Propositional and First-order Logic

1 Problem 1

Russell and Norvig, Exercise 7.4.

Prove each of the following assertions:

a. α is valid if and only if $True \Rightarrow \alpha$.

b. For any α, $False \Rightarrow \alpha$.

c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.
ANSWER.
Remember, $\alpha \models \beta$ if and only if in very model in which α is true, β is also true. Therefore,
a. A valid sentence is one that is true in all models. The sentence $True$ is also valid in all models. So if α is valid then the entailment holds(because both $True$ and α hold in every model), and if the entailment holds then α must be valid, because it must be true in all models, because it must be true in all models in which $True$ holds.
b. $False$ does not hold in any model, so α trivial holds in every model that $False$ holds in.
c. $\alpha \models \beta$ holds in those models where β holds or where $\neg\alpha$ holds. That is precisely the case if $\alpha \Rightarrow \beta$ is valid.
d. This follows from applying **c** in both directions.
e. This reduces to **c**, because $\alpha \wedge \neg\beta$ is unsatisfiable just when $\alpha \Rightarrow \beta$ is valid.

2 Problem 2

Russell and Norvig, Exercise 7.9.
(Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.
ANSWER.
See *ComputerV.S.Agent* example.

3   Problem 3

Write the following sentences in FOL

a. For every action, there is an equal and opposite reaction.

ANSWER. $\forall x \ Action(x) \Rightarrow \exists y \ Reaction(y, x) \wedge Equal(magnitude(x), magnitude(y)) \wedge Opposite(direction(x), direction(y))$

b. Everyone loves someone who loves everyone.

ANSWER. $\forall x, \exists y \ Loves(x, y) \wedge \forall z \ Loves(y, z)$

c. We are living in a green earth.
ANSWER. $\exists x \ Earth(x) \wedge Green(x) \wedge \forall y \ Among - Us(y) \Rightarrow Live - In(y, x)$