A Framework for Relating Timed Transition Systems and Preserving TCTL Model Checking

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Abstract. Many formal translations between time dependent models have been proposed over the years. While some of them produce timed bisimilar models, others preserve only reachability or (weak) trace equivalence. We suggest a general framework for arguing when a translation preserves Timed Computation Tree Logic (TCTL) or its safety fragment. The framework works at the level of timed transition systems, making it independent of the modeling formalisms and applicable to many of the translations published in the literature. Finally, we present a novel translation from extended Timed-Arc Petri Nets to Networks of Timed Automata and using the framework argue that it preserves the full TCTL. The translation has been implemented in the verification tool TAPAAL.

1 Introduction

Time dependent formal models like Timed Automata (TA) [1], Time Petri Nets (TPN) [15] and Timed-Arc Petri Nets (TAPN) [6] have received a significant attention in the theory of embedded systems. While originally developed by different communities of researchers, there has recently been devoted considerable effort towards establishing formal relationships among the different models. To this end, several translations have been developed (see e.g. [5, 6, 8, 9, 10, 11, 14, 17] or [16, 19] for a more complete overview) and some of them have been implemented in verification tools like Romeo [12], TAPAAL [9] or the TIOA Toolkit [2].

Many of these translations utilize similar tricks that allow for the simulation of one system by another. Typically, a single step in one formalism is simulated by a sequence of steps in the other. We identify a general class of translations that preserve Timed Computation Tree Logic (TCTL) (see e.g. [16]), a logic suitable for practical specification of many useful temporal properties. Our main goal is to provide a framework directly applicable to e.g. tool developers. The theory was motivated by the translations presented in [9] and [10]. Unlike much work on TCTL where only infinite alternating runs are considered [16] or the details are simply not discussed [7, 10], we consider also finite maximal runs that appear in the presence of stuck computations or time invariants (strict or nonstrict) and treat the semantics in its full generality as used in some state-of-the-art verification tools like UPPAAL [3]. This is particularly important for liveness properties. While some translations in the literature preserve some variant of timed bisimilarity [8, 10, 11, 14], other translations preserve only reachability or trace equivalence [4, 9]. Our framework allows us to argue that several such translations preserve the full TCTL or at least its safety fragment. In this paper we focus only on the interleaving semantics.

To illustrate the applicability of the framework, we propose a novel, full TCTL-preserving translation from extended timed-arc Petri nets to UPPAAL networks of timed automata. Earlier translations either caused exponential blow-up in the size [8, 17, 18], preserved only safety properties [9], or where not suitable for implementation in tools due to an inefficient use of clocks and communication primitives [18]. The translation from TAPN to UPPAAL timed automata presented in this paper is the first to run in polynomial time while preserving the full TCTL. We implemented the translation in the tool TAPAAL [9] and the initial experiments confirm its efficiency also in practice.

Full version of this paper with complete proofs can be found in [13].

2 Preliminaries

We let $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{R}$ and $\mathbb{R}_{\geq 0}$ denote the sets of natural numbers, non-negative integers, real numbers and non-negative real numbers, respectively. A *timed transition system* (TTS) is a quadruple $T = (S, \longrightarrow, \mathcal{AP}, \mu)$ where S is a set of states (or processes), $\longrightarrow \subseteq S \times S \cup S \times \mathbb{R}_{\geq 0} \times S$ is a transition relation, $\mathcal{AP}$ is a set of atomic propositions, and $\mu : S \longrightarrow 2^{\mathcal{AP}}$ is a function assigning sets of true atomic propositions to states.

We write $s \longrightarrow s'$ whenever $(s, s') \in \longrightarrow$ and call them *discrete transitions*, and $s \xrightarrow{d} s'$ whenever $(s, d, s') \in \longrightarrow$ and call them *delay transitions*. We implicitly assume the standard axioms of time additivity, time continuity, zero delay and time determinism (see e.g [5] or [13]). By $s[d]$ we denote the state s' (if it exists) such that $s \xrightarrow{d} s'$ (time determinism ensures the uniqueness of $s[d]$). We write $s \longrightarrow$ if $s \longrightarrow s'$ for some $s' \in S$ and $s \not\longrightarrow$ otherwise. Similarly for $\xrightarrow{d}$. A *run* $\rho = s_0 \xrightarrow{d_0} s_0[d_0] \longrightarrow s_1 \xrightarrow{d_1} s_1[d_1] \longrightarrow s_2 \xrightarrow{d_2} \ldots$ is a (finite or infinite) alternating sequence of time delays and discrete actions.

The set of time intervals $\mathcal{I}$ is defined by the abstract syntax

$$I ::= [a, a] \mid [a, b] \mid [a, b) \mid (a, b] \mid (a, b) \mid [a, \infty) \mid (a, \infty)$$

where $a \in \mathbb{N}_0, b \in \mathbb{N}$ and $a < b$.

We shall now introduce the syntax and semantics of Timed Computation Tree Logic (TCTL). The presentation is inspired by [16]. Let $\mathcal{AP}$ be a set of *atomic propositions*. The set of TCTL formulae $\Phi(\mathcal{AP})$ over $\mathcal{AP}$ is given by

$$\varphi ::= \wp \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid E(\varphi_1\, U_I\, \varphi_2) \mid A(\varphi_1\, U_I\, \varphi_2) \mid E(\varphi_1\, R_I\, \varphi_2) \mid A(\varphi_1\, R_I\, \varphi_2)$$

where $\wp \in \mathcal{AP}$ and $I \in \mathcal{I}$. Formulae without any occurrence of the operators $A(\varphi_1\, U_I\, \varphi_2)$ and $E(\varphi_1\, R_I\, \varphi_2)$ form the *safety fragment* of TCTL.

The intuition of the until and release TCTL operators (formalized later on) is as follows: $E(\varphi_1\, U_I\, \varphi_2)$ is true if there exists a maximal run such that φ_2

eventually holds within the interval I, and until it does, φ_1 continuously holds; $E(\varphi_1\, R_I\, \varphi_2)$ is true if there exists a maximal run such that either φ_2 always holds within the interval I or φ_1 occurred previously. As we aim to apply our framework to concrete case studies with possible tool support, we need to handle maximal runs in their full generality. Hence we have to consider all possibilities in which a run can be "stuck". In this case, we annotate the last transition of such a run with one of the three special ending symbols (denoted δ in the definition below).

A *maximal run* ρ is either

(i) an infinite alternating sequence of the form $\rho = s_0 \xrightarrow{d_0} s_0[d_0] \longrightarrow s_1 \xrightarrow{d_1} s_1[d_1] \longrightarrow s_2 \xrightarrow{d_2} s_2[d_2] \longrightarrow \ldots$, or
(ii) a finite alternating sequence of the form $\rho = s_0 \xrightarrow{d_0} s_0[d_0] \longrightarrow s_1 \xrightarrow{d_1} s_1[d_1] \longrightarrow \ldots \longrightarrow s_n \xrightarrow{\delta}$ where $\delta \in \{\infty, d_n^{\leq}, d_n^{<}\}$ for some $d_n \in \mathbb{R}_{\geq 0}$ s.t.
   - if $\delta = \infty$ then $s_n \xrightarrow{d} s_n[d]$ for all $d \in \mathbb{R}_{\geq 0}$,
   - if $\delta = d_n^{\leq}$ then $s_n \overset{d}{\not\longrightarrow}$ for all $d > d_n$ and $s_n \xrightarrow{d_n} s_n[d_n]$ s.t. $s_n[d_n] \not\longrightarrow$, and
   - if $\delta = d_n^{<}$ then $s_n \overset{d}{\not\longrightarrow}$ for all $d \geq d_n$, and there exists d_s, $0 \leq d_s < d_n$, such that for all d, $d_s \leq d < d_n$, we have $s_n \xrightarrow{d} s_n[d]$ and $s_n[d] \not\longrightarrow$.

By $\mathit{MaxRuns}(T, s)$ we denote the set of maximal runs in a TTS T starting at s.

Intuitively, the three conditions in case (ii) describe all possible ways in which a finite run can terminate. First, a run can end in a state where the time diverges. The other two cases define a run which ends in a state from which no discrete transition is allowed after some time delay, but time cannot diverge either (typically caused by the presence of invariants in the model). These cases differ in whether the bound on the maximal time delay can be reached or not.

Let us now introduce some notation for a given maximal run $\rho = s_0 \xrightarrow{d_0} s_0[d_0] \longrightarrow s_1 \xrightarrow{d_1} s_1[d_1] \longrightarrow s_2 \xrightarrow{d_2} \ldots$. First, $r(i, d)$ denotes the total time elapsed from the beginning of the run up to some delay $d \in \mathbb{R}_{\geq 0}$ after the i'th discrete transition. Formally, $r(i, d) = \left(\sum_{j=0}^{i-1} d_j\right) + d$. Second, we define a predicate $\mathit{valid}_\rho : \mathbb{N} \times \mathbb{R}_{\geq 0} \times \mathcal{I} \to \{\mathit{true}, \mathit{false}\}$ such that $\mathit{valid}_\rho(i, d, I)$ checks whether the total time for reaching the state $s_i[d]$ in ρ belongs to the time interval I, formally

$$\mathit{valid}_\rho(i, d, I) = \begin{cases} d \leq d_i \wedge r(i, d) \in I & \text{if } d_i \in \mathbb{R}_{\geq 0} \\ r(i, d) \in I & \text{if } d_i = \infty \\ d \leq d_n \wedge r(i, d) \in I & \text{if } d_i = d_n^{\leq} \\ d < d_n \wedge r(i, d) \in I & \text{if } d_i = d_n^{<}\ . \end{cases}$$

Third, we define a function $\mathit{history}_\rho : \mathbb{N} \times \mathbb{R}_{\geq 0} \to 2^{\mathbb{N} \times \mathbb{R}_{\geq 0}}$ s.t. $\mathit{history}_\rho(i, d)$ returns the set of pairs (j, d') that constitute all states $s_j[d']$ in ρ preceding $s_i[d]$, formally $\mathit{history}_\rho(i, d) = \{(j, d') \mid 0 \leq j < i \wedge 0 \leq d' \leq d_j\} \cup \{(i, d') \mid 0 \leq d' < d\}$.

Now we can define the satisfaction relation $s \models \varphi$ for a state $s \in S$ in a TTS $T = (S, \longrightarrow, \mathcal{AP}, \mu)$ and a TCTL formula φ.

$$
\begin{array}{ll}
s \models \wp & \text{iff } \wp \in \mu(s) \\
s \models \neg\varphi & \text{iff } s \not\models \varphi \\
s \models \varphi_1 \wedge \varphi_2 & \text{iff } s \models \varphi_1 \text{ and } s \models \varphi_2 \\
s \models E(\varphi_1 \, U_I \, \varphi_2) & \text{iff } \exists \rho \in \mathit{MaxRuns}(T, s) \, . \\
& \quad \exists i \geq 0 \, . \, \exists d \in \mathbb{R}_{\geq 0} \, . \, \big[\mathit{valid}_\rho(i, d, I) \wedge s_i[d] \models \varphi_2 \wedge \\
& \quad \forall (j, d') \in \mathit{history}_\rho(i, d) \, . \, s_j[d'] \models \varphi_1\big] \\
s \models E(\varphi_1 \, R_I \, \varphi_2) & \text{iff } \exists \rho \in \mathit{MaxRuns}(T, s) \, . \\
& \quad \forall i \geq 0 \, . \, \forall d \in \mathbb{R}_{\geq 0} \, . \, \mathit{valid}_\rho(i, d, I) \Rightarrow \\
& \quad \big[s_i[d] \models \varphi_2 \vee \exists (j, d') \in \mathit{history}_\rho(i, d) \, . \, s_j[d'] \models \varphi_1\big]
\end{array}
$$

The operators $A(\varphi_1 \, U_I \, \varphi_2)$ and $A(\varphi_1 \, R_I \, \varphi_2)$ are defined analogously by replacing the quantification $\exists \rho \in \mathit{MaxRuns}(T, s)$ with $\forall \rho \in \mathit{MaxRuns}(T, s)$.

As expected, the until and release operators are dual [13].

Lemma 1. *Let $T = (S, \longrightarrow, \mathcal{AP}, \mu)$ be a TTS and $s \in S$. Then $s \models A(\varphi_1 \, R_I \, \varphi_2)$ iff $s \models \neg E(\neg\varphi_1 \, U_I \, \neg\varphi_2)$, and $s \models A(\varphi_1 \, U_I \, \varphi_2)$ iff $s \models \neg E(\neg\varphi_1 \, R_I \, \neg\varphi_2)$.*

3 Framework Description

In this section, we shall present a general framework for arguing when a simulation of one time dependent system by another preserves satisfiability of TCTL formulae. We define the notion of one-by-many correspondence, a relation between two TTSs A and B. If A is in one-by-many correspondence with B then every transition in A can be simulated by a sequence of transitions in B. Further, every TCTL formula φ can be algorithmically translated into a formulate $tr(\varphi)$ s.t. $A \models \varphi$ iff $B \models tr(\varphi)$. In the rest of this section, we shall use A and B to refer to the original and the translated system, respectively.

3.1 One-By-Many Correspondence

As the system B is simulating a single transition of A by a sequence of transitions, the systems A and B are comparable only in the states before and after this sequence was performed. We say that B is *stable* in such states and introduce a fresh atomic proposition called *stable* to explicitly identify this situation. States that do not satisfy the proposition *stable* are called *intermediate states*. We now define three conditions that B should possess in order to apply to our framework. The third condition is optional and necessary only for the preservation of liveness TCTL properties. A TTS $(S, \rightarrow, \mathcal{AP}, \mu)$ s.t. $\mathit{stable} \in \mathcal{AP}$ is

– *delay-implies-stable* if for any $s \in S$, it holds that $s \xrightarrow{d}$ for some $d > 0$ implies $s \models \mathit{stable}$,

- *delay-preserves-stable* if for any $s \in S$ such that $s \models \mathit{stable}$, if $s \xrightarrow{d} s[d]$ then $s[d] \models \mathit{stable}$ for all $d \in \mathbb{R}_{\geq 0}$, and
- *eventually-stable* if for any $s_0 \in S$ such that $s_0 \models \mathit{stable}$ and for any infinite sequence of discrete transitions $\rho = s_0 \longrightarrow s_1 \longrightarrow s_2 \longrightarrow s_3 \longrightarrow s_4 \longrightarrow \dots$ or any finite nonempty sequence of discrete transitions $\rho = s_0 \longrightarrow s_1 \longrightarrow \cdots \longrightarrow s_n \not\longrightarrow$ there exists an index $i \geq 1$ such that $s_i \models \mathit{stable}$. We call such a sequence a *maximal discrete sequence*.

We write $s \rightsquigarrow s'$ if there is an alternating sequence $s = s_0 \longrightarrow s_1 \xrightarrow{0} s_1 \longrightarrow s_2 \xrightarrow{0} s_2 \longrightarrow \cdots \xrightarrow{0} s_{n-1} \longrightarrow s_n = s'$ such that $s \models \mathit{stable}$, $s' \models \mathit{stable}$, and $s_j \not\models \mathit{stable}$ for $1 \leq j \leq n-1$.

Remark 1. For technical convenience, we introduced zero delays in the definition of $\rightsquigarrow$ in order to preserve the alternating nature of the sequence. Note that this is not restrictive as for any $s \in S$ we always have $s \xrightarrow{0} s$.

Definition 1. *Let $A = (S, \rightarrow_A, \mathcal{AP}_A, \mu_A)$ and $B = (T, \rightarrow_B, \mathcal{AP}_B, \mu_B)$ be two TTSs s.t. $\mathit{stable} \in \mathcal{AP}_B$ and B is* delay-implies-stable *and* delay-preserves-stable *TTS. A relation $\mathcal{R} \subseteq S \times T$ is a* one-by-many correspondence *if there exists a function $tr_p : \mathcal{AP}_A \longrightarrow \mathcal{AP}_B$ such that whenever $s \,\mathcal{R}\, t$ then*

1. *$t \models \mathit{stable}$,*
2. *$s \models \wp$ iff $t \models tr_p(\wp)$ for all $\wp \in \mathcal{AP}_A$,*
3. *if $s \longrightarrow s'$ then $t \rightsquigarrow t'$ and $s' \,\mathcal{R}\, t'$,*
4. *if $s \xrightarrow{d} s[d]$ then $t \xrightarrow{d} t[d]$ and $s[d] \,\mathcal{R}\, t[d]$ for all $d \in \mathbb{R}_{\geq 0}$,*
5. *if $t \rightsquigarrow t'$ then $s \longrightarrow s'$ and $s' \,\mathcal{R}\, t'$ and*
6. *if $t \xrightarrow{d} t[d]$ then $s \xrightarrow{d} s[d]$ and $s[d] \,\mathcal{R}\, t[d]$ for all $d \in \mathbb{R}_{\geq 0}$.*

If B is moreover an eventually-stable *TTS, then we say that $\mathcal{R}$ is a* complete one-by-many correspondence. *We write $s \Rrightarrow t$ (resp. $s \Rrightarrow_c t$) if there exists a relation $\mathcal{R}$ which is a one-by-many correspondence (resp. a complete one-by-many correspondence) such that $s \,\mathcal{R}\, t$.*

Example 1. Consider the TTSs A, B and C in Figure 1 where the sets of propositions for A, B and C are $\mathcal{AP}_A = \{p, q\}$ and $\mathcal{AP}_B = \mathcal{AP}_C = \{p, q, \mathit{stable}\}$. Then $\{(s_0[d], t_0[d]) \mid 0 \leq d \leq 4.4\} \cup \{(s_1, t_1), (s_2, t_4), (s_3, t_6), (s_2, t_7)\}$ is a complete one-by-many correspondence which implies that $s_0 \Rrightarrow_c t_0$ and $\{(s_0[d], u_0[d]) \mid 0 \leq d \leq 4.4\} \cup \{(s_1, u_1), (s_2, u_4), (s_3, u_7)\}$ is a one-by-many correspondence which implies that $s_0 \Rrightarrow u_0$. Notice that the system C is not *eventually-stable* since the two maximal discrete sequences $u_1 \longrightarrow u_5 \longrightarrow u_6 \longrightarrow u_6 \longrightarrow u_6 \longrightarrow u_6 \longrightarrow \cdots$ and $u_1 \longrightarrow u_2 \longrightarrow u_3$ do not contain any stable states.

Consider now the maximal run $\rho = s_0 \xrightarrow{4.4} s_1 \longrightarrow s_2 \xrightarrow{0^{\leq}}$ in the system A. This run witnesses that $s_0 \models E(\neg q \, R_{[3,5]} \, q)$. Similarly, the maximal run $\rho' = t_0 \xrightarrow{4.4} t_1 \rightsquigarrow t_4 \xrightarrow{0^{\leq}}$ witnesses that $t_0 \models E((\neg q \wedge \mathit{stable}) \, R_{[3,5]} \, (q \vee \neg \mathit{stable}))$. $\square$

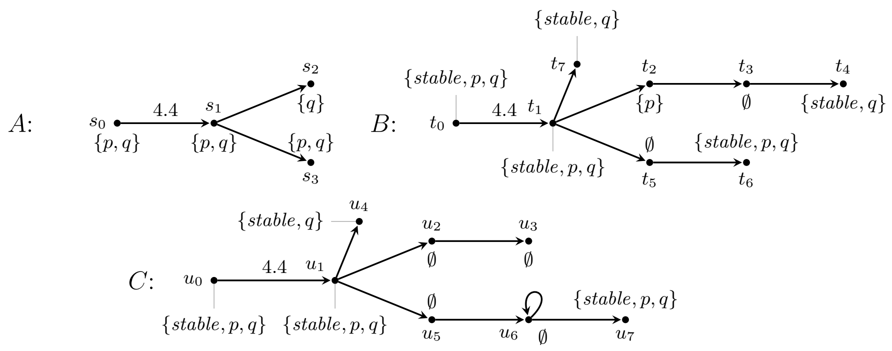


Fig. 1. Three TTSs such that $s_0 \mathrel{\underline{\rightrightarrows}}_c t_0$ and $s_0 \mathrel{\underline{\rightrightarrows}} u_0$.

Now we translate TCTL formulae. Let $\mathcal{AP}_A$ and $\mathcal{AP}_B$ be sets of atomic propositions such that $stable \in \mathcal{AP}_B$ and let $tr_p : \mathcal{AP}_A \longrightarrow \mathcal{AP}_B$ be a function translating atomic propositions. We define $tr : \Phi(\mathcal{AP}_A) \to \Phi(\mathcal{AP}_B)$ as follows:

$$\begin{aligned}
tr(\wp) &= tr_p(\wp)\\
tr(\neg\varphi_1) &= \neg tr(\varphi_1)\\
tr(\varphi_1 \wedge \varphi_2) &= tr(\varphi_1) \wedge tr(\varphi_2)\\
tr(E(\varphi_1\, U_I\, \varphi_2)) &= E((tr(\varphi_1) \vee \neg stable)\, U_I\, (tr(\varphi_2) \wedge stable))\\
tr(A(\varphi_1\, U_I\, \varphi_2)) &= A((tr(\varphi_1) \vee \neg stable)\, U_I\, (tr(\varphi_2) \wedge stable))\\
tr(E(\varphi_1\, R_I\, \varphi_2)) &= E((tr(\varphi_1) \wedge stable)\, R_I\, (tr(\varphi_2) \vee \neg stable))\\
tr(A(\varphi_1\, R_I\, \varphi_2)) &= A((tr(\varphi_1) \wedge stable)\, R_I\, (tr(\varphi_2) \vee \neg stable))
\end{aligned}$$

We are now ready to state the main result of this section.

Theorem 1. *Let $A = (S, \to_A, \mathcal{AP}_A, \mu_A)$ and $B = (T, \to_B, \mathcal{AP}_B, \mu_B)$ be two TTSs such that $stable \in \mathcal{AP}_B$ and let $s_0 \in S$ and $t_0 \in T$. If $s_0 \mathrel{\underline{\rightrightarrows}}_c t_0$ then for any TCTL formula φ, $s_0 \models \varphi$ if and only if $t_0 \models tr(\varphi)$. If $s_0 \mathrel{\underline{\rightrightarrows}} t_0$ then the claim holds only for any formula φ from the safety fragment of TCTL.*

Proof (Sketch). The proof is by structural induction on φ and relies on the fact that for every maximal run in A there is a maximal run in B s.t. they are related by $\mathrel{\underline{\rightrightarrows}}$ in all stable states of B. In the opposite direction, every maximal run in B has a corresponding maximal run in A, provided that the correspondence relation is complete. The main technical complication is that we handle the maximal runs in their full generality (see [13] for details). $\square$

3.2 Overall Methodology

We finish this section by recalling the steps needed in order to apply the framework to a particular translation between two time-dependent systems. Assume that we designed an algorithm that for a given system A constructs a system B together with the notion of stable states in the system B.

1. Show that B is a *delay-implies-stable* and *delay-preserves-stable* TTS (and optionally an *eventually-stable* TTS).
2. Define a proposition translation function $tr_p : \mathcal{AP}_A \longrightarrow \mathcal{AP}_B$.
3. Define a relation $\mathcal{R}$ and show that it fulfills conditions 1–6 of Definition 1.

Theorem 1 now allows us to conclude that the translation preserves the full TCTL (or its safety fragment if $\mathcal{R}$ is only a one-by-many correspondence).

4 Translation from Bounded TAPN to NTA

This section describes a translation from extended timed-arc Petri nets to networks of timed automata (NTA). We start with the definitions of the models.

4.1 Extended Timed-Arc Petri Nets

We shall now define timed-arc Petri nets with invariants, inhibitor arcs and transport arcs. Recall the set of time intervals $\mathcal{I}$ defined in Section 2. The predicates $r \in I$ for $I \in \mathcal{I}$ and $r \in \mathbb{R}_{\geq 0}$ are defined in the expected way. By $\mathcal{I}_{\text{Inv}}$ we denote a subset of $\mathcal{I}$ of intervals containing 0 and call them *invariant intervals.*

Definition 2. *A* timed-arc Petri net with invariants, inhibitor arcs and transport arcs *(TAPN) is a tuple* $N = (P, T, F, c, F_{tarc}, c_{tarc}, F_{inhib}, c_{inhib}, \iota)$ *where*

- P *is a finite set of* places*,*
- T *is a finite set of* transitions *such that* $P \cap T = \emptyset$*,*
- $F \subseteq (P \times T) \cup (T \times P)$ *is a set of* normal arcs*,*
- $c : F|_{P \times T} \longrightarrow \mathcal{I}$ *assigns time intervals to arcs from places to transitions,*
- $F_{tarc} \subseteq (P \times T \times P)$ *is a set of* transport arcs *that satisfy for all* $(p, t, p') \in F_{tarc}$ *and all* $r \in P$*:* $(p, t, r) \in F_{tarc} \Rightarrow p' = r$*,* $(r, t, p') \in F_{tarc} \Rightarrow p = r$*,* $(p, t) \notin F$*, and* $(t, p') \notin F$*,*
- $c_{tarc} : F_{tarc} \longrightarrow \mathcal{I}$ *is a function assigning time intervals to transport arcs,*
- $F_{inhib} \subseteq P \times T$ *is a set of* inhibitor arcs *satisfying for all* $(p, t) \in F_{inhib}$ *and all* $p' \in P$*:* $(p, t) \notin F$ *and* $(p, t, p') \notin F_{tarc}$*,*
- $c_{inhib} : F_{inhib} \longrightarrow \mathcal{I}$ *assigns time intervals to inhibitor arcs, and*
- $\iota : P \longrightarrow \mathcal{I}_{inv}$ *is a function assigning* invariants *to places.*

The *preset* of $t \in T$ is defined as ${}^\bullet t = \{p \in P \mid (p, t) \in F \vee \exists p' \in P \,.\, (p, t, p') \in F_{tarc}\}$ and the *postset* of t is $t^\bullet = \{p \in P \mid (t, p) \in F \vee \exists p' \in P \,.\, (p', t, p) \in F_{tarc}\}$.

A *marking* on a TAPN N is a function $M : P \longrightarrow \mathcal{B}(\mathbb{R}_{\geq 0})$, where $\mathcal{B}(\mathbb{R}_{\geq 0})$ is the set of finite multisets of non-negative real numbers s.t. for every place $p \in P$ and every token $x \in M(p)$ it holds that $x \in \iota(p)$. The set of all markings on N is denoted by $\mathcal{M}(N)$. Note that in TAPN each token has its own age. A *marked* TAPN is a pair (N, M_0) where N is a TAPN and M_0 is an *initial marking* on N with all tokens of age 0. A transition $t \in T$ is *enabled* in marking M if

- for all $p \in {}^\bullet t$ s.t. $(p, t) \in F$ there is a token x of an age in the time interval on the arc from p to t: $\forall p \in {}^\bullet t$ s.t. $(p, t) \in F \,.\, \exists x \in M(p) \,.\, x \in c(p, t)$,

– for all $p \in {}^\bullet t$ s.t. $(p, t, p') \in F_{tarc}$ the age of the token x in p satisfies the invariant at p': $\forall p \in {}^\bullet t$ s.t. $(p, t, p') \in F_{tarc}$. $\exists x \in M(p)$. $x \in c_{tarc}(p, t, p') \wedge x \in \iota(p')$,
– for all $p \in P$ s.t. $(p, t) \in F_{inhib}$ there is no token with age in the interval on the inhibitor arc: $\forall p \in P$ s.t. $(p, t) \in F_{inhib}$. $\neg\exists x \in M(p)$. $x \in c_{inhib}(p, t)$.

Definition 3 (Firing Rule). *If t is enabled in a marking M then it can be* fired *producing a marking M' defined as $M'(p) = \big(M(p) \setminus C_t^-(p)\big) \cup C_t^+(p)$ for all $p \in P$ where*

– *for every $p \in P$ such that $(p, t) \in F$*
  $C_t^-(p) = \{x\}$ where $x \in M(p)$ and $x \in c(p, t)$,
– *for every $p \in P$ such that $(t, p) \in F$*
  $C_t^+(p) = \{0\}$, and
– *for every $p, p' \in P$ such that $(p, t, p') \in F_{tarc}$*
  $C_t^-(p) = \{x\} = C_t^+(p')$ where $x \in M(p)$, $x \in c_{tarc}(p, t, p')$ and $x \in \iota(p')$, and
– *in all other cases we set the above sets to $\emptyset$.*

Note that there may be multiple choices for $C_t^-(p)$ and $C_t^+(p)$ and the minus and union operators are interpreted over multisets.

Definition 4 (Time Delay). *A* time delay *$d \in \mathbb{R}_{\geq 0}$ is allowed in a marking M if $(x + d) \in \iota(p)$ for all $p \in P$ and all $x \in M(p)$, i.e. by delaying d time units no tokens violate the invariants on places. By delaying d time units we reach a marking M' defined as $M'(p) = \{x + d \mid x \in M(p)\}$ for all $p \in P$.*

A TAPN $N = (P, T, F, c, F_{tarc}, c_{tarc}, F_{inhib}, c_{inhib}, \iota)$ generates a TTS $T(N) = (\mathcal{M}(N), \longrightarrow, \mathcal{AP}, \mu)$ where states are markings on N, $M \longrightarrow M'$ if by firing some transition t in marking M we reach the marking M', and $M \xrightarrow{d} M'$ if by delaying d time units in marking M we get to marking M'. The set of atomic propositions $\mathcal{AP}$ and the labeling function μ are defined as $\mathcal{AP} \overset{def}{=} \{(p \bowtie n) \mid p \in P, n \in \mathbb{N}_0$ and $\bowtie \in \{<, \leq, =, \geq, >\}\}$ and $\mu(M) \overset{def}{=} \{(p \bowtie n) \mid |M(p)| \bowtie n$ and $\bowtie \in \{<, \leq, =, \geq, >\}\}$. The idea here is that the proposition $(p \bowtie n)$ is true in a marking M if and only if the number of tokens in the place p satisfies the constraint with respect to n.

4.2 Networks of Timed Automata

We shall now introduce networks of timed automata in the UPPAAL style [3]. UPPAAL timed automata can perform handshake and broadcast communication and manipulate finite data structures. We define only those features that are needed for our translation, namely broadcast communication and integer variables (used only for counting). These features are only a syntactic sugar and the expressive power is identical to the timed automata model by Alur and Dill [1].

Let $C = \{c_1, c_2, \ldots\}$ be a finite set of real-valued *clocks*. A *clock constraint* (or guard) is a boolean expression defined by the abstract syntax: $g_1, g_2 ::= \mathit{true} \mid$

$c \bowtie n \mid g_1 \wedge g_2$ where $c \in C$, $n \in \mathbb{N}_0$ and $\bowtie \in \{\leq, <, ==, >, \geq\}$. For *invariant clock constraints*, we require $\bowtie \in \{\leq, <\}$. The set of all clock constraints and invariant clock constraints over C are denoted by $\mathcal{G}(C)$ and $\mathcal{G}_{inv}(C)$, respectively.

A (clock) *valuation* is a function $v : C \to \mathbb{R}_{\geq 0}$ that for every clock $c \in C$ returns the value of c. Let $d \in \mathbb{R}_{\geq 0}$. We define the valuation $(v+d)$ after delaying d time units by $(v + d)(c) = v(c) + d$ for every $c \in C$. Let $r \subseteq C$. We define the valuation $v[r]$ after the clocks in r are reset by $v[r](c) = 0$ if $c \in r$ and $v[r](c) = v(c)$ if $c \in C \setminus r$. The satisfaction relation $v \models g$ (i.e. when a valuation satisfies a guard g) is defined in the natural way.

We will now define the concept of integer variables. Let X be a finite set of *integer variables*. The set VE(X) of arithmetic expressions over X is given by the abstract syntax $expr ::= m \mid x{+}{+} \mid x{-}{-}$ where $m \in \mathbb{Z}$ and $x \in X$. The set VG(X) of *variable guards* is a boolean combination of the predicates $expr \bowtie expr$ where $expr \in VE(X)$ and $\bowtie \in \{<, \leq, ==, \geq, >\}$.

Variable *assignments* are expressions of the form $x := expr$ where $x \in X$ and $expr \in \mathrm{VE}(X)$. The set of all variables assignments over X is denoted by VA(X). A set $\mathcal{A} \subseteq \mathrm{VA}(X)$ of variable assignments is called *non-conflicting* if for every $x \in X$ whenever $(x := expr_1) \in \mathcal{A}$ and $(x := expr_2) \in \mathcal{A}$ then $expr_1 = expr_2$.

Finally, we will define a *variable valuation* as a total mapping $z : X \longrightarrow \mathbb{Z}$ that for a variable $x \in X$ returns its current value. This mapping is naturally extended to all variable expressions in VE(X). The satisfaction relation $z \models \phi$ is true if the variable guard $\phi \in$ VG(X) evaluates to true under the valuation z. Let $\mathcal{A}$ be a finite non-conflicting set of variable assignments and let z be a variable valuation. We define $z[\mathcal{A}]$ as a variable valuation updated with the assignments from $\mathcal{A}$ by $z[\mathcal{A}](x) = z(expr)$ if $(x := expr) \in \mathcal{A}$ and $z[\mathcal{A}](x) = z(x)$ otherwise.

We can now define the notion of a timed automaton.

Definition 5 (Timed Automaton). *A* timed automaton *is a tuple* $(L, \ell_0, Act, C, X, \longrightarrow, I_C, I_X)$ *where* L *is a finite set of* locations *and* $\ell_0 \in L$ *is the* initial location*,* Act *is a finite set of* actions*,* C *is a finite set of* clocks*,* X *is a finite set of* integer variables*,* $\longrightarrow \subseteq L \times \mathcal{G}(C) \times VG(X) \times Act \times 2^C \times 2^{VA(X)} \times L$ *is a finite set of* edges *s.t. whenever* $(\ell, g, \phi, a, r, \mathcal{A}, \ell') \in \longrightarrow$ *then* $\mathcal{A}$ *is finite and non-conflicting set of variables assignments,* $I_C : L \to \mathcal{G}_{inv}(C)$ *is a function assigning* clock invariants *to locations, and* $I_X : L \to VG(X)$ *is a function assigning* variable invariants *to locations.*

We write $\ell \xrightarrow{g,\phi,a,r,\mathcal{A}} \ell'$ instead of $(\ell, g, \phi, a, r, \mathcal{A}, \ell') \in \longrightarrow$, where ℓ is a source location, g is a clock guard, ϕ is a variable guard, a is an action, r is a set of clocks to be reset, $\mathcal{A}$ is a finite non-conflicting set of variable assignments and ℓ' is a target location.

We can now define a network (parallel composition) of timed automata communicating via broadcast. Let *Broad* be a finite set of *broadcast channel names* and let τ denote the *internal action* τ performed by a single component. The set of actions is $Act = \{a¡, a¿ \mid a \in Broad\} \cup \{\tau\}$. The intuition is that $a¡$ indicates initiation of broadcasting on a channel a and all automata where the action $a¿$ is enabled must participate in the broadcast communication.

Definition 6 (Network of Timed Automata). *Let $A_1, A_2, \ldots, A_n$ for some $n \in \mathbb{N}$ be timed automata over a fixed set of actions Act, clocks C and integer variables X such that $A_i = (L_i, Act, C, X, \longrightarrow_i, I_C^i, I_X^i, \ell_0^i)$ for all $1 \leq i \leq n$. A* network of timed automata *(NTA) is a parallel composition $A_1 \parallel A_2 \parallel \ldots \parallel A_n$.*

A *configuration* of an NTA is a tuple $(\ell_1, \ell_2, \ldots, \ell_n, z, v)$ where $\ell_i \in L_i$ for all $1 \leq i \leq n$, z is a variable valuation over X and v is a clock valuation over C such that for every i, $1 \leq i \leq n$, it holds that $z \models I_X^i(\ell_i)$ and $v \models I_C^i(\ell_i)$. The set of all configurations of a given NTA A is denoted by $Conf(A)$.

We can now define the precise semantics of networks of timed automata as TTSs. Let $A = A_1 \parallel A_2 \parallel \ldots \parallel A_n$, where $A_i = (L_i, Act, C, X, \longrightarrow_i, I_C^i, I_X^i, \ell_0^i)$ be an NTA. The TTS generated by A is $T(A) = (Conf(A), \longrightarrow, \mathcal{AP}, \mu)$ such that the transition relation consists of

- *Ordinary transitions:* $(\ell_1, \ldots, \ell_i, \ldots, \ell_n, z, v) \longrightarrow (\ell_1, \ldots, \ell_i', \ldots, \ell_n, z', v')$ if there is an edge $\ell_i \xrightarrow{g,\phi,\tau,r,\mathcal{A}}_i \ell_i'$ in the i'th automaton such that $v \models g$, $z \models \phi$, $v' = v[r]$, $z' = z[\mathcal{A}]$, $v' \models I_C^i(\ell_i') \wedge \bigwedge_{j \neq i} I_C^j(\ell_j)$ and $z' \models I_X^i(\ell_i') \wedge \bigwedge_{j \neq i} I_X^j(\ell_j)$,
- *Broadcast synchronization transitions:* $(\ell_1, \ldots, \ell_n, z, v) \longrightarrow (\ell_1', \ldots, \ell_n', z', v')$ if there is $a \in Broad$ and
  - there exists an i, $1 \leq i \leq n$, s.t. $\ell_i \xrightarrow{g_i,\phi_i,a¡,r_i,\mathcal{A}_i}_i \ell_i'$ is an edge in the i'th automaton where $v \models g_i$ and $z \models \phi_i$,
  - let $\mathcal{J}$ be the set of all j, $1 \leq j \neq i \leq n$, s.t. $\ell_j \xrightarrow{g_j,\phi_j,a¿,r_j,\mathcal{A}_j}_j \ell_j'$ in the j'th automaton where $v \models g_j$ and $z \models \phi_j$,
  - for all $j \in \mathcal{J}$ we set ℓ_j', $\mathcal{A}_j$ and r_j according to the edge $\ell_j \xrightarrow{g_j,\phi_j,a¿,r_j,\mathcal{A}_j}_j \ell_j'$ (note that there may be multiple edges to choose from),
  - for all $j \notin \mathcal{J}$, $1 \leq j \neq i \leq n$, we let $\ell_j' = \ell_j$, $\mathcal{A}_j = \emptyset$ and $r_j = \emptyset$,
  - $z' = (\ldots((\ldots(((z[\mathcal{A}_i])[\mathcal{A}_1])[\mathcal{A}_2])\ldots[\mathcal{A}_{i-1}])[\mathcal{A}_{i+1}])\ldots[\mathcal{A}_{n-1}])[\mathcal{A}_n]$ such that $z' \models \bigwedge_{k=1}^n I_X^k(\ell_k')$,
  - $v' = v[R]$ where $R = \bigcup_{k=1}^n r_k$ such that $v' \models \bigwedge_{k=1}^n I_C^k(\ell_k')$, and
- *Delay transitions:* $(\ell_1, \ldots, \ell_n, z, v) \xrightarrow{d} (\ell_1, \ldots, \ell_n, z, v + d)$ if $d \in \mathbb{R}_{\geq 0}$ s.t. $v + d \models \bigwedge_{i=1}^n I_i(\ell_i)$.

We let $\mathcal{AP} \overset{\text{def}}{=} \{(\#\ell \bowtie m) \mid \ell \in \cup_{i=1}^n L_i, m \in \mathbb{N}_0 \text{ and } \bowtie \in \{<, \leq, =, \geq, >\}\}$, and $\mu : Conf(A) \longrightarrow 2^{\mathcal{AP}}$ is defined such that a proposition $(\#\ell \bowtie m)$ is true in a given configuration if and only if the number of parallel components that are currently in the location ℓ satisfies the constraint with respect to m.

The initial configuration is $(\ell_0^1, \ell_0^2, \ldots, \ell_0^n, z_0, v_0)$ where $v_0(c) = 0$ for all $c \in C$. We require that z_0 satisfies the variable invariants of all initial locations.

Remark 2. Note that during a broadcast, the assignments on the edge of the sender are evaluated first, followed by the assignments of the receivers that are evaluated in the order from A_1 to A_n.

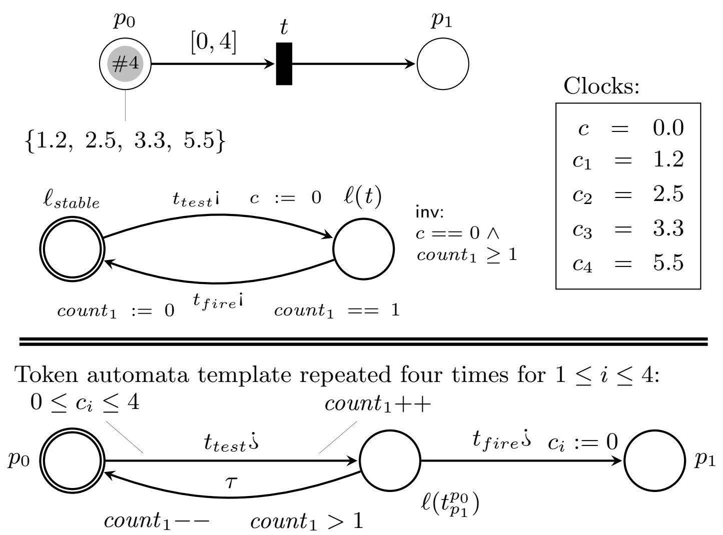


Fig. 2. A simple TAPN and the translated NTA.

4.3 The Translation

We will now present the translation from k-bounded TAPN (where the maximum number of tokens in every reachable marking is at most k) to NTA. For each token in the net, we create a parallel component in the network of timed automata. Since we cannot dynamically instantiate new timed automata, we need to have a constant number of tokens in the net at all times. As we assume that the net is k-bounded, it is enough to construct k automata to simulate each token. In each of these automaton there is a location corresponding to each place in the net. Whenever a TA is in one of these locations, it simulates a token in the corresponding place. Moreover, each automaton has a local clock which represents the age of the token. All automata simulating the tokens have the same structure, the only difference being their initial location, which corresponds to the tokens' initial position in the net. Because there may not always be exactly k tokens present during the execution of the net, we add a new location $\ell_{capacity}$ where the automata representing currently unused tokens are waiting.

In addition to these "token" automata we create a single control automaton. The purpose is to simulate the firing of transitions and to move tokens around via broadcasts initiated by the control automaton. This automaton has a location ℓ_{stable} which acts as a mutex in the sense that the control automaton moves out of this location once the simulation of a transition begins and returns back once the simulation of the transition ends. Moreover, each time the automaton is in ℓ_{stable}, the token automata in the composed NTA correspond to a marking in the TAPN. We will first show how the translation works on two examples.

Example 2. Figure 2 shows a simple TAPN with a single transition and four tokens of different ages. The translated NTA consists of five automata, one control automaton (topmost automaton) and four token automata, one for each token. Notice that in this example we have refrained from drawing the $\ell_{capacity}$ location as it is not used.

The translated NTA works as follows. First, the control automaton broadcasts on the channel t_{test}. Any token automaton with its clock in the interval $[0, 4]$ is forced to participate in the broadcast; in our case three token automata will participate. We use integer variables to count the number of token automata that took part in the broadcast. Because the preset of t has size one, we only need one counter variable $count_1$. Once the token automata synchronized in the broadcast, they move to the intermediate locations $\ell(t_{p_1}^{p_0})$ and during the update each increments $count_1$ by one; in our case the value of $count_1$ will become three. This means that the invariant on $\ell(t)$ in the control automaton is satisfied. In other words, we know that there are enough tokens with appropriate ages in the input places for t to fire. Notice that if there were not enough tokens in some of the input places, then the invariant on $\ell(t)$ was not satisfied and the broadcast could not take place at all. This is one of the crucial aspects to realize in order to see why this translation preserves liveness properties.

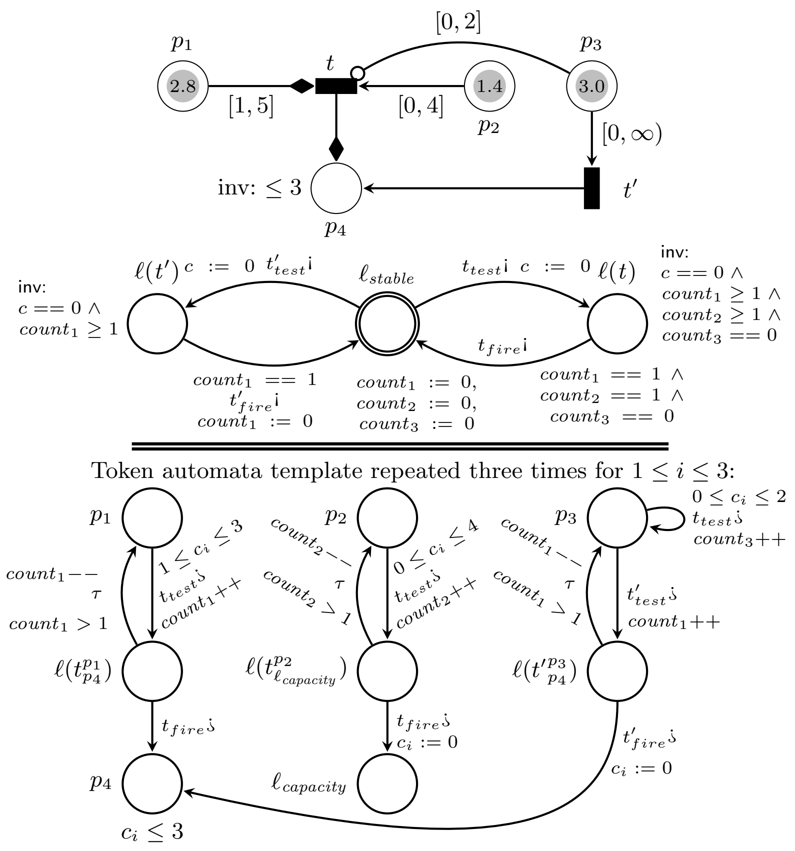
Now the value of $count_1$ is three and the control automaton may not broadcast on the t_{fire} channel yet since the guard ensures that this is only possible when exactly one token automaton remains in its intermediate place. Therefore, we are forced to move two of the token automata back to p_0 via the τ-transitions. This is possible only as long as $count_1$ is strictly greater than one. Hence exactly one token automaton has to remain in its intermediate place before the control automaton can broadcast on the t_{fire} channel and finalize the simulation of firing t. Note that due to the invariant $c == 0$ in the control automaton, no time delay is possible during the simulation of the transition. □

After demonstrating the basic idea of the broadcast translation, let us discuss a slightly more elaborate example using all of the features of the TAPN model.

Example 3. Consider the TAPN model in Figure 3 that uses transport arcs (the pair of arcs with filled tips from p_1 to p_4) for moving tokens while preserving their ages, an inhibitor arc (the arc with the circle tip) and an invariant in place p_4. The NTA created by our algorithm is below the net. As before, the template is repeated three times, once for each token, the only difference being the initial location (p_1, p_2 and p_3, respectively) and the name of the clock (c_1, c_2 and c_3, respectively).

We see that the control automaton has a test-fire loop for every transition in the TAPN model. There are some special constructions worth mentioning. First of all, consider the inhibitor arc from p_3 to t. This arc is encoded using a self-loop participating in the t_{test} broadcast transition. We use a counter variable to count the number of automata that take this edge. We simply encode the requirement that there is no token in the interval $[0, 2]$ by adding the invariant $count_3 == 0$ on the location $\ell(t)$.

A second observation is the guard on the edge from p_1 to $\ell(t_{p_4}^{p_1})$. It is evident that this does not match the interval $[1, 5]$ located on the arc from p_1 to t in the TAPN model. The guard $1 \leq c_i \leq 3$ is in fact the intersection of the interval



p_1
2.8
$[1,5]$
t
$[0,2]$
1.4
$[0,4]$
p_2
p_3
3.0
$[0,\infty)$
t'
inv: ≤ 3
p_4
$\ell(t')$ $c := 0$ $t'_{test}!$
ℓ_{stable}
$t_{test}!$ $c := 0$ $\ell(t)$
inv: $c == 0 \wedge count_1 \geq 1$
inv: $c == 0 \wedge count_1 \geq 1 \wedge count_2 \geq 1 \wedge count_3 == 0$
$t_{fire}!$
$count_1 == 1$ $t'_{fire}!$ $count_1 := 0$
$count_1 := 0,$ $count_2 := 0,$ $count_3 := 0$
$count_1 == 1 \wedge count_2 == 1 \wedge count_3 == 0$
Token automata template repeated three times for $1 \leq i \leq 3$:
p_1
$1 \leq c_i \leq 3$ $t_{test}?$ $count_1{+}{+}$
$count_1{-}{-}$ τ $count_1 > 1$
$\ell(t^{p_1}_{p_4})$
$t_{fire}?$
p_4
$c_i \leq 3$
p_2
$count_2{-}{-}$ τ $count_2 > 1$
$0 \leq c_i \leq 4$ $t_{test}?$ $count_2{+}{+}$
$\ell(t^{p_2}_{\ell_{capacity}})$
$t_{fire}?$ $c_i := 0$
$\ell_{capacity}$
p_3
$0 \leq c_i \leq 2$ $t_{test}?$ $count_3{+}{+}$
$count_1{-}{-}$ τ $count_1 > 1$
$t'_{test}?$ $count_1{+}{+}$
$\ell(t'^{p_3}_{p_4})$
$t'_{fire}?$ $c_i := 0$

Fig. 3. A TAPN and the translated NTA.

$[1,5]$ and the invariant ≤ 3 on the place p_4. This is because the age of the token consumed in p_1 will be preserved once moved to p_4 and by intersecting the intervals we avoid possible deadlocks. One may think that it is enough to add the invariant ≤ 3 on the intermediate place, however, this may result in incorrect behavior. If there were two tokens in p_1 with ages 4 and 2, the broadcast on t_{test} would be blocked. This is because invariants block the entire broadcast transition even if only a single automaton with a satisfied guard cannot participate due to the violation of the invariant in its target location.

For our specific example, we need at least one token of age $[1,3]$ in p_1, at least one token of age $[0,4]$ in p_2 and zero tokens of age $[0,2]$ in p_3 in order for t to be enabled, which is precisely encoded in the invariant on $\ell(t)$. The reader may also notice that different transitions share counter variables. The variable $count_1$ is used in the simulation of both t and t' but they are used in a non-conflicting way, in the sense that we are never simulating t and t' at the same time. We also see that during the simulation of t' we do not take the invariant of the target location into account since the arc from t' to p_4 is a normal arc and produces a token of age zero which always satisfies any invariant. $\square$

Algorithm 1: Translation from k-bounded TAPN to NTA.

```
Input: A $k$-bounded TAPN $N = (P, T, F, c, F_{tarc}, c_{tarc}, F_{inhib}, c_{inhib}, \iota)$ with a marking $M_0$
Output: NTA $P_{TA} = A \| A_1 \| A_2 \| \ldots \| A_k$ s.t. $A = (L, \mathcal{A}ct, C, X, \longrightarrow, I_C, I_X, \ell_0)$ and
        $A_i = (L_i, \mathcal{A}ct, C, X, \longrightarrow_i, I^i_C, I^i_X, \ell^i_0)$
begin
    for $i := 1$ to $k$ do $L_i := P \cup \{\ell_{capacity}\}$
    $L := \{\ell_{stable}\}$; $\mathcal{A}ct := \{t_{test}\text{¡}, t_{test}\text{¿}, t_{fire}\text{¡}, t_{fire}\text{¿} \mid t \in T\} \cup \{\tau\}$
    $C := \{c, c_1, c_2, \ldots, c_k\}$; $X := \{count_i \mid 1 \leq i \leq NumVars(N)\}$
    forall $t \in T$ do
        $j := 0$; $varInv_t := true$; $varGuard_t := true$
        while $|Pairing(t)| > 0$ do
            $j := j + 1$; Remove some $(p, I, p', type)$ from $Pairing(t)$
            for $i := 1$ to $k$ do
                $L_i := L_i \cup \{\ell(t^p_{p'})\}$
                Add $p \xrightarrow{g, true,\, t_{test}\text{¿},\, \emptyset,\, count_j{+}{+}}_i \ell(t^p_{p'})$ s.t. $g := c \in I$ if $type = normal$ else
                $g := c \in I \cap \iota(p')$
                Add $\ell(t^p_{p'}) \xrightarrow{true,\, true,\, t_{fire}\text{¿},\, R,\, \emptyset}_i p'$ s.t. $R = \{c_i\}$ if $type = normal$ else
                $R = \emptyset$
                Add $\ell(t^p_{p'}) \xrightarrow{true,\, count_j > 1,\, \tau,\, \emptyset,\, count_j{-}{-}}_i p$
            $varInv_t := varInv_t \wedge count_j \geq 1$; $varGuard_t := varGuard_t \wedge count_j == 1$
        forall $p \in P$ where $(p, t) \in F_{inhib}$ do
            $j := j + 1$;  for $i := 1$ to $k$ do  Add $p \xrightarrow{c_i \in c_{inhib}(p,t),\, true,\, t_{test}\text{¿},\, \emptyset,\, count_j{+}{+}}_i p$
            $varInv_t := varInv_t \wedge count_j == 0$; $varGuard_t := varGuard_t \wedge count_j == 0$
        $L := L \cup \{\ell(t)\}$; Add $\ell_{stable} \xrightarrow{true,\, true,\, t_{test}\text{¡},\, \{c\},\, \emptyset} \ell(t)$ and
        $\ell(t) \xrightarrow{true,\, varGuard_t,\, t_{fire}\text{¡},\, \emptyset,\, \{count_i := 0 \mid 1 \leq i \leq j\}} \ell_{stable}$
    for $i := 1$ to $k$ do
        $I^i_C(p) := \begin{cases} c_i \leq a & \text{if } p \in P \text{ and } \iota(p) = [0, a] \\ c_i < b & \text{if } p \in P \text{ and } \iota(p) = [0, b) \\ true & \text{if } p \in L_i \setminus P \end{cases}$      $I^i_X(p) := true$ for $p \in L_i$
    $I_C(p) := \begin{cases} true & \text{if } p = \ell_{stable} \\ c \leq 0 & \text{if } p \in L \setminus \{\ell_{stable}\} \end{cases}$      $I_X(p) := \begin{cases} varInv_t & \text{if } p = \ell(t) \text{ for } t \in T \\ true & \text{if } L \setminus \{\ell(t) \mid t \in T\} \end{cases}$
    $i := 0$;  forall $p \in P$ do  forall $Token \in M_0(p)$ do $\ell^i_0 := p; i := i + 1$
    for $i := |M_0| + 1$ to $k$ do $\ell^i_0 := \ell_{capacity}$
    $\ell_0 := \ell_{stable}$
end
```

We shall now proceed to present the translation algorithm. For every transition t we assume an a priori fixed set $Pairing(t)$, motivated by [9], where

$$
\begin{aligned}
Pairing(t) = {} & \{(p, I, p', tarc) \mid (p, t, p') \in F_{tarc} \wedge I = c_{tarc}(p, t, p')\} \cup \\
& \{(p_1, I_1, p'_1, normal), \ldots, (p_m, I_m, p'_m, normal) \mid \\
& \quad \{p_1, \ldots, p_\ell\} = \{p \mid (p, t) \in F\}, \{p'_1, \ldots, p'_{\ell'}\} = \{p \mid (t, p) \in F\}, \\
& \quad m = \max(\ell, \ell'), I_i = c(p_i, t) \text{ if } 1 \leq i \leq \ell \text{ else } I_i = [0, \infty), \\
& \quad p_i = \ell_{capacity} \text{ if } \ell < i \leq m, p'_i = \ell_{capacity} \text{ if } \ell' < i \leq m\}.
\end{aligned}
$$

The set $Pairing(t)$ simply pairs input and output places of t in order to fix the paths on which tokens will travel when firing t. It also records the time interval on the input arc and the type of the arc (*normal* for normal arcs and *tarc* for transport arcs). As an example, a possible pairing for the transition t in Figure 3 is $\mathit{Pairing}(t) = \{(p_1, [1,5], p_4, \mathit{tarc}), (p_2, [0,4], \ell_{capacity}, \mathit{normal})\}$. We also let $\mathit{NumVars}(N) \overset{\text{def}}{=} \max_{t \in T}(|\mathit{Pairing}(t)| + |\{(p,t) \mid (p,t) \in F_{inhib}\}|)$ denote the maximum number of integer variables needed in the translation. The translation is given in Algorithm 1. Note that it works in polynomial time.

Let (N, M_0) be a marked k-bounded TAPN and let P_{TA} be the NTA constructed by Algorithm 1 with initial configuration s_0. We can now apply our general framework to argue that they are in complete one-by-many correspondence. First, we define the *stable* proposition as $(\#\ell_{stable} = 1)$, which is true iff the control automaton is in its location ℓ_{stable}. From the construction it is clear that $T(P_{TA})$ is a *delay-implies-stable*, *delay-preserves-stable* and *eventually-stable* TTS. Next, a TAPN proposition $(p \bowtie n)$ is translated into $(\#p \bowtie n)$.

Let $M = \{(p_1, r_1), (p_2, r_2), \ldots, (p_n, r_n)\}$ be a marking of N such that $n \leq k$, where (p_i, r_i) is a token located in the place p_i with age $r_i \in \mathbb{R}_{\geq 0}$. Further, let $s = (\ell, \ell_1, \ell_2, \ldots, \ell_k, z, v)$ be a configuration of P_{TA}. We define a relation $\mathcal{R}$ such that $(M, s) \in \mathcal{R}$ iff there exists an injection $h : \{1, 2, \ldots, n\} \longrightarrow \{1, 2, \ldots, k\}$ such that $\ell = \ell_{stable}$, $\ell_{h(i)} = p_i$ and $v(c_{h(i)}) = r_i$ for all i where $1 \leq i \leq n$, $\ell_j = \ell_{capacity}$ for all $j \in \{1, 2, \ldots, k\} \setminus \mathit{range}(h)$ and $\mathit{count}_i = 0$ for all $1 \leq i \leq \mathit{NumVars}(N)$. Intuitively, if $(M, s) \in \mathcal{R}$ then for every token in M there is a TA where its location and clock valuation matches the token data and vice versa. The relation $\mathcal{R}$ is a complete one-by-many correspondence (see [13] for details). By applying Theorem 1 we can now conclude the following.

Theorem 2. *Let N be a k-bounded TAPN and let P_{TA} be the NTA constructed by Algorithm 1. Then $N \models \varphi$ iff $P_{TA} \models tr(\varphi)$ for any TCTL formula φ.*

5 Conclusion

We have introduced a general framework for arguing when a translation between two timed transition systems preserves TCTL model checking. The framework generalizes earlier translations like [9] and [10] that dealt with concrete models. Apart from [9, 10], the framework is applicable also to other translations like [8, 11, 14, 18]. We have further described a novel reduction from bounded timed-arc Petri nets with transport/inhibitor arcs and invariants on places to networks of timed automata in the UPPAAL style to which the framework is applicable. Compared to earlier translations, we considered a more general class of nets and showed that also liveness TCTL properties are preserved. The translation works in polynomial time and was implemented in the verification tool TAPAAL [9].

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