Inference in First-order Logic

1 Problem 1

Russell and Norvig, Exercise 9.18.
From "Horses are animals," it follows that "The head of a horse is the head of an animal." Demonstrate that this inference is valid by carrying out the following steps:

a. Translate the premise and the conclusion into the language of first-order logic. Use three predicates: $HeadOf(h, x)$(meaning "h is the head of x"), $Horse(x)$, and $Animal(x)$.

ANSWER:
Knowledge base:
$C1 : \forall x \;\; Horse(x) \Rightarrow Animal(x)$
Conclusion:
$G : \forall x, h \;\; Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y \;\; Animal(y) \wedge HeadOf(h, y)$

b. Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.

c. Use resolution to show that the conclusion follows from the premise.

ANSWER:
We get $C2 : \neg Horse(x) \vee Animal(x)$ by converting $C1$ into normal form.
$\neg G$: $\exists x, h \;\; \neg[Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y \;\; Animal(y) \wedge HeadOf(h, y)]$
Implication elimination: $\exists x, h \;\; \neg\{\neg[Horse(x) \wedge HeadOf(h, x)] \vee [\exists y \;\; Animal(y) \wedge HeadOf(h, y)]\}$
Move $\neg$ inwards: $\exists x, h \;\; \{[Horse(x) \wedge HeadOf(h, x)] \wedge [\forall y \;\; \neg Animal(y) \vee \neg HeadOf(h, y)]\}$
Skolemization: $\{[Horse(G) \wedge HeadOf(H, G)] \wedge [\forall y \;\; \neg Animal(y) \vee \neg HeadOf(H, y)]\}$
Hence, we get the normal forms:
$C3 : Horse(G)$
$C4 : HeadOf(H, G)$
$C5 : \neg Animal(y) \vee \neg HeadOf(H, y)$

Then, we resolve $C4$ and $C5$ to yield $\neg Animal(y)$. Resolve this with $C2$ to give $\neg Horse(G)$. Resolve this with $C3$ to obtain a contradiction.

2 Problem 2

Use first-order refutation resolution to prove the following theorem:

Knowledge Base: For every married couple, there is some habit of the husband's that the wife does not like. Thomas is Kristina's husband.

Theorem: Kristina does not like all of Thomas's habits

ANSWER.

For every married couple, there is some habit of the husband's that the wife does not like.

$\forall husband, wife\ \ Husband{-}Of(husband, wife) \Rightarrow \exists habit\ \ Has(husband, habit) \wedge \neg Likes(wife, habit)$

1. Conversion to normal form

Implication out.$\forall husband, wife\ \ \neg Husband{-}Of(husband, wife) \vee \exists habit\ \ Has(husband, habit) \wedge \neg Likes(wife, habit)$

Skolemize. $\forall husband, wife\ \ \neg Husband{-}Of(husband, wife) \vee Has(husband, habit(husband, wife)) \wedge \neg Likes(wife, habit(husband, wife))$

Distribute law. $(\forall husband, wife\ \ \neg Husband{-}Of(husband, wife) \vee Has(husband, habit(husband, wife))) \wedge (\neg Husband - Of(husband, wife) \vee \neg Likes(wife, habit(husband, wife)))$

Rename variable.$(\forall husband, wife\ \ \neg Husband{-}Of(husband_1, wife_1) \vee Has(husband_1, habit(husband_1, wife_1)$
$\wedge (\neg Husband{-}Of(husband_2, wife_2) \vee \neg Likes(wife_2, habit(husband_2, wife_2)))$

2. Then we have two clauses:

C1:$\neg Husband{-}Of(husband_1, wife_1) \vee Has(husband_1, habit(husband_1, wife_1))$

C2:$\neg Husband{-}Of(husband_2, wife_2) \vee \neg Likes(wife_2, habit(husband_2, wife_2))$

Thomas is Kristina's husband.

C3:$husband - of(thomas, kristina)$

The goal: Kristina does not like all of Thomas's habits

$Has(thomas, badhabit) \wedge \neg Likes(kristina, badhabit)$

The negation gives

C4: $\neg Has(thomas, badhabit) \vee Likes(kristina, badhabit)$

Then, we use resolution

C5: $Has(thomas, habit(thomas, kristina))$ by unifying **C1** with **C3**

C6: $Likes(kristina, habit(thomas, kristina))$ by unifying **C4** with **C5**

C7: $\neg Husband - Of(thomas, kristina)$ by unifying **C2** with **C6**

C8: $false$ by unifying **C3** with **C7**