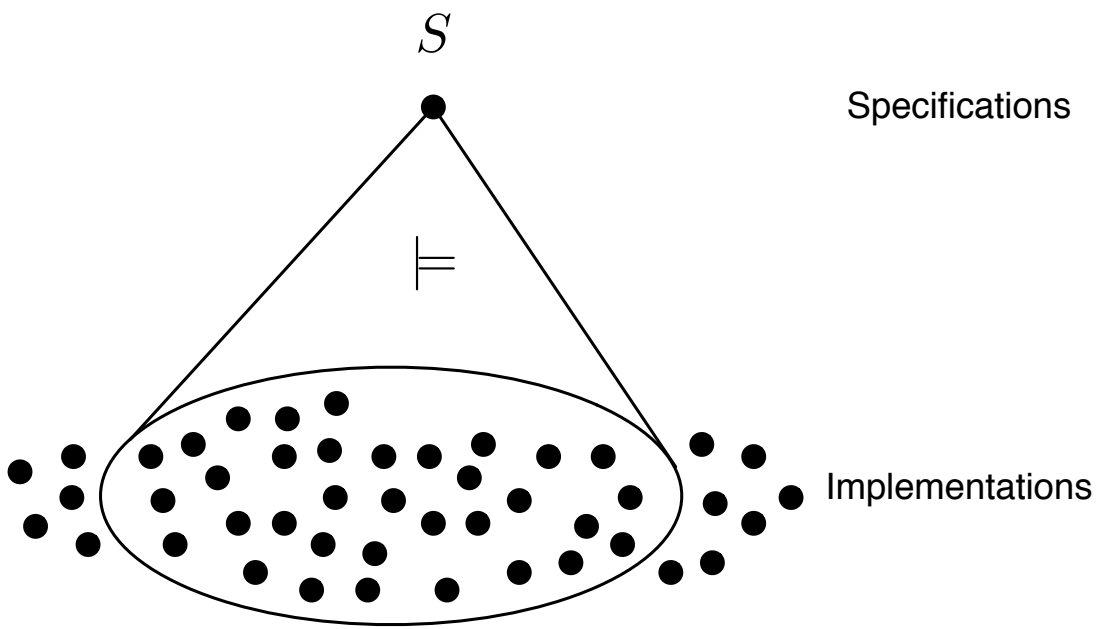
Solution:

- Component-based design
  - Reduced complexity in components
  - Stepwise refinement
  - Independent implementation

Motivation

Specification theories

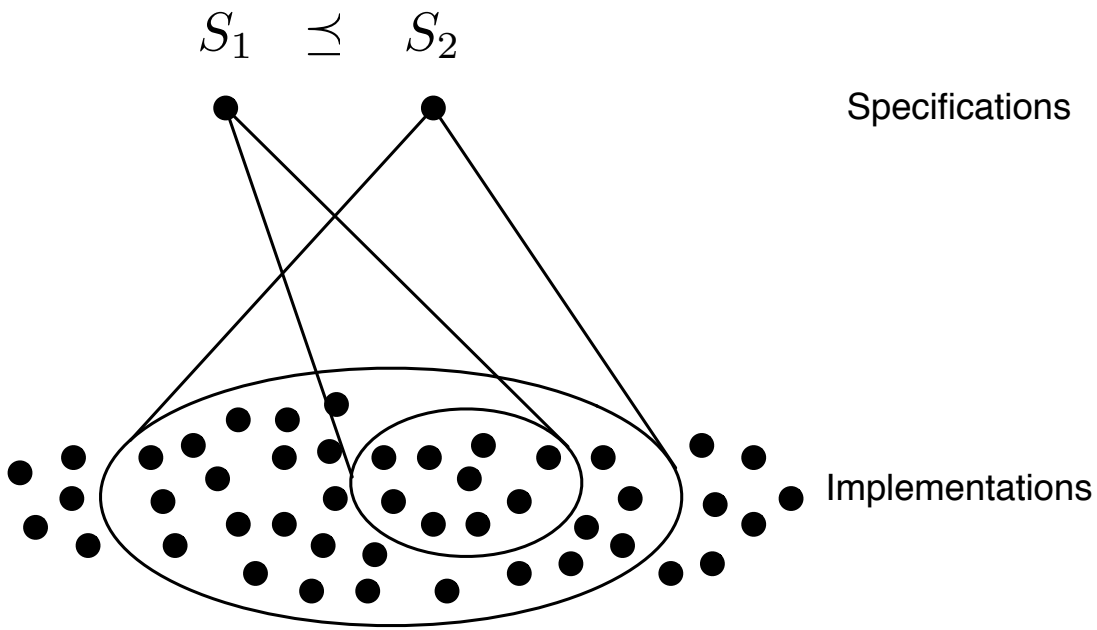
- Satisfaction



Motivation

Specification theories

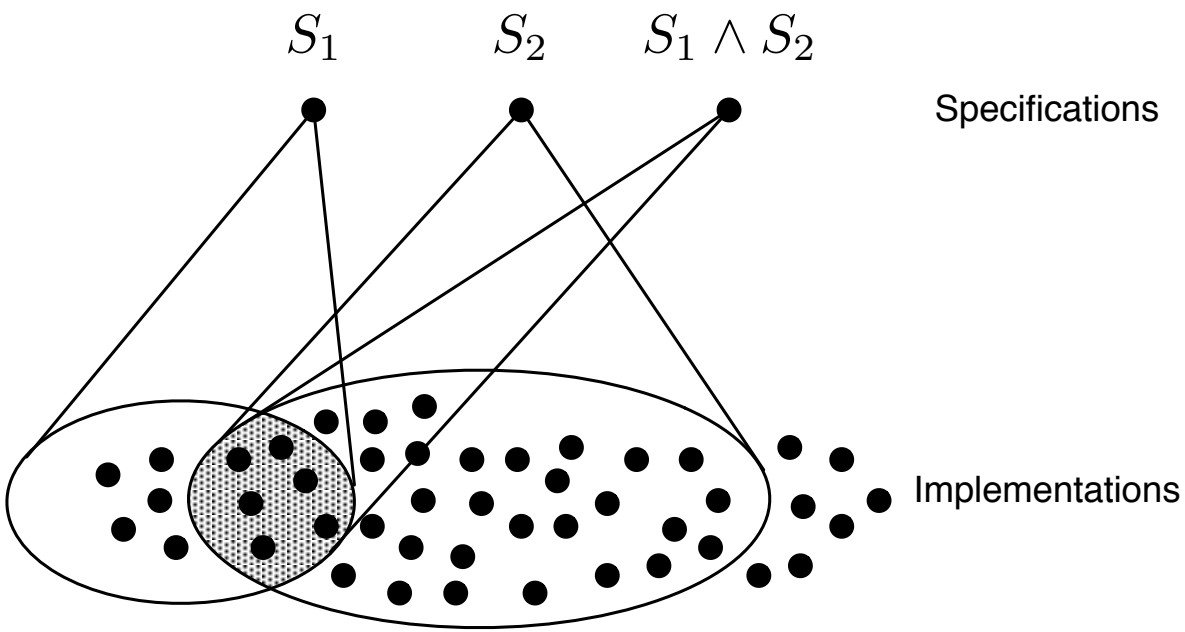
- Refinement



Motivation

Specification theories

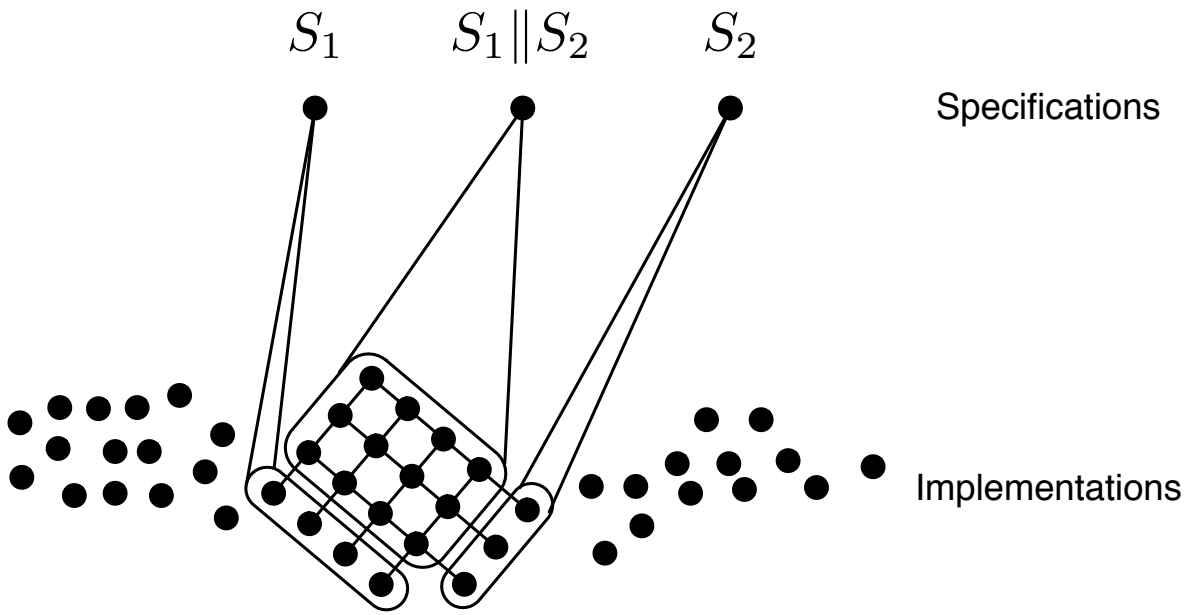
- Conjunction



Motivation

Specification theories

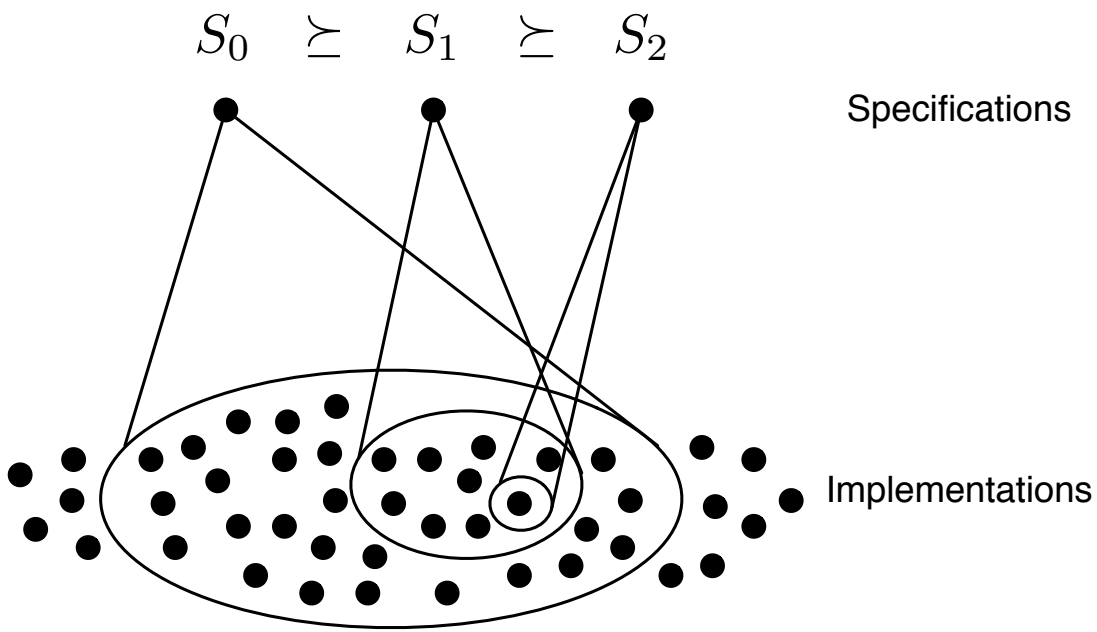
- Parallel composition



Motivation

Specification theories

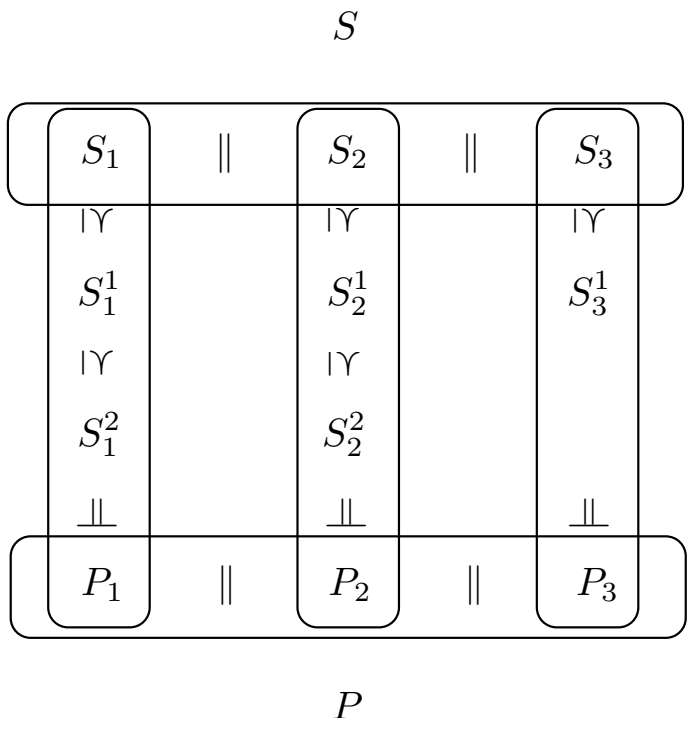
- Stepwise refinement



Motivation

Specification theories

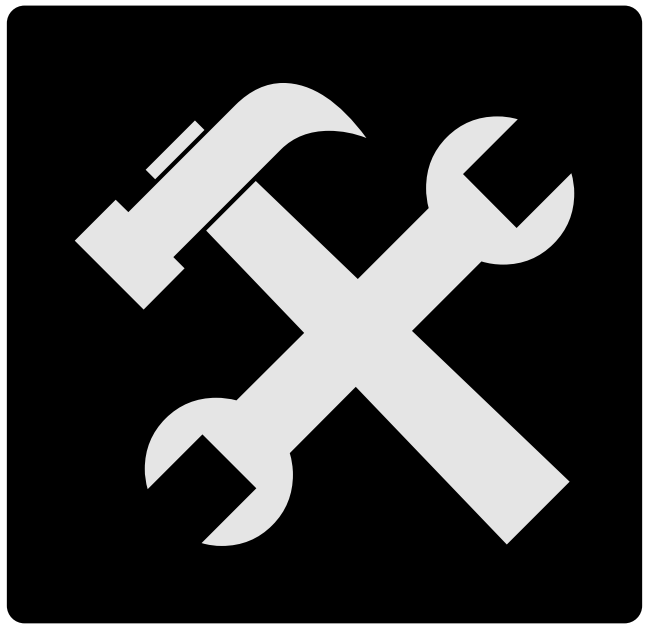
- Independent implementability



Motivation

Specification theories

- Tool support



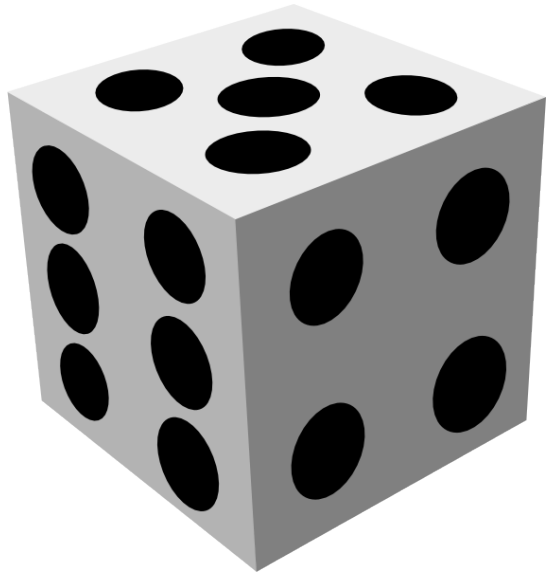
Motivation

Probabilities: in general

- Probabilities abstracts away hidden information
- Allows for quantitative reasoning
  - E.g. what is the probability to avoid a specific error?

Motivation

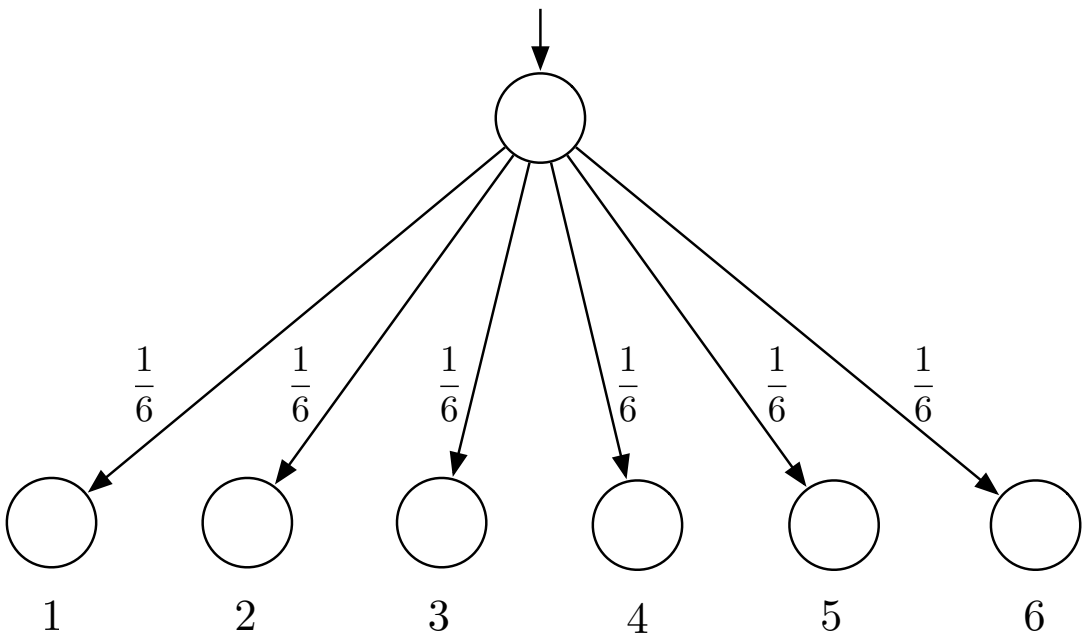
Probabilities: an informal example

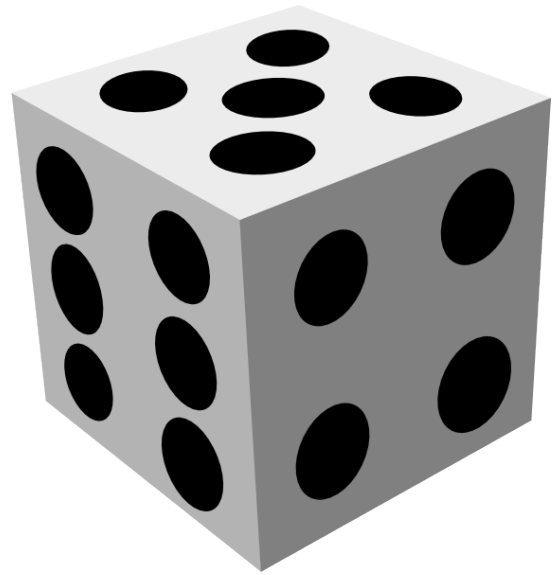


- How could this be modelled?

Motivation

Probabilities: an informal example

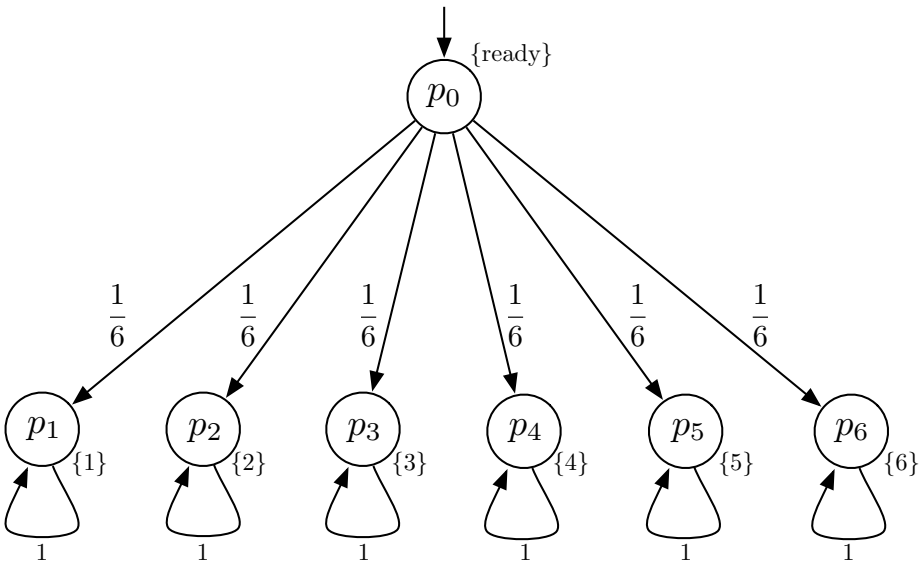




- Distribution is learned by e.g. sampling

Motivation

Probabilities: an informal example



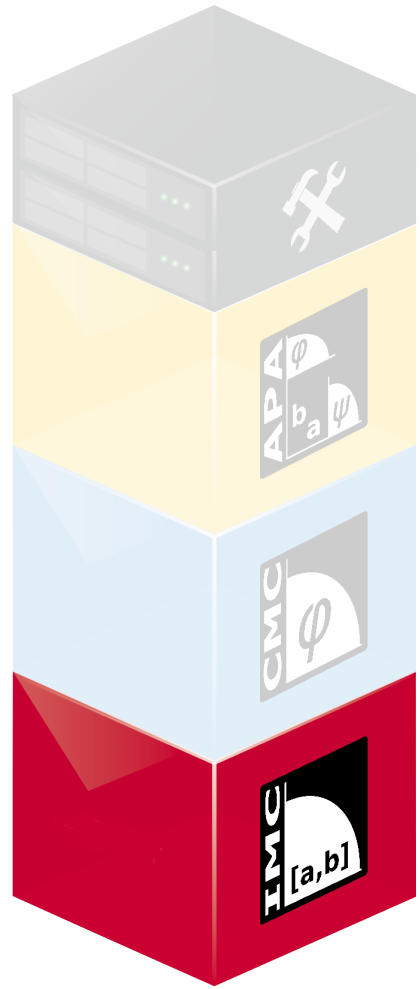
Definition

A *Markov Chain* consists of

- P: set of states, $p_0 \in P$
- A: atomic propositions,
- $V_C : P \to 2^A$: state valuation function
- $\pi : P \to \mathit{Dist}(Q)$: probability distribution assignment

Markov 1906

Interval Markov Chains



Paper A (**JLAP**, LATA'11)

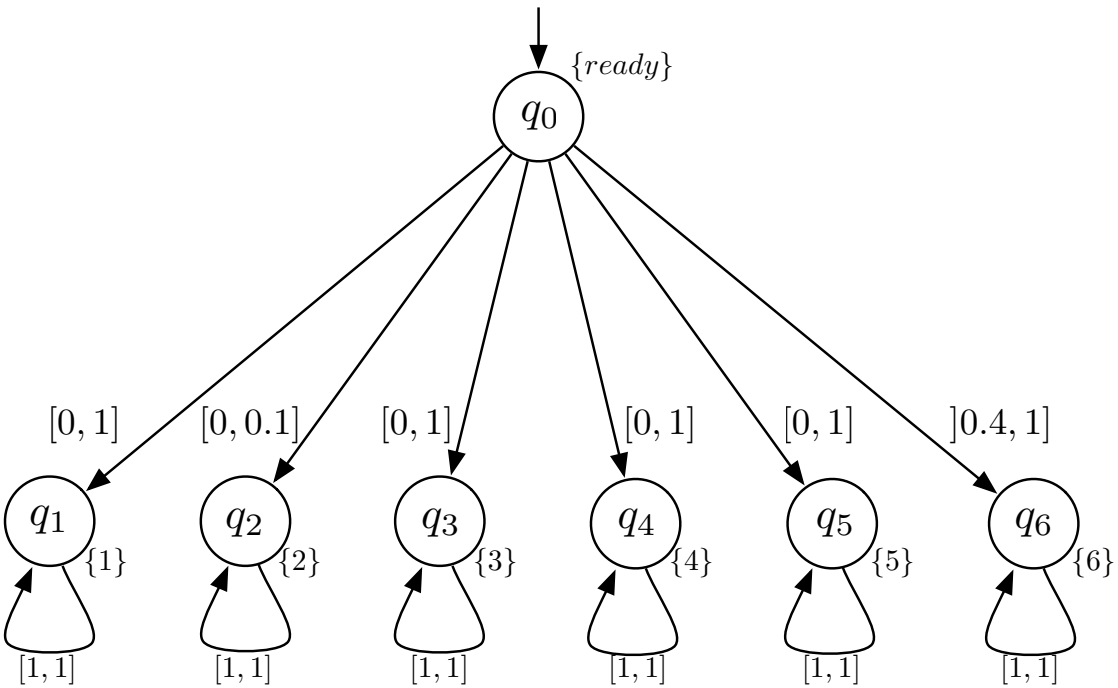
Interval Markov Chains

Motivation

- We modelled the behaviour of a dice
- How could we specify a dice?

Interval Markov Chains

Motivation



Interval Markov Chains

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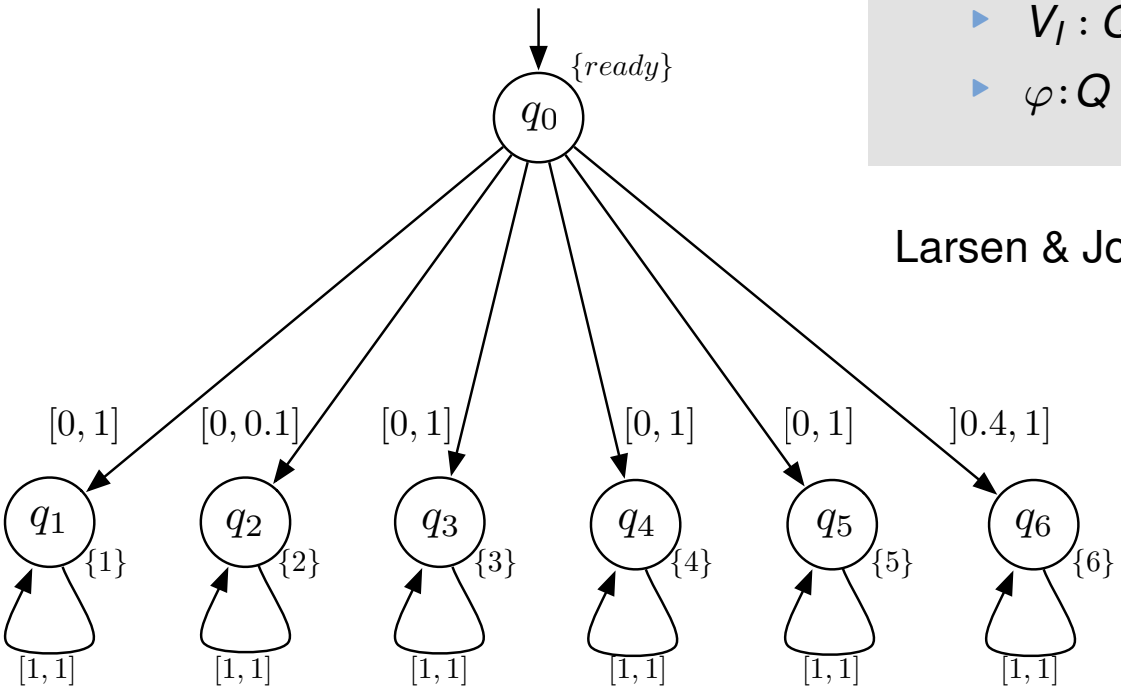


Definition

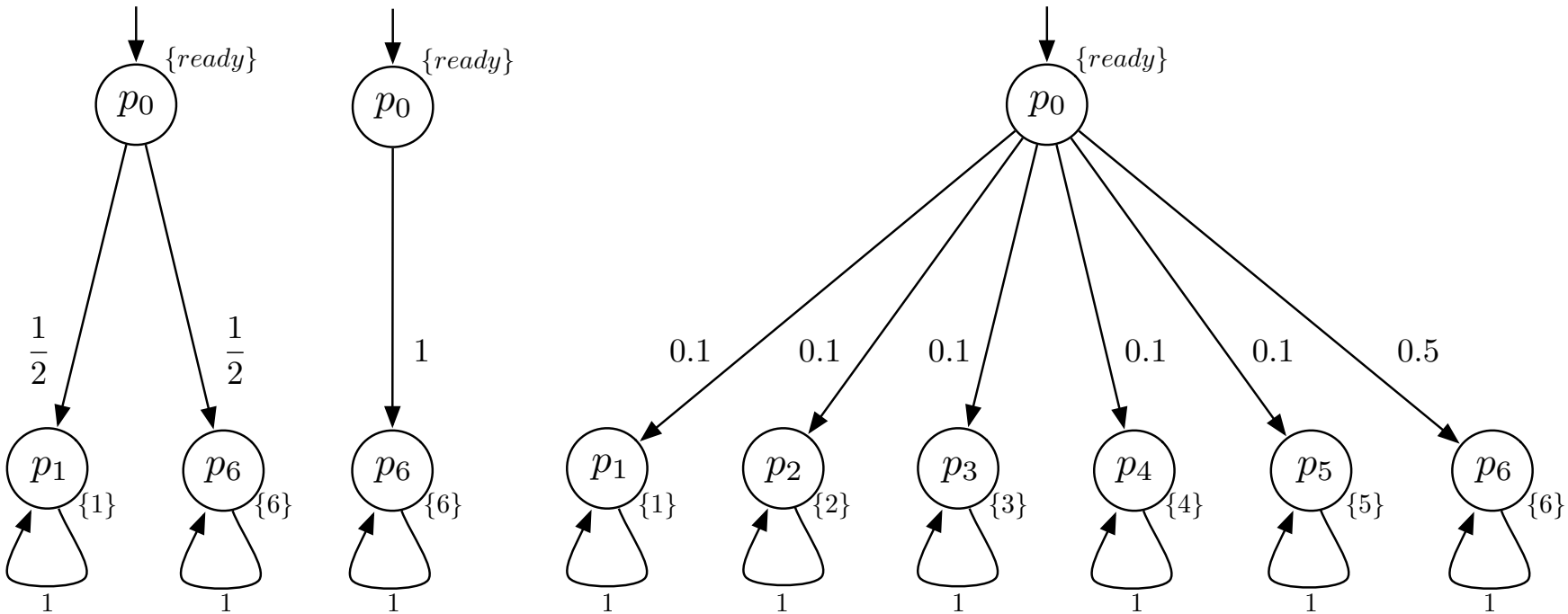
An *Interval Markov Chain* consists of

- Q: set of states, $q_0 \in Q$
- A: atomic propositions
- $V_l : Q \to 2^A$: state valuation function
- $\varphi : Q \to (Q \to \text{Intervals}_{[0,1]})$.

Larsen & Jonsson 1991

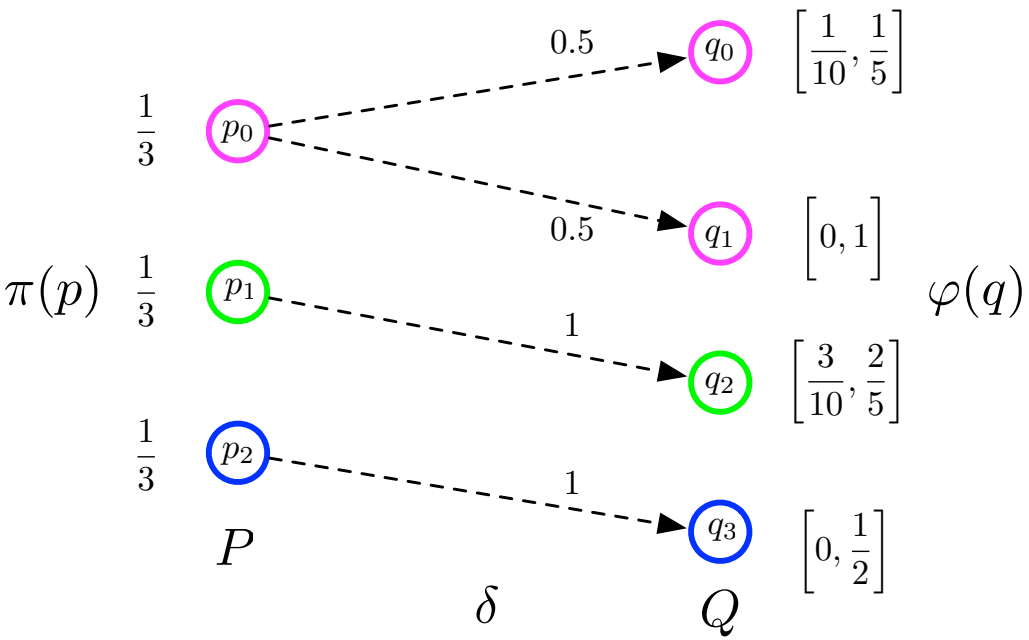
Interval Markov Chains

Motivation



Interval Markov Chains

Satisfaction - redistribution



Definition

$p \in C$ MC, $q \in I$ IMC
$\delta : P \to (Q \to [0,1])$: correspondence function
$\mathcal{R} \subseteq C \times I$

$\pi(p) \lhd^{\delta}_{\mathcal{R}} \varphi(q)$, if and only if

- if $\pi(p)(p') > 0$, then $\delta(p')$ is a distribution on Q,
- $\sum_{p' \in P} \pi(p)(p')\delta(p')(q') \in \varphi(q)(q')$, and
- if $\delta(p')(q') > 0$, then $p' \mathcal{R} q'$.

Interval Markov Chains

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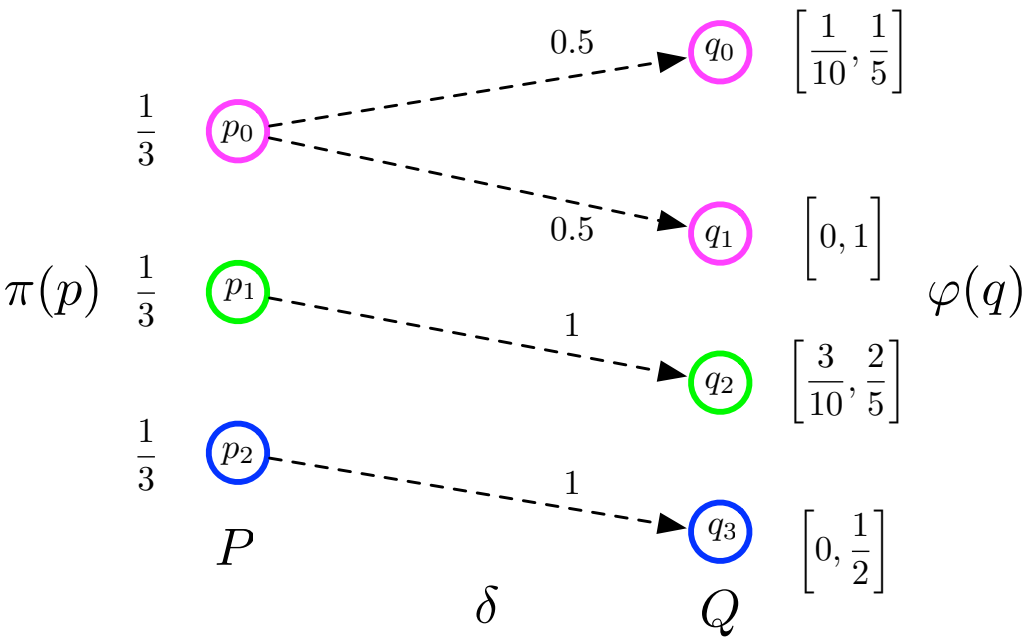

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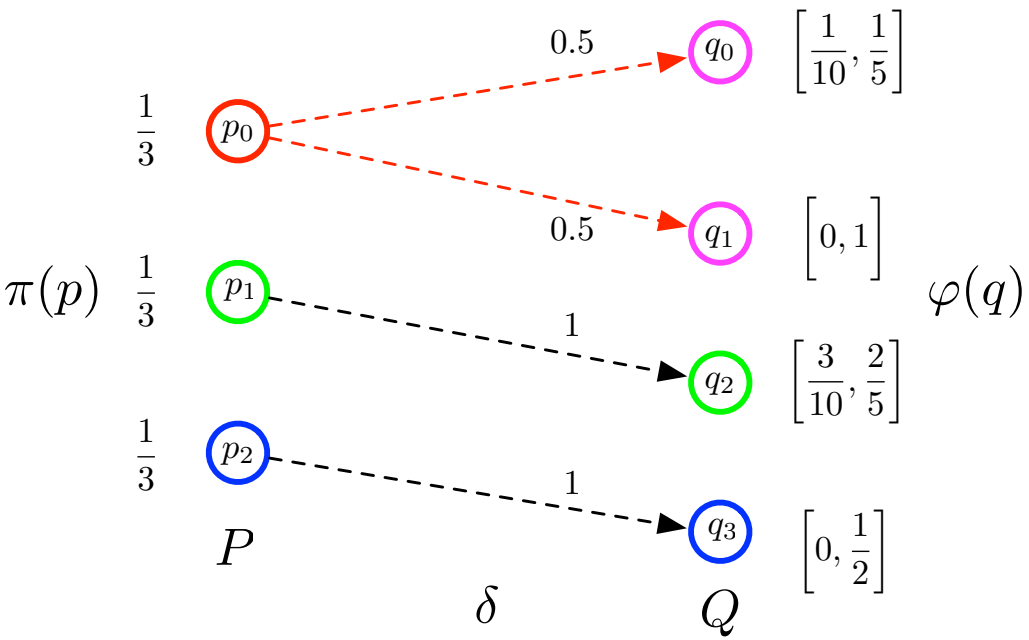

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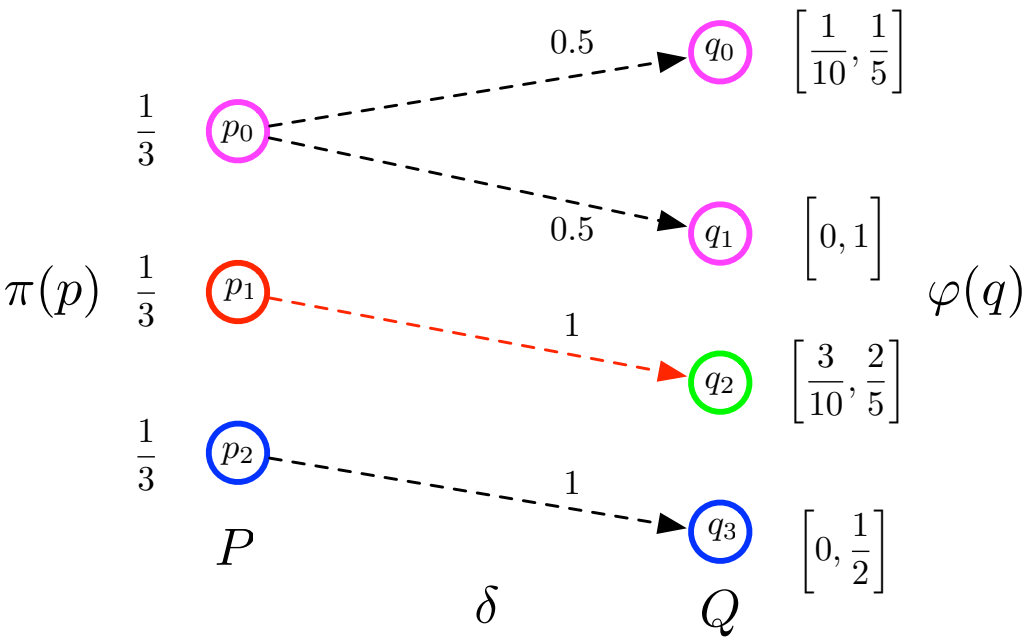

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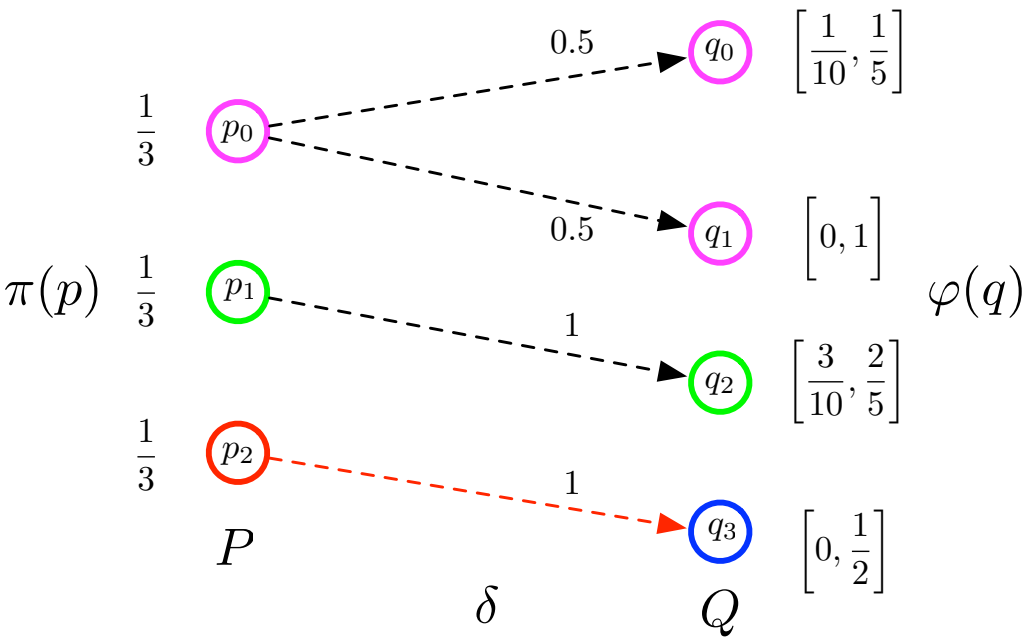

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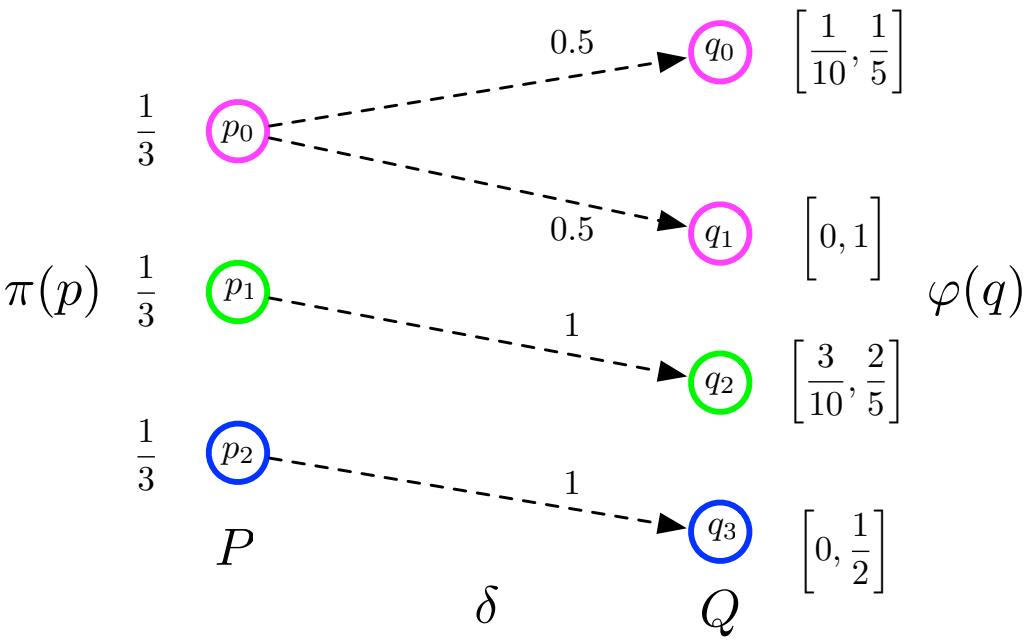

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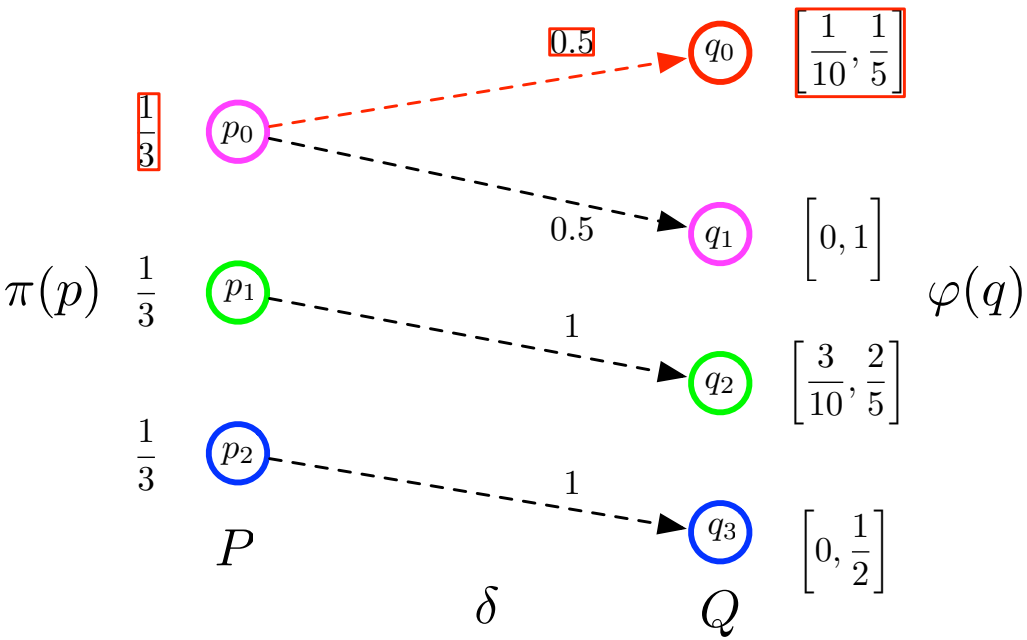

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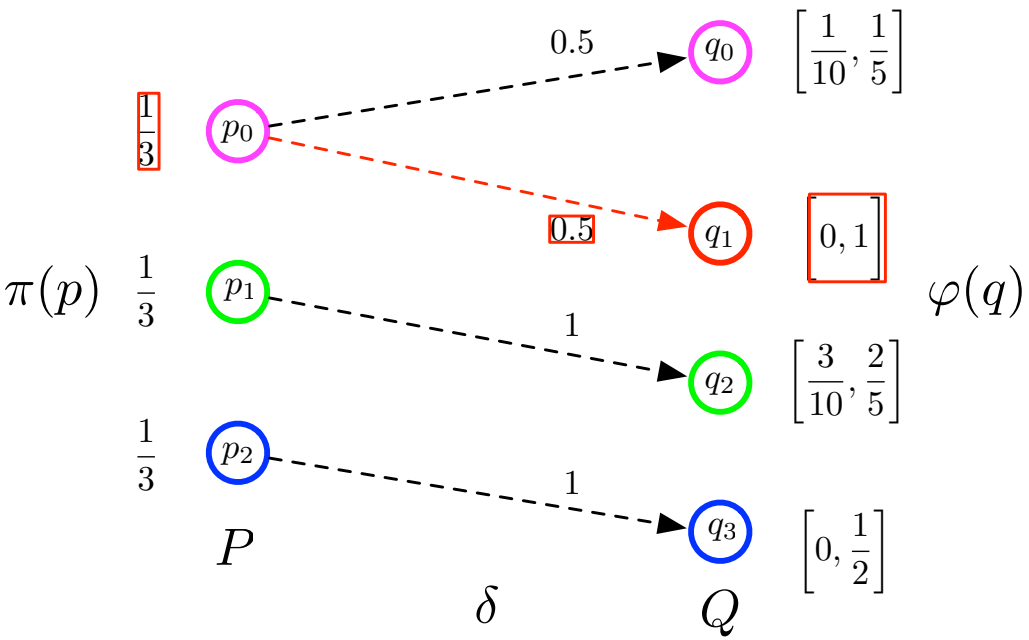

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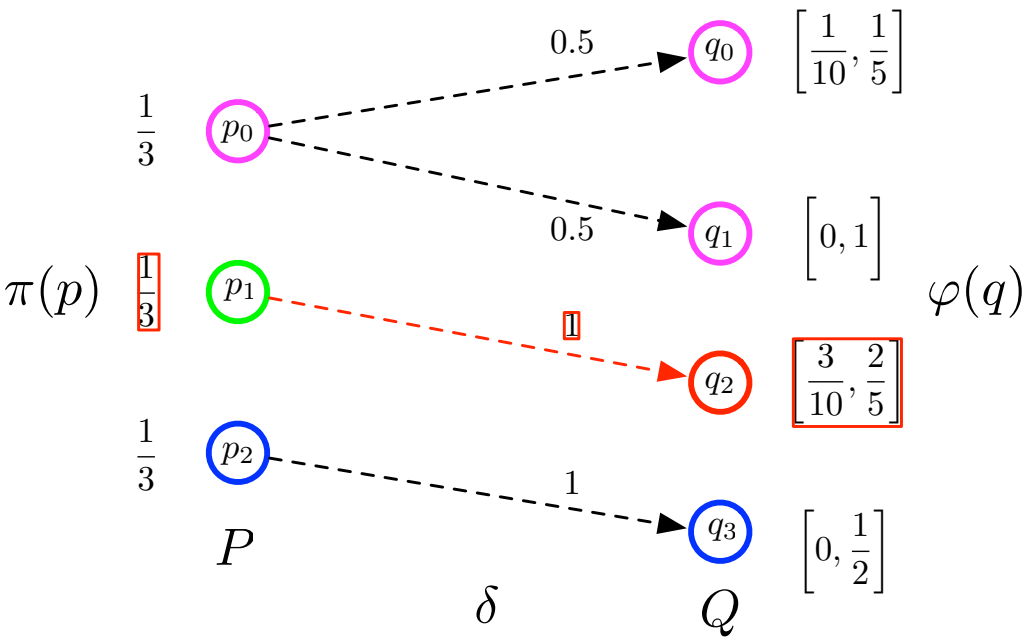

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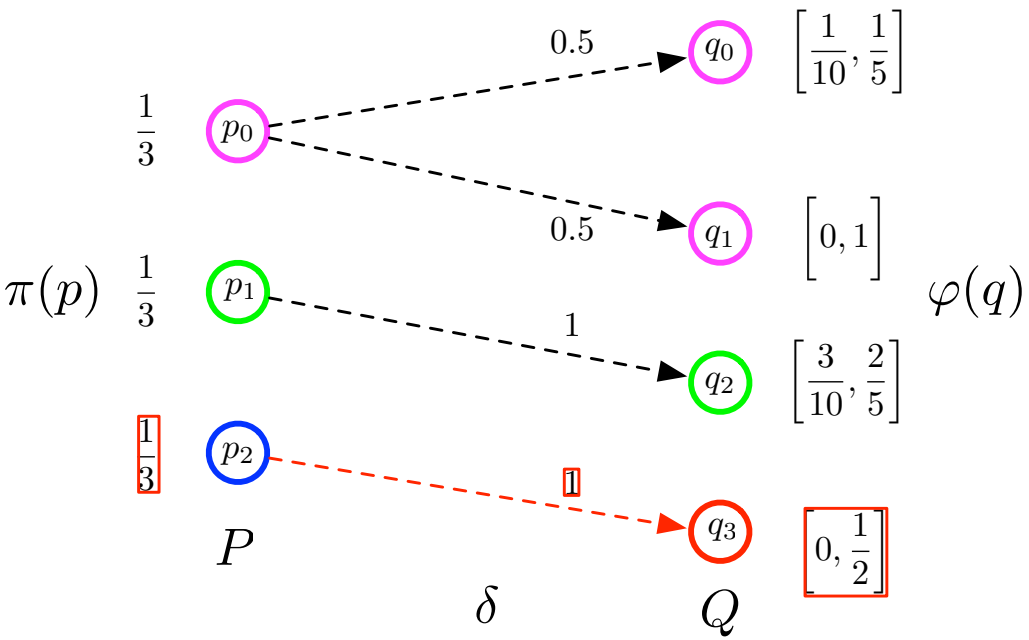

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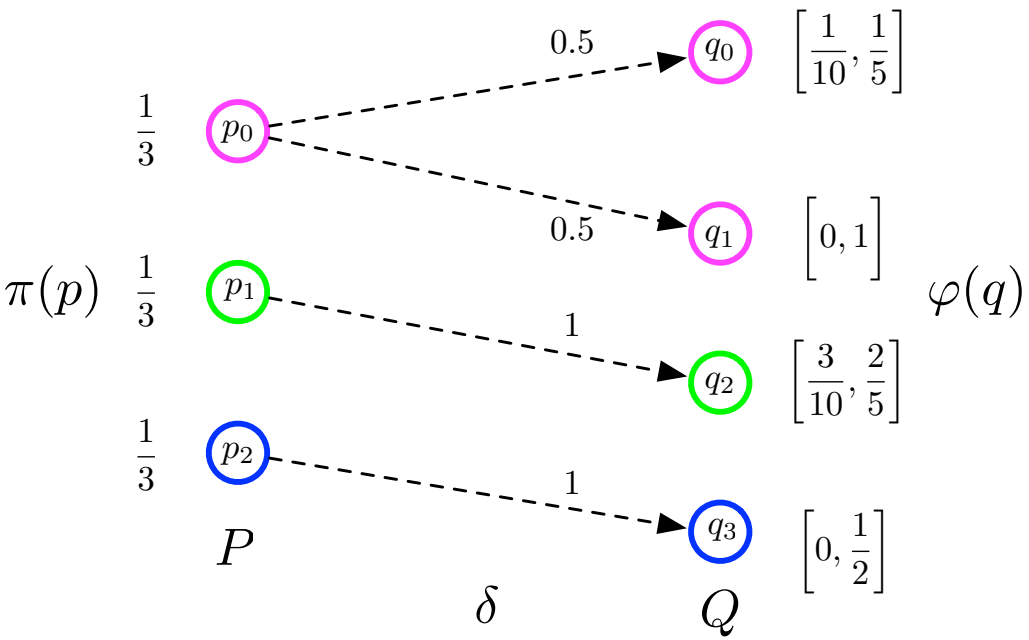

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Interval Markov Chains

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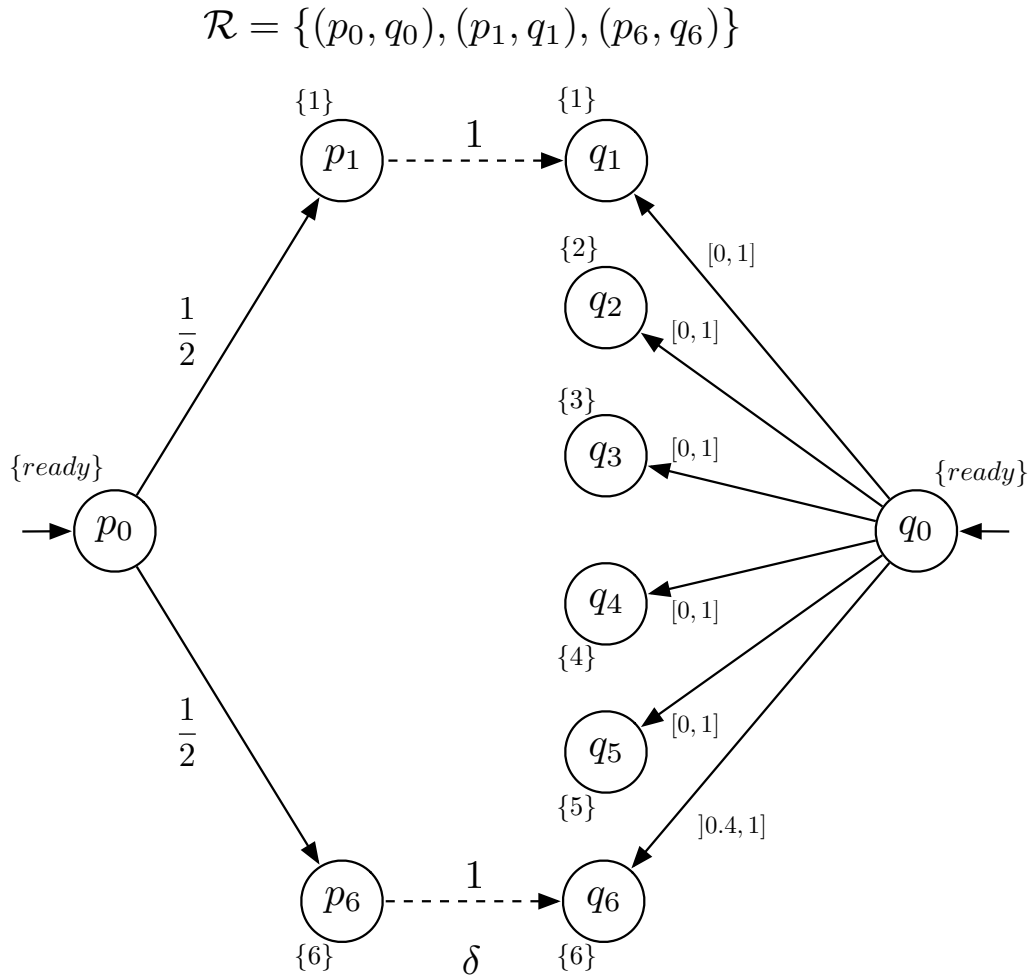


Definition

C be a MC, I IMC
$\mathcal{R} \subseteq C \times I$ is a *satisfaction relation* if and only if whenever $p \mathcal{R} q$ then

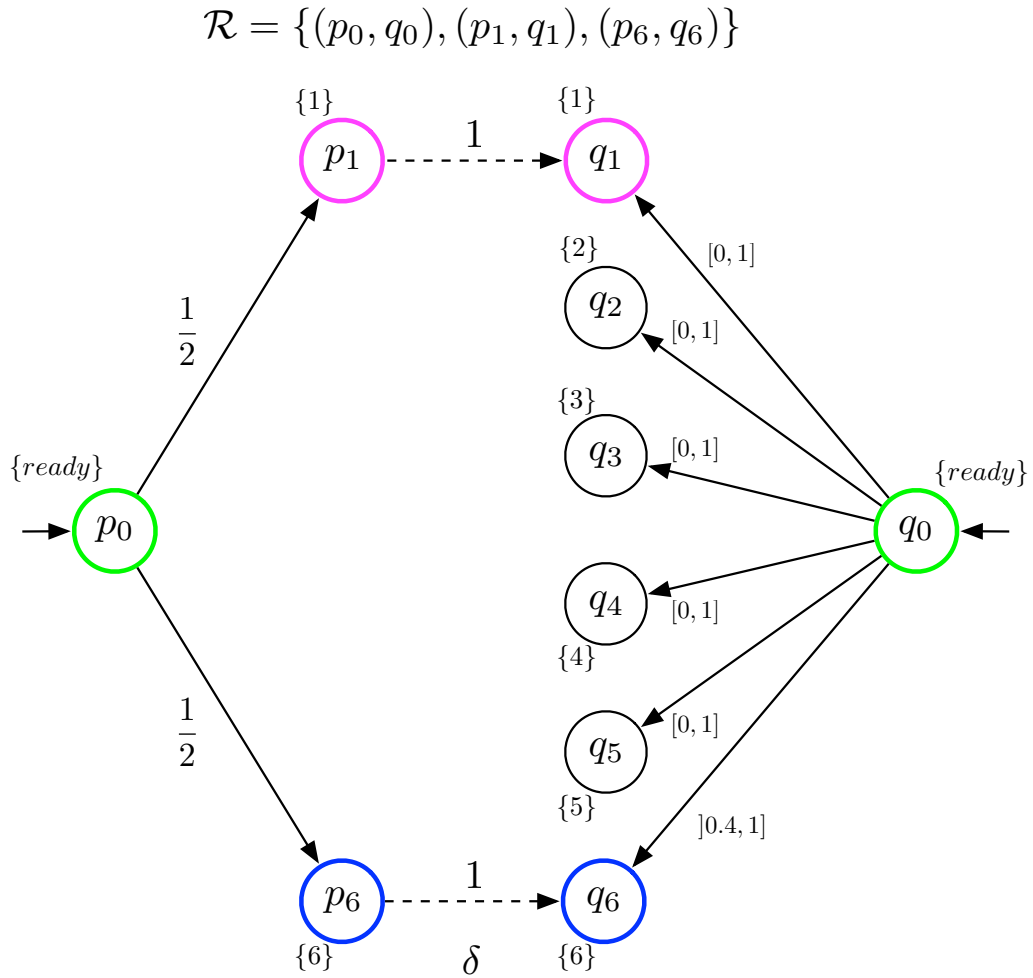
- $V_C(p) = V_I(q)$, and
- $\exists \delta : P \to (Q \to [0, 1])$ such that $\pi(p) \lhd^{\delta}_{\mathcal{R}} \varphi(q)$.

$C \models I \iff \exists\ \mathcal{R}$ such that $(p_0, q_0) \in \mathcal{R}$. We write that $C \in [\![I]\!]$.

Larsen & Jonsson 1991

Interval Markov Chains

Satisfaction



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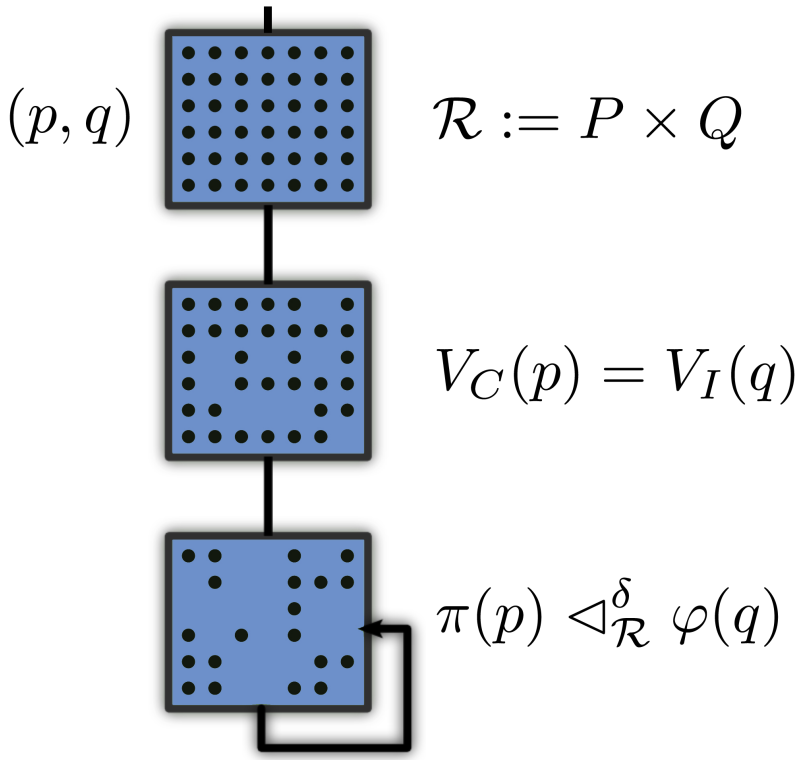
Larsen & Jonsson 1991

Interval Markov Chains

Satisfaction: towards an algorithm

$\models$



Interval Markov Chains

Refinement relations

Assume I_1 and I_2 IMCs and $(q, s) \in Q \times S$.

- Strong refinement $\preceq_S$: $\exists \delta \forall \sigma \in \varphi_1(q) : \sigma \lhd^{\delta}_{\mathcal{R}} \varphi_2(s)$
- Weak refinement $\preceq$: $\forall \sigma \in \varphi_1(q) \exists \delta : \sigma \lhd^{\delta}_{\mathcal{R}} \varphi_2(s)$
- Thorough refinement $\preceq_T$: $[\![I_1]\!] \subseteq [\![I_2]\!]$

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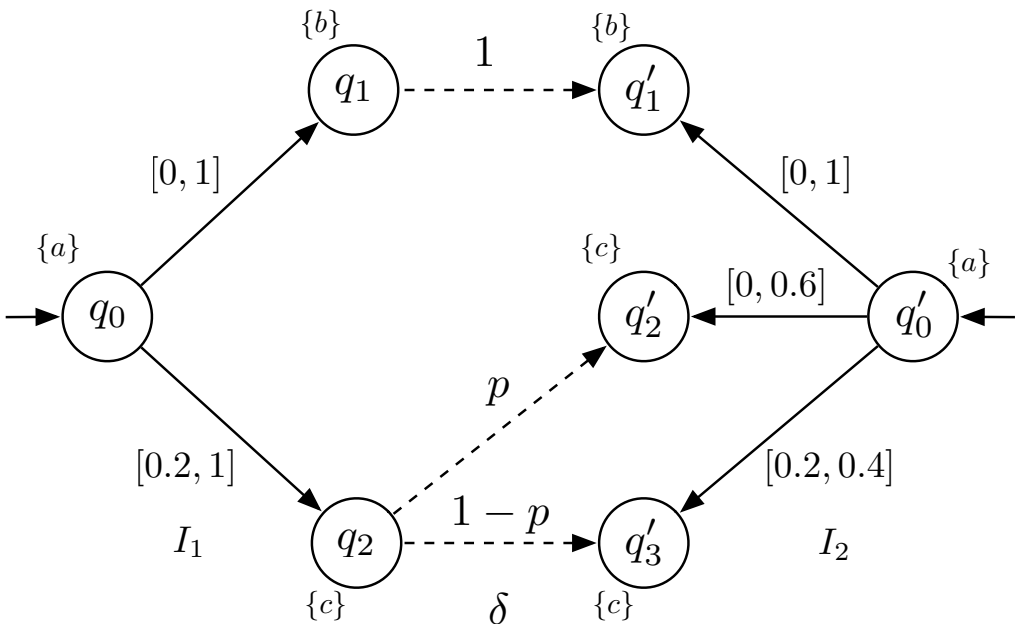


Interval Markov Chains

Results

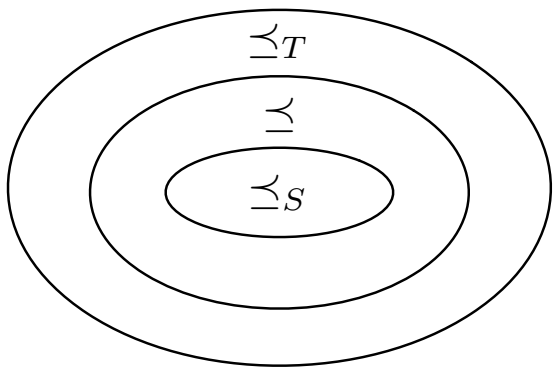
Theorem

Let I_1 and I_2 be IMCs. We have that

$$I_1 \preceq_S I_2 \Rightarrow I_1 \preceq I_2 \Rightarrow I_1 \preceq_T I_2,$$

and under determinism,

$$I_1 \preceq_S I_2 \Leftrightarrow I_1 \preceq I_2 \Leftrightarrow I_1 \preceq_T I_2.$$

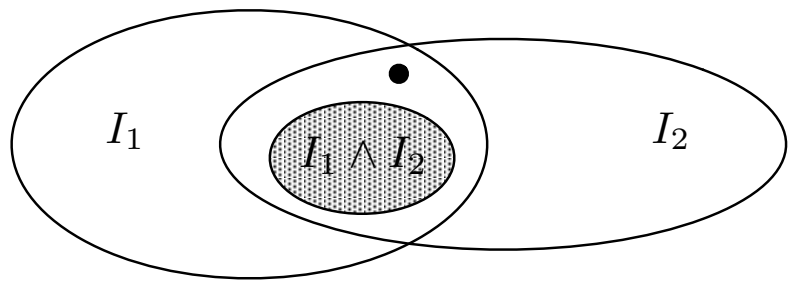


Interval Markov Chains

Closure problems: conjunction

Requirement for conjunction operator:

- $I_1 \wedge I_2 \preceq I_1$ and $I_1 \wedge I_2 \preceq I_2$, and
- for all I_3, if $I_3 \preceq I_1$ and $I_3 \preceq I_2$, then $I_3 \preceq I_1 \wedge I_2$.

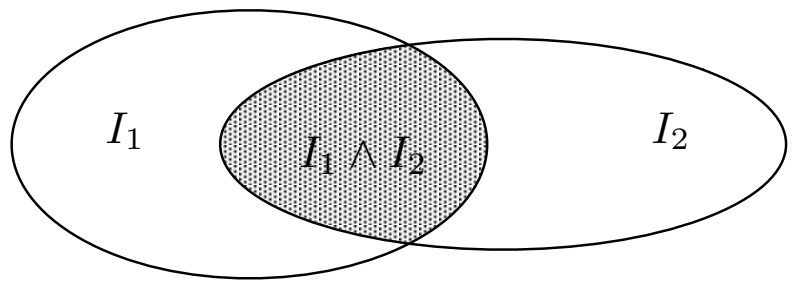


Interval Markov Chains

Closure problems: conjunction

Requirement for conjunction operator:

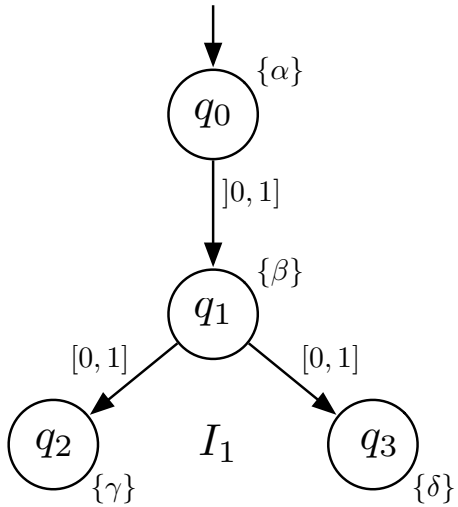
- $I_1 \wedge I_2 \preceq I_1$ and $I_1 \wedge I_2 \preceq I_2$, and
- for all I_3, if $I_3 \preceq I_1$ and $I_3 \preceq I_2$, then $I_3 \preceq I_1 \wedge I_2$.

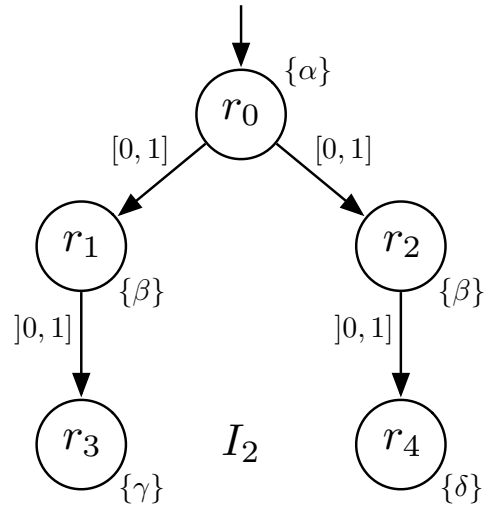


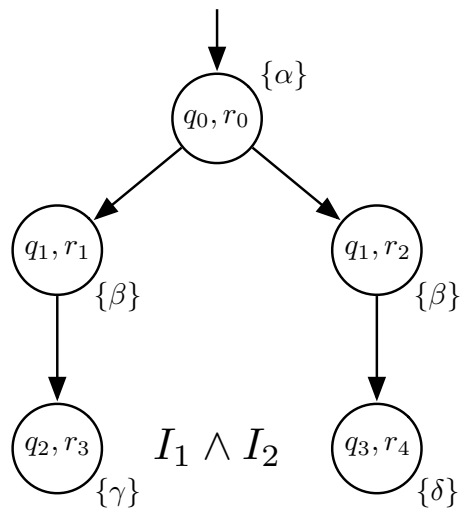
$$[\![I_1 \wedge I_2]\!] = [\![I_1]\!] \cap [\![I_2]\!]$$

Interval Markov Chains

Closure problems: conjunction

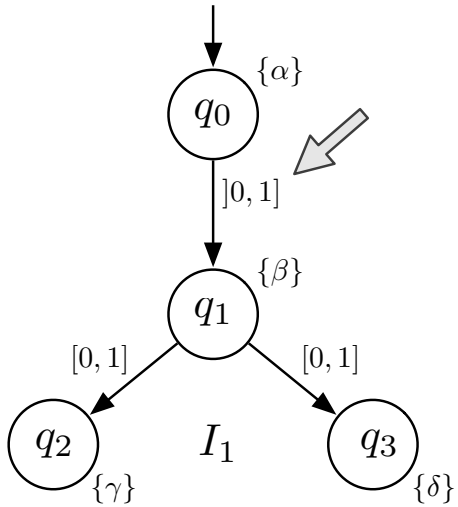


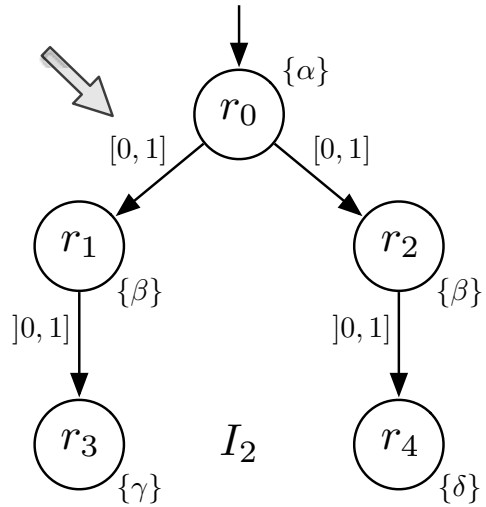


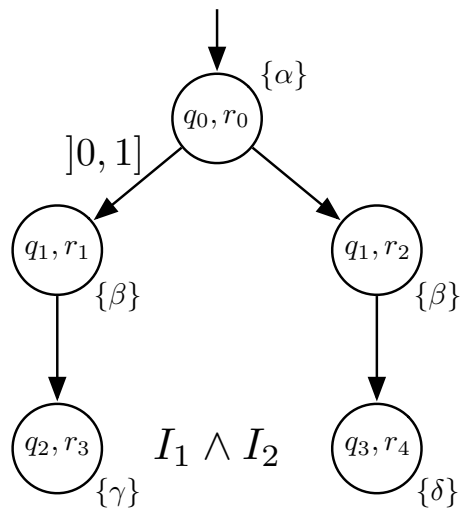


Interval Markov Chains

Closure problems: conjunction

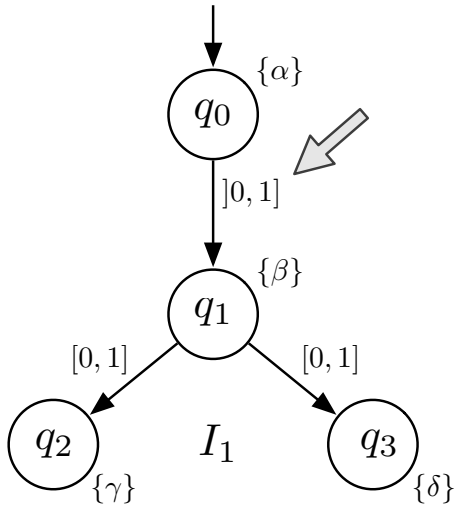


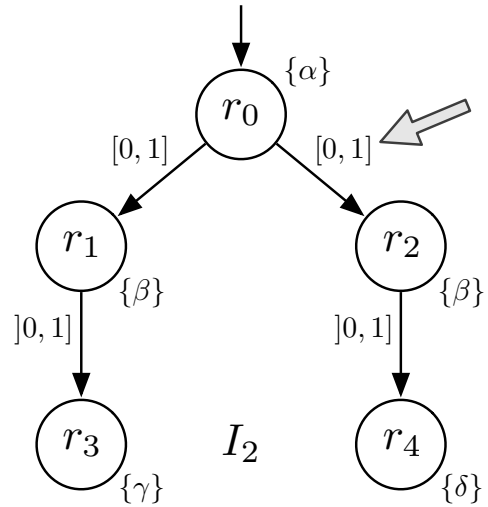


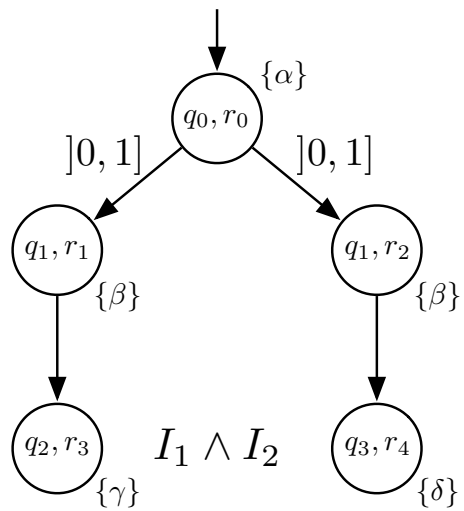


Interval Markov Chains

Closure problems: conjunction

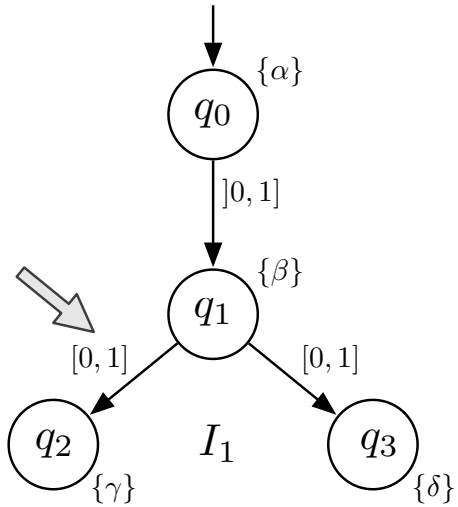


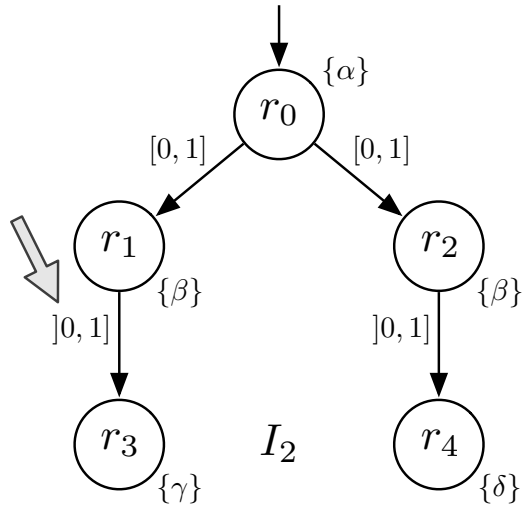


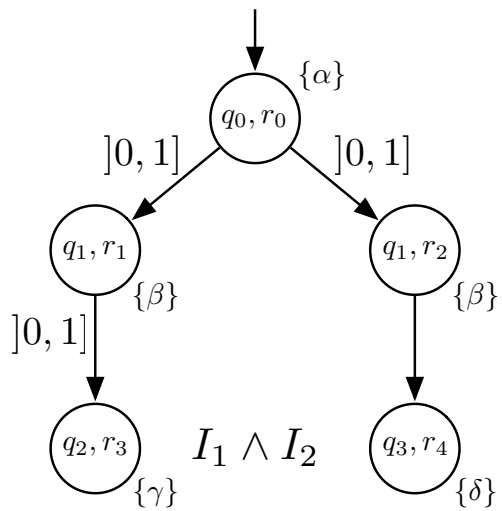


Interval Markov Chains

Closure problems: conjunction

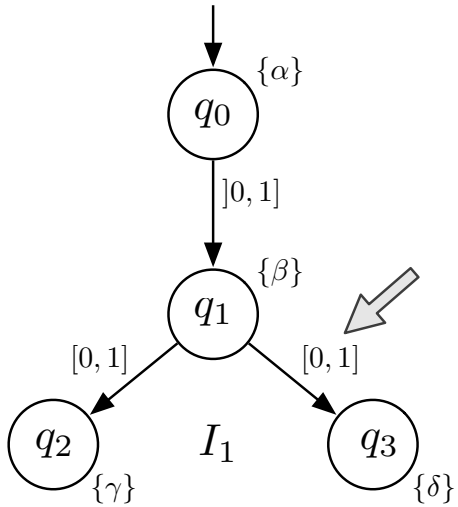


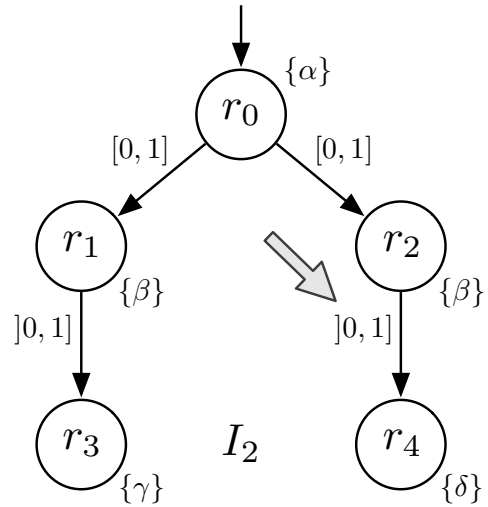


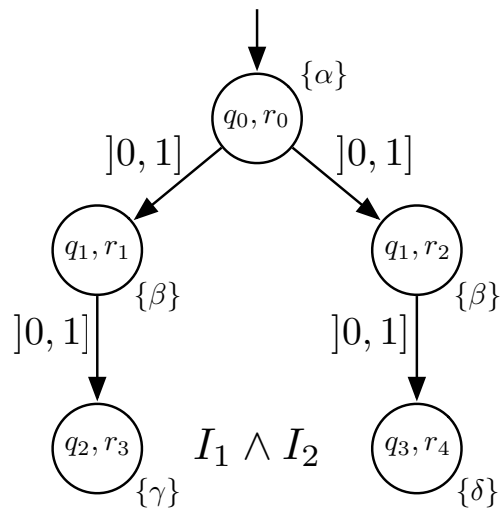


Interval Markov Chains

Closure problems: conjunction

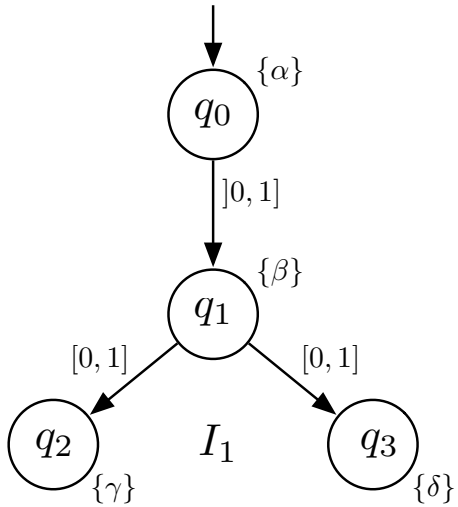


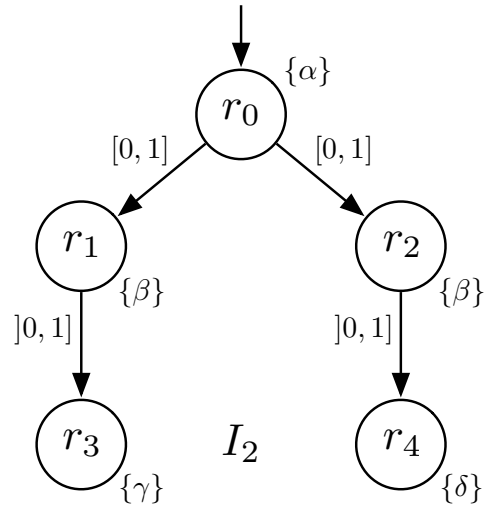


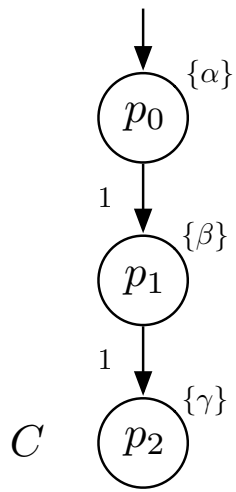


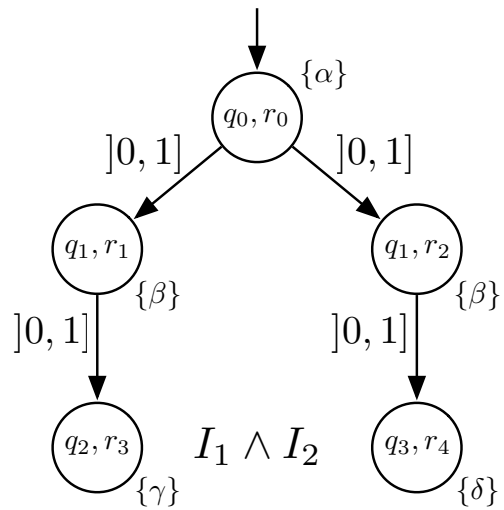
Interval Markov Chains

Closure problems: conjunction





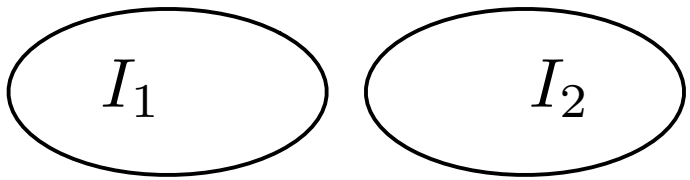




Interval Markov Chains

Common implementation

- Common implementation

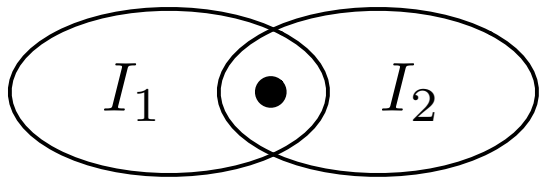


- Consistency
- Pruning

Interval Markov Chains

Common implementation

- Common implementation



- Consistency
- Pruning

Interval Markov Chains

Complexity results

Theorem

Checking for the existence of a common implementation is EXPTIME-complete

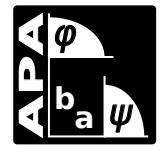
Theorem

Checking consistency is in PTIME

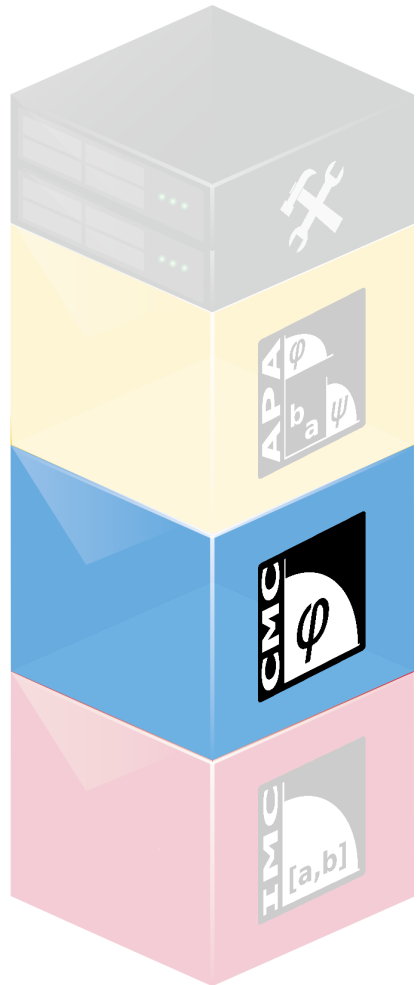
Theorem

Checking thorough refinement is EXPTIME-complete

Conclusion – IMC

			
Satisfaction	✓		
Refinement	✓		
Consistency	✓		
Pruning	✓		
Conjunction			
Parallel Composition			

Constraint Markov Chains

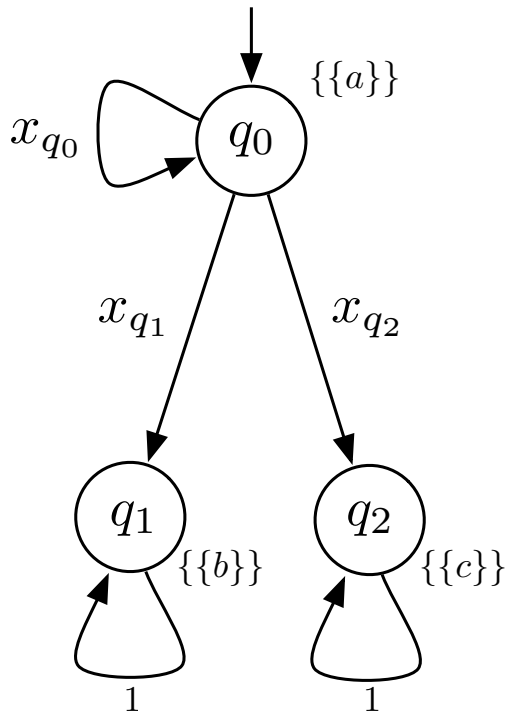


Paper B (**TCS**, QEST'10)

Paper C (**PEVA**, –"–)

Constraint Markov Chains

Motivation



$$\varphi(q_0)(x) \equiv 0.1 \leq x_{q_0} \leq 0.2 \vee 0.7 \leq x_{q_0} \leq 0.9$$

Definition

A *Constraint Markov Chain* consists of

- Q: set of states, $q_0 \in Q$
- A: atomic propositions
- $V : Q \to 2^{2^A}$: admissible state valuations
- $\varphi : Q \to (Dist(Q) \to \{0, 1\})$: constraint function

Constraint Markov Chains

Results

Theorem

Let S_1 and S_2 be CMCs. We have that

$$S_1 \preceq_S S_2 \Rightarrow S_1 \preceq S_2 \Rightarrow S_1 \preceq_T S_2,$$

and under determinism and single valuations in initial states,

$$S_1 \preceq_S S_2 \Leftrightarrow S_1 \preceq S_2 \Leftrightarrow S_1 \preceq_T S_2.$$

Constraint Markov Chains

Closure properties: results

- Conjunction

Theorem

$$[\![S_1 \wedge S_2]\!] = [\![S_1]\!] \cap [\![S_2]\!]$$

- Parallel composition

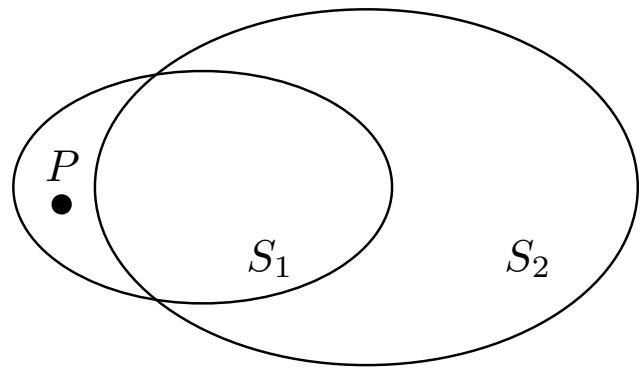
Theorem

If $S_1 \preceq S_3$ *and* $S_2 \preceq S_4$*, then* $S_1 \| S_2 \preceq S_3 \| S_4$

Constraint Markov Chains

Counter-example generation and weak refinement

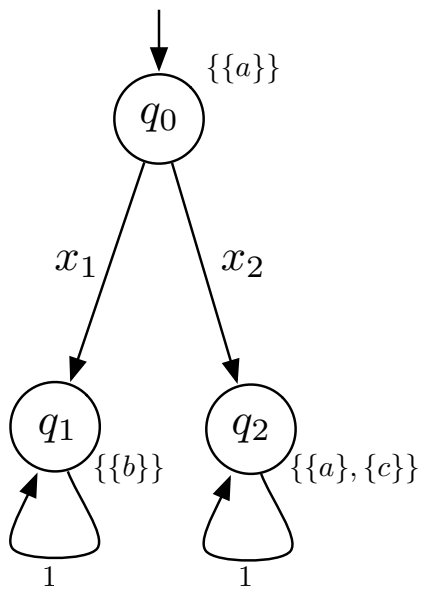
- $S_1 \not\leq S_2$ (deterministic and in single valuation normal form)
- Counter-example P: $P \in [\![S_1]\!]$ and $P \notin [\![S_2]\!]$
- Uses equivalences of refinement relations



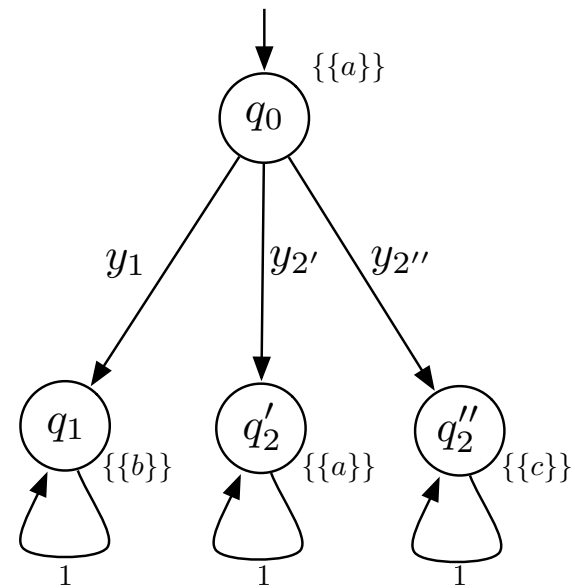
Constraint Markov Chains

Further results

- Normalization: $[\![S]\!] = [\![\mathcal{N}(S)]\!]$



$$\varphi(q_0)(x) \equiv x_1 \geq 0.7 \wedge x_2 \leq 0.4 \wedge x_1 + x_2 = 1$$

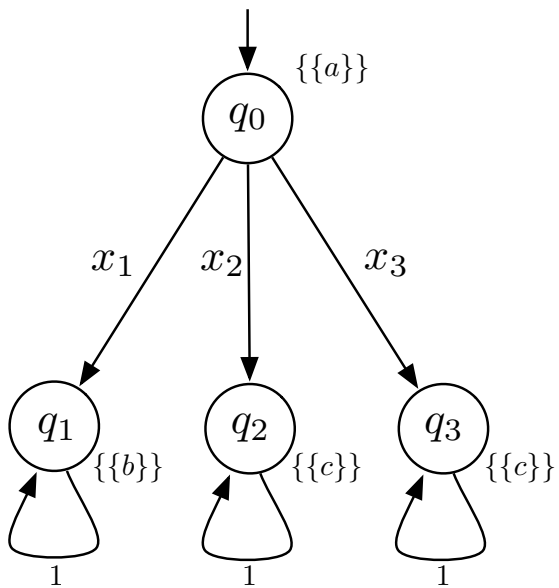


$$\varphi(q_0)(y) \equiv y_1 \geq 0.7 \wedge y_{2'} + y_{2''} \leq 0.4 \wedge y_1 + y_{2'} + y_{2''} = 1$$

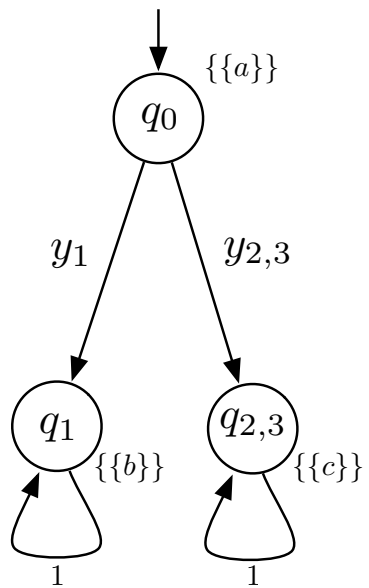
Constraint Markov Chains

Further results

- Determinization: $S \preceq \rho(S)$



$\varphi(q_0)(x) \equiv x_1 + x_2 + x_3 = 1$

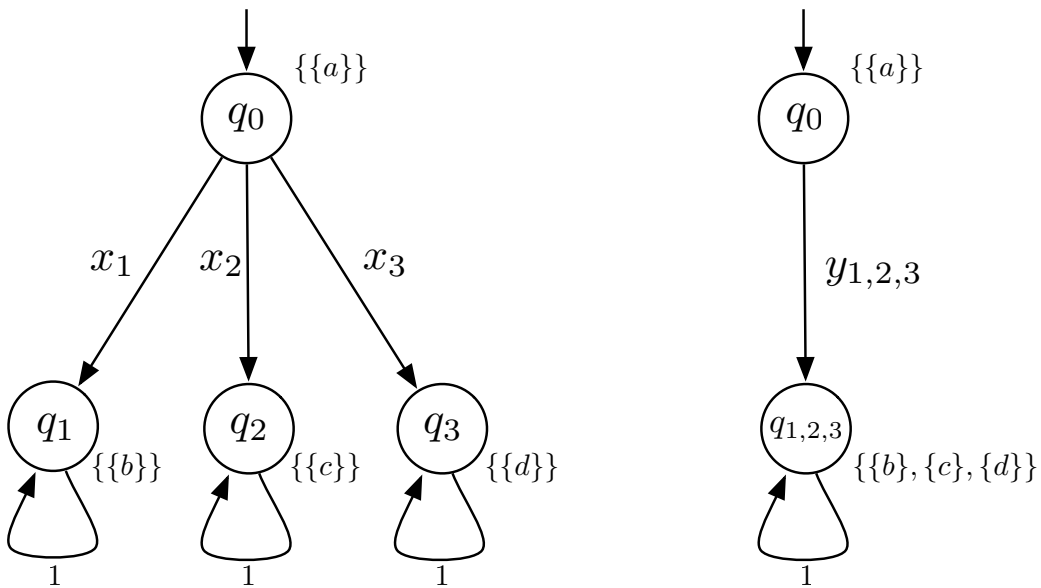


$\varphi(q_0)(y) \equiv x_1 + y_{2,3} = 1$

Constraint Markov Chains

Further results

- State abstraction: $S \preceq \alpha(S)$

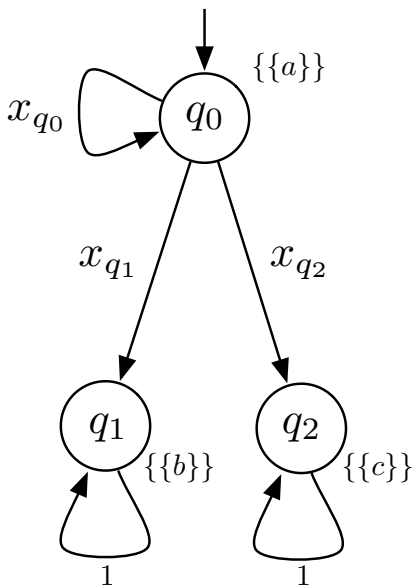


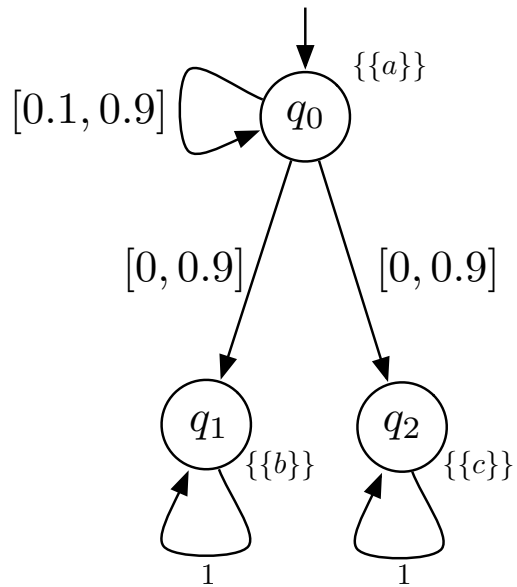
$\varphi(q_0)(x) \equiv x_1 + x_2 + x_3 = 1$ $\varphi(q_0)(y) \equiv y_{1,2,3} = 1$

Constraint Markov Chains

Further results

- Constraint abstraction: $S \leq \chi(S)$
- Minimality: $S \leq I$, then $\chi(S) \leq I$



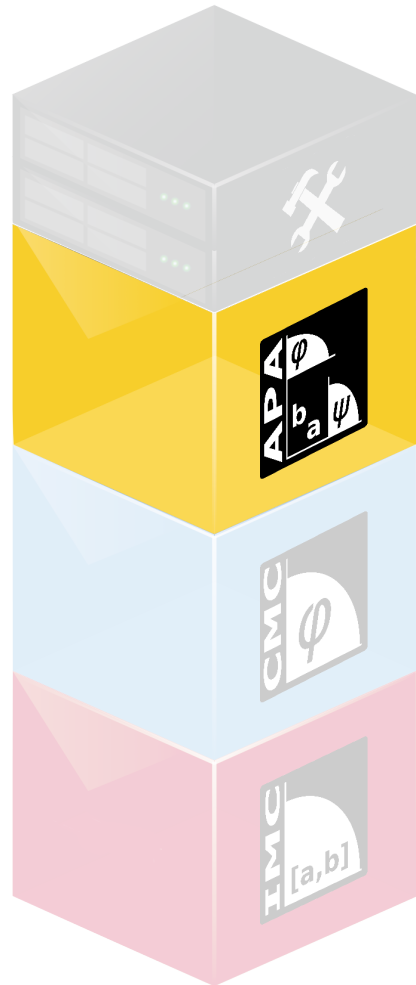


$$\varphi(q_0)(x) \equiv 0.1 \leq x_{q_0} \leq 0.2 \vee 0.7 \leq x_{q_0} \leq 0.9$$

Conclusion – CMC

	IMC [a,b]	CMC φ	APA φ b a ψ
Satisfaction	✓	✓	
Refinement	✓	✓	
Consistency	✓	✓	
Pruning	✓	✓	
Normalization		✓	
Constraint-abstraction		✓	
State-abstraction		✓	
Conjunction		✓	
Parallel Composition		✓	

Abstract Probabilistic Automata



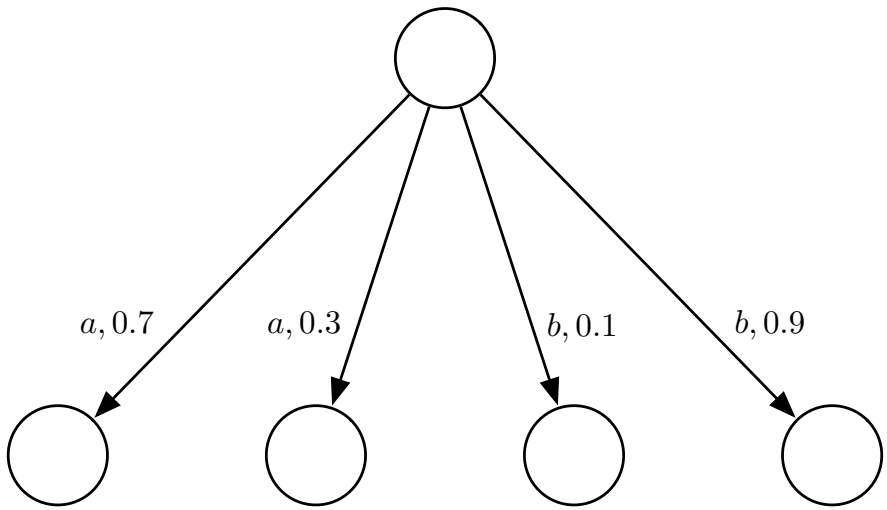
Paper D (**VMCAI'11**)

Paper E (**ACSD'11**)

Extending the notion of MCs

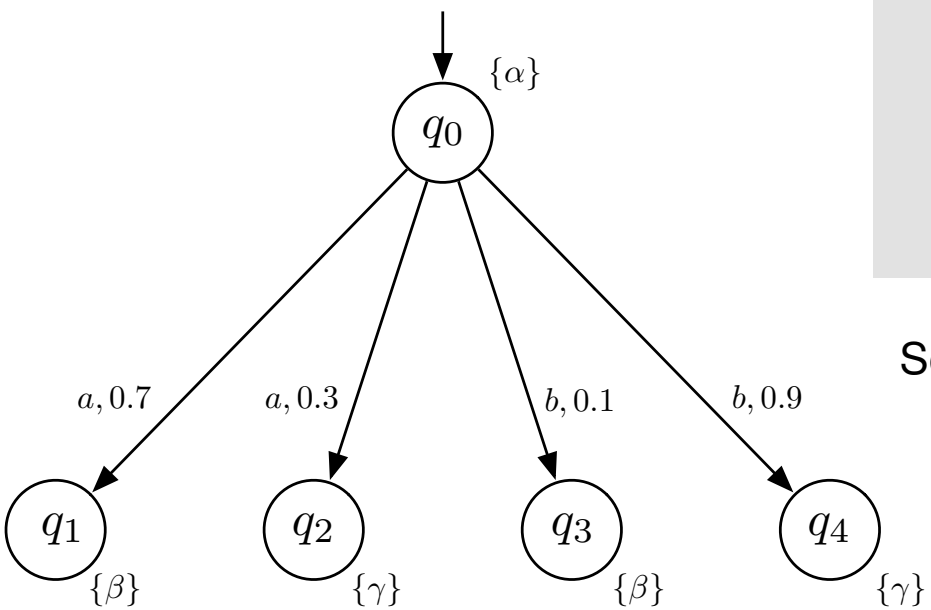
Probabilistic automata

- MCs: precisely one outgoing transitions from each state
- Non-determinism



Extending the notion of MCs

Motivation



Definition

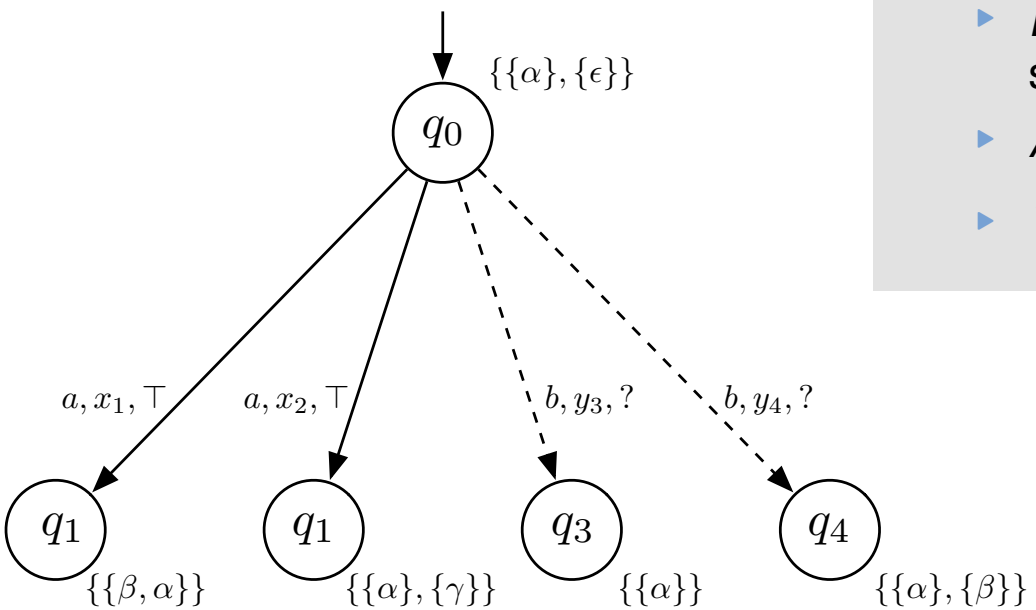
A *Probabilistic Automaton* consists of

- S: set of states, $s_0 \in S$
- A: actions
- L: $S \times A \times Dist(S) \to \{\bot, \top\}$: transition function
- AP: atomic propositions
- V: $S \to 2^{AP}$: state-labeling function

Segala & Lynch 1994

Abstract Probabilistic Automata

Motivation



$$\varphi_x \equiv (x_1 = 1 \vee x_2 = 1) \wedge x_1 + x_2 = 1$$
$$\varphi_y \equiv y_3 \geq 0.1 \wedge y_3 + y_4 = 1$$

Definition

An *Abstract Probabilistic Automaton* consists of

- S: set of states, $s_0 \in S$
- A: actions
- $L : S \times A \times C(S) \longrightarrow \{\bot, ?, \top\}$: state-constraint function
- AP: atomic propositions,
- V: $S \to 2^{2^{AP}}$: state-labeling function.

Abstract Probabilistic Automata

Weak refinement

Definition

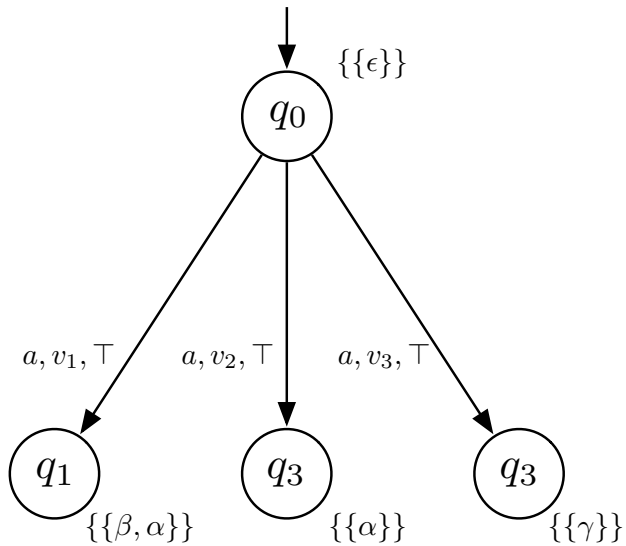
N, N' APAs
$\mathcal{R} \subseteq N \times N'$ is a weak refinement relation if and only if, for all $(s, s') \in \mathcal{R}$,

- $L'(s', a, \varphi') = \top \Rightarrow \exists \varphi \in C(S), L(s, a, \varphi) = \top$ and $\forall \mu \in Sat(\varphi) \exists \delta : S \to (S' \to [0,1]), \mu \lhd^{\delta}_{\mathcal{R}} \varphi'$,
- $L(s, a, \varphi) \neq \bot \Rightarrow \exists \varphi' \in C(S'), L'(s', a, \varphi') \neq \bot$ and $\forall \mu \in Sat(\varphi) \exists \delta : S \to (S' \to [0,1]), \mu \lhd^{\delta}_{\mathcal{R}} \varphi'$, and
- $V(s) \subseteq V'(s')$.

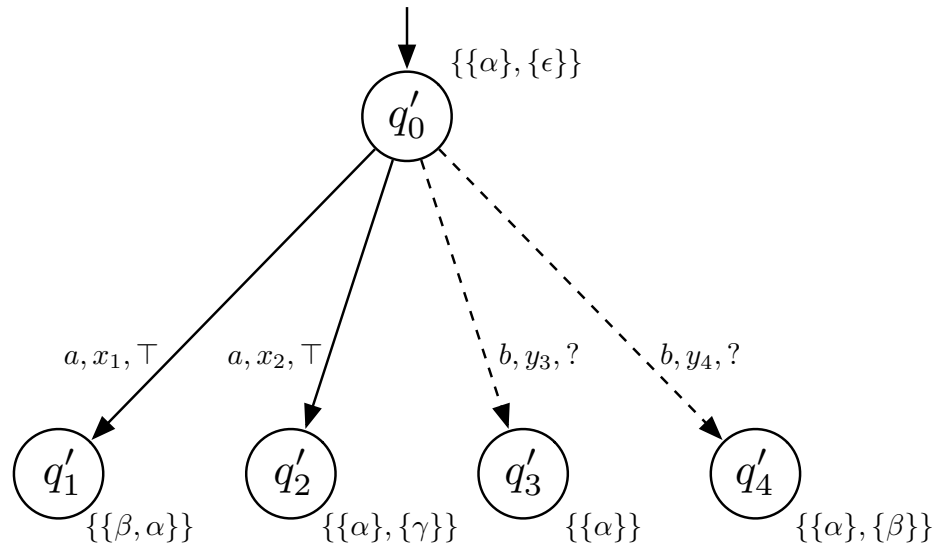
$N \preceq N' \iff \exists \mathcal{R},\ (s_0, s'_0) \in \mathcal{R}$.

Abstract Probabilistic Automata

Refinement: example



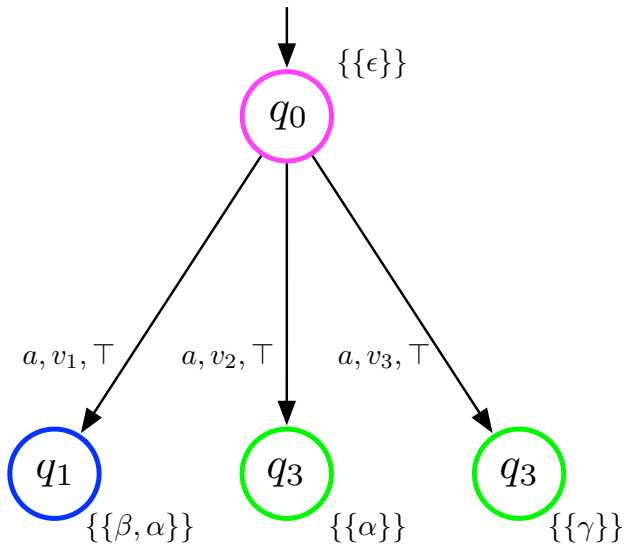
$$\varphi_v \equiv (v_1 = 1 \vee v_2 + v_3 = 1) \wedge v_1 + v_2 + v_3 = 1$$



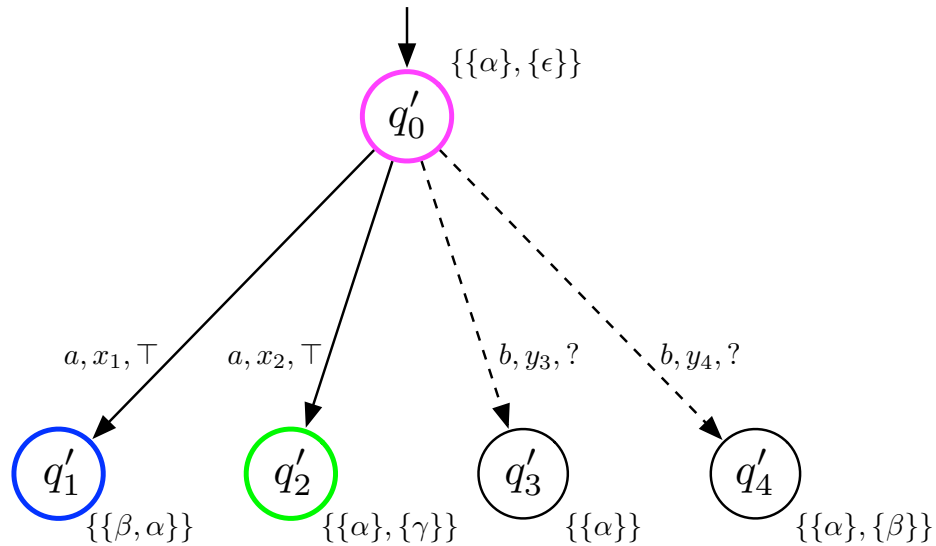
$$\varphi_x \equiv (x_1 = 1 \vee x_2 = 1) \wedge x_1 + x_2 = 1$$
$$\varphi_y \equiv y_3 \geq 0.1 \wedge y_3 + y_4 = 1$$

Abstract Probabilistic Automata

Refinement: example



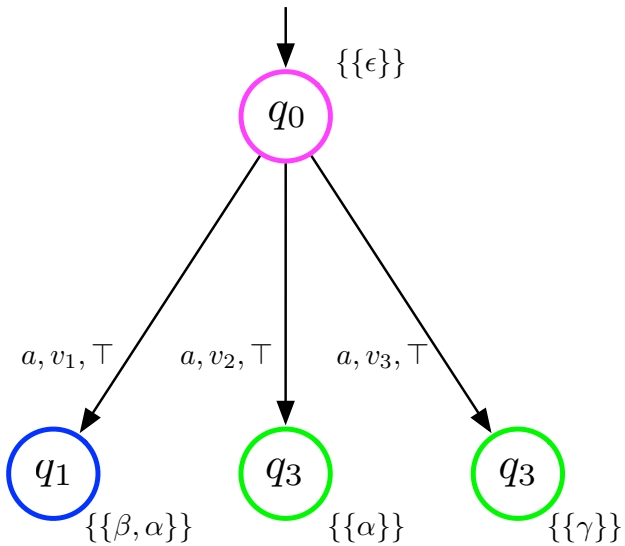
$$\varphi_v \equiv (v_1 = 1 \vee v_2 + v_3 = 1) \wedge v_1 + v_2 + v_3 = 1$$



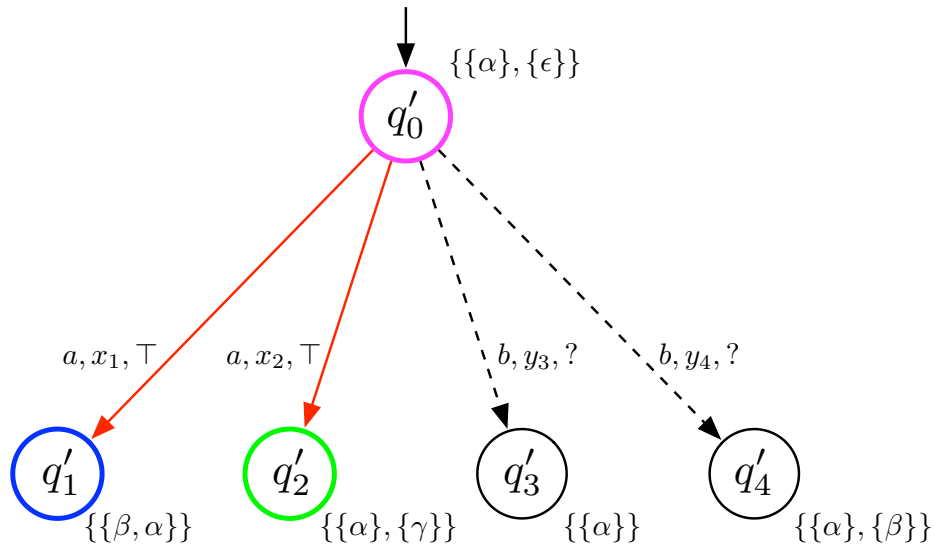
$$\varphi_x \equiv (x_1 = 1 \vee x_2 = 1) \wedge x_1 + x_2 = 1$$
$$\varphi_y \equiv y_3 \geq 0.1 \wedge y_3 + y_4 = 1$$

Abstract Probabilistic Automata

Refinement: example



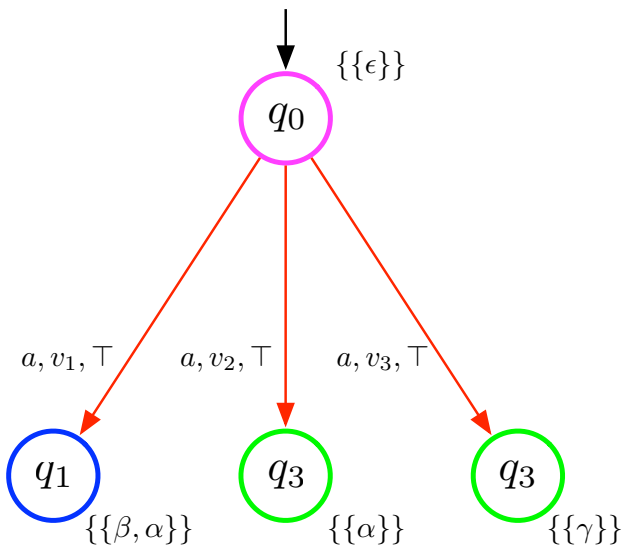
$$\varphi_v \equiv (v_1 = 1 \vee v_2 + v_3 = 1) \wedge v_1 + v_2 + v_3 = 1$$



$$\varphi_x \equiv (x_1 = 1 \vee x_2 = 1) \wedge x_1 + x_2 = 1$$
$$\varphi_y \equiv y_3 \geq 0.1 \wedge y_3 + y_4 = 1$$

Abstract Probabilistic Automata

Refinement: example



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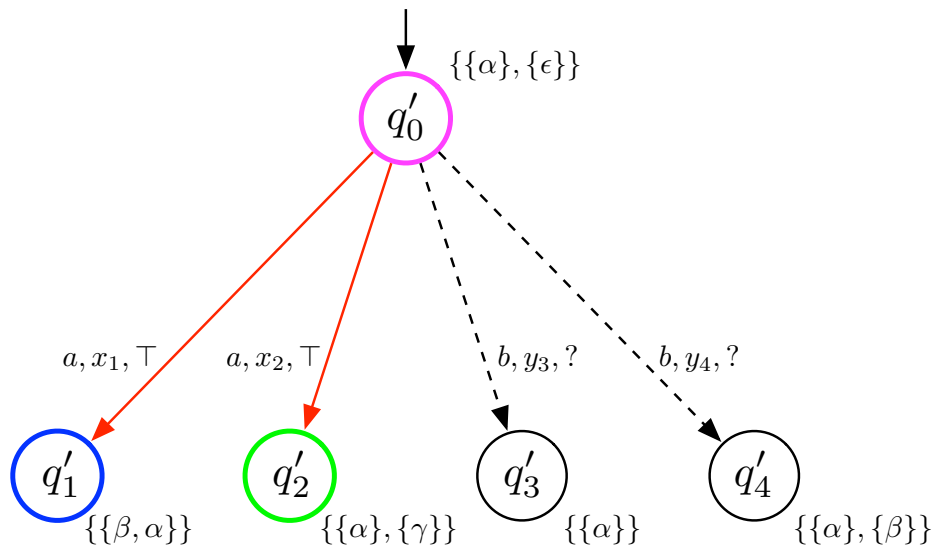

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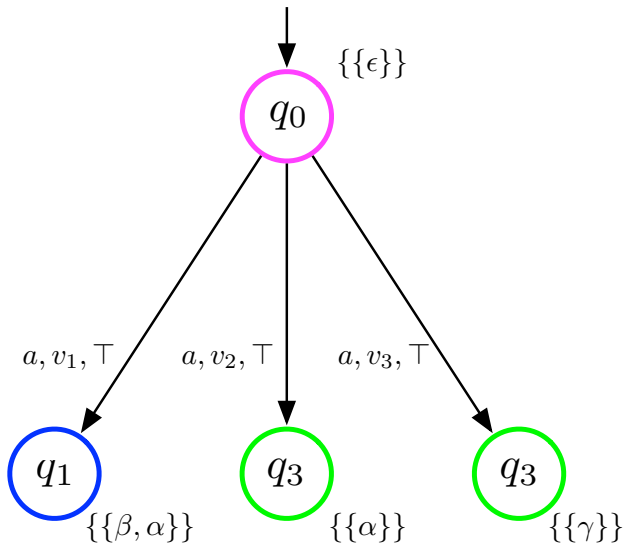
Abstract Probabilistic Automata

Refinement: example



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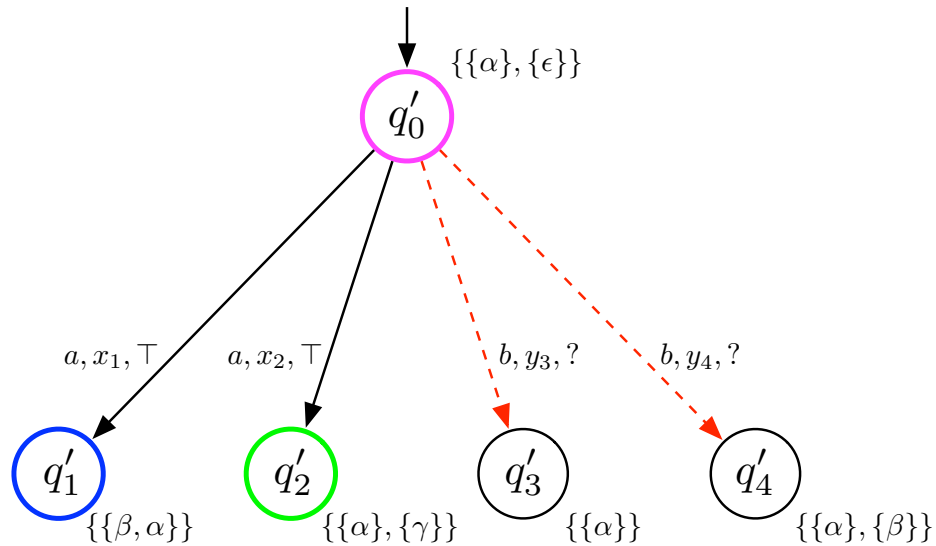

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Abstract Probabilistic Automata

Refinement relations

- Weak refinement $\preceq$
- Strong refinement $\preceq_S$
- Thorough refinement $\preceq_T$
- Weak weak refinement $\preceq_W$

Abstract Probabilistic Automata

Refinement relations

- Weak refinement $\preceq$
- Strong refinement $\preceq_S$
- Thorough refinement $\preceq_T$
- Weak weak refinement $\preceq_W$
  - $L'(s', a, \varphi') = \top \Rightarrow \exists \varphi \in C(S), L(s, a, \varphi) = \top$ and $\forall \mu \in Sat(\varphi) \exists \delta : S \to (S' \to [0,1]), \mu \lhd^{\delta}_{\mathcal{R}} \varphi'$
  - $L(s, a, \varphi) \neq \bot \Rightarrow \forall \mu \in Sat(\varphi) \exists \varphi' \in C(S'), L'(s', a, \varphi') \neq \bot$ and $\exists \delta : S \to (S' \to [0,1]), \mu \lhd^{\delta}_{\mathcal{R}} \varphi'$

Abstract Probabilistic Automata

Results

Theorem

Let N_1 and N_2 be APAs. We have that

$$N_1 \preceq_S N_2 \Rightarrow N_1 \preceq N_2 \Rightarrow N_1 \preceq_W N_2 \Rightarrow N_1 \preceq_T N_2,$$

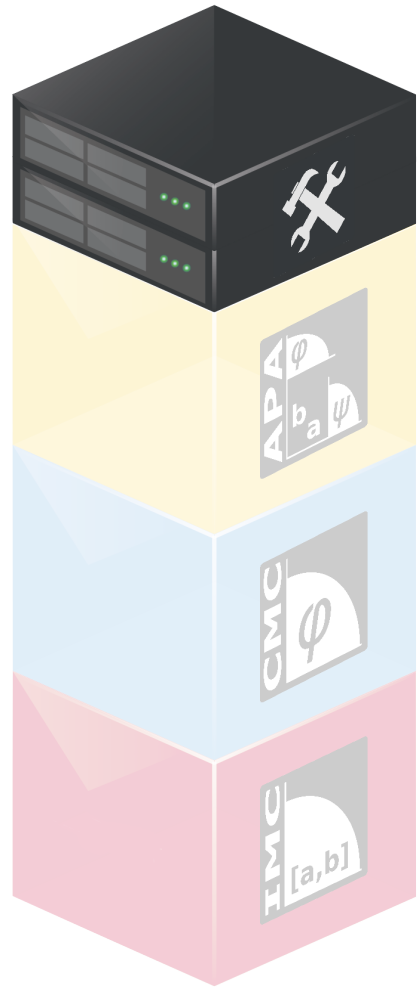
and under determinism and single valuations in initial states,

$$N_1 \preceq_S N_2 \Leftrightarrow N_1 \preceq N_2 \Leftrightarrow N_1 \preceq_W N_2 \Leftrightarrow N_1 \preceq_T N_2.$$

Conclusion – APA

	IMC [a,b]	CMC φ	APA φ b a ψ
Satisfaction	✓	✓	✓
Refinement	✓	✓	✓
Consistency	✓	✓	✓
Pruning	✓	✓	✓
Normalization		✓	✓
Constraint-abstraction		✓	✓
State-abstraction		✓	✓
Conjunction		✓	✓
Parallel Composition		✓	✓

APAC



Paper C

Paper E

Paper F (**QEST'11**)

APAC

Features

- Language for specifying CMCs and APAs
- Written in C# and uses the SMT solver Z3

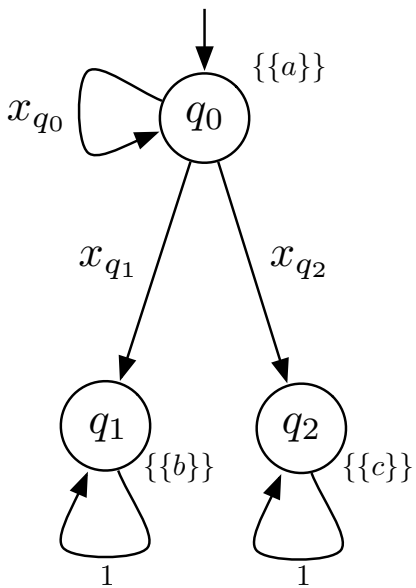
APAC

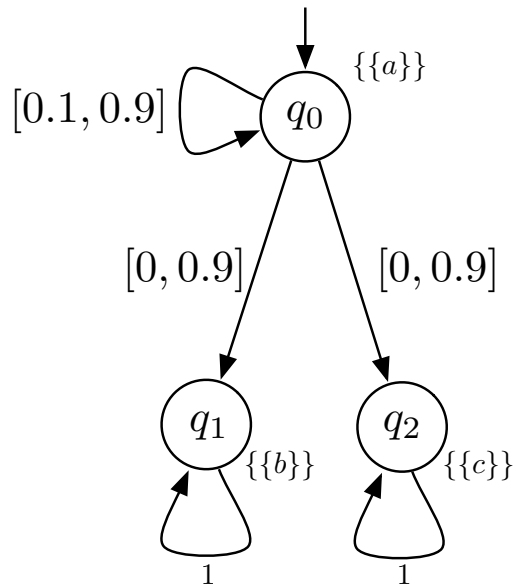
Z3

- Simplifies expressions by e.g. quantifier elimination
- Example: $\exists x : ax^2 + bx + c = 0 \rightsquigarrow b^2 - 4ac \geq 0$
  - Model instantiation
- Solving equations
- Z3 supports linear arithmetic for reals

APAC

- Constraint abstraction





$$\varphi(q_0)(x) \equiv 0.1 \leq x_{q_0} \leq 0.2 \vee 0.7 \leq x_{q_0} \leq 0.9$$

APAC

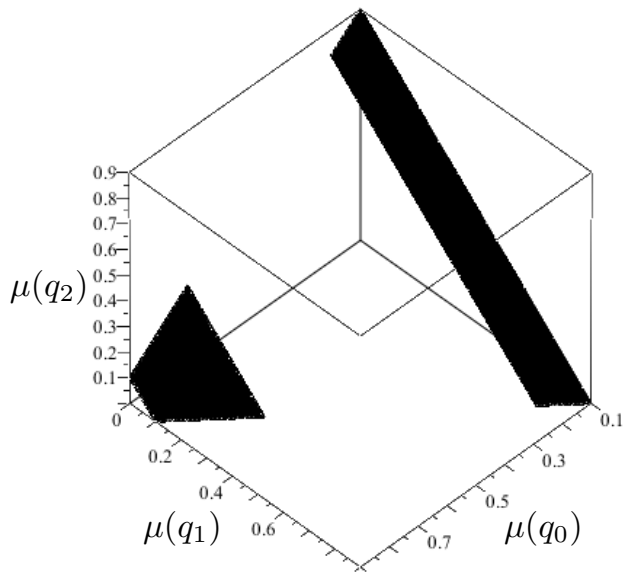
Constraint abstraction

Let $u \in Q_1$.

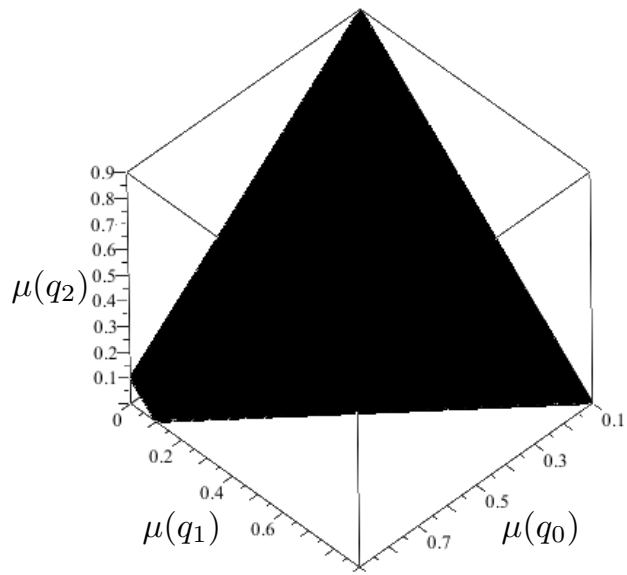
$$
\begin{aligned}
&\forall v \in Q_1 : (0 \leq L_v \leq U_v \leq 1) \wedge \\
&(\forall x \in \mathit{Dist}(Q_1) : \varphi(u)(x) \Rightarrow \forall v \in Q_1 : L_v \leq x(v) \leq U_v) \wedge \\
&\forall (L'_1, U'_1, \ldots, L'_k, U'_k) \in [0,1]^{|Q_1|} : \\
&\quad \Big[((\forall x \in \mathit{Dist}(Q_1) : \varphi(u)(x) \Rightarrow \forall v \in Q_1 : L'_v \leq x(v) \leq U'_v) \wedge \\
&\quad (\forall v \in Q_1 : 0 \leq L'_v \leq U'_v \leq 1)) \Rightarrow (\forall v \in Q_1 : L'_v \leq L_v \wedge U_v \leq U'_v) \Big].
\end{aligned}
$$

APAC

Constraint abstraction



$$\varphi(q_0)(x) \equiv 0.1 \leq x_{q_0} \leq 0.2 \vee 0.7 \leq x_{q_0} \leq 0.9$$



$$\varphi(q_0)(q_0) = [0.1, 0.9]$$
$$\varphi(q_0)(q_1) = [0, 0.9]$$
$$\varphi(q_0)(q_2) = [0, 0.9]$$

Conclusion

	IMC [a,b]	CMC φ	APA φ b a ψ
Satisfaction	✓	✓	✓
Refinement	✓	✓	✓
Consistency	✓	✓	✓
Pruning	✓	✓	✓
Normalization		✓	✓
Constraint-abstraction		✓	✓
State-abstraction		✓	✓
Conjunction		✓	✓
Parallel Composition		✓	✓

Conclusion

	IMC [a,b]	CMC φ	APA φ b a ψ
Satisfaction	✓	🛠	🛠
Refinement	✓	🛠	🛠
Consistency	✓	🛠	🛠
Pruning	✓	🛠	🛠
Normalization		🛠	🛠
Constraint-abstraction		🛠	🛠
State-abstraction		🛠	🛠
Conjunction		🛠	🛠
Parallel Composition		✓	✓

Conclusion

Recent, current and future work

- New notions of satisfaction and refinement:
  - Stuttering satisfaction
- A modal logic for APAs
- A generalisation of counter-example generation
- Improvements to APAC (+GUI, +internet-based execution)

Conclusion

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- New notions of satisfaction and refinement:
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