Propositional and First-order Logic

1   Problem 1

Russell and Norvig, Exercise 7.4.

Prove each of the following assertions:

a. α is valid if and only if $True \Rightarrow \alpha$.

b. For any α, $False \Rightarrow \alpha$.

c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.

2   Problem 2

Russell and Norvig, Exercise 7.9.
(Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

3   Problem 3

Write the following sentences in FOL

a. For every action, there is an equal and opposite reaction.

b. Everyone loves someone who loves everyone.

c. We are living in a green earth.